- These slides/notes represent only part of the course, and were accompanied by face-to-face explanations on white-board and additional topics / learning materials.
- In preparation of these slides I have also benefited from various books and online material.
- Some of the slides contain animations which may not be visible in pdf version.
- Corrections, comments, feedback may be sent to https://www.linkedin.com/in/naveedrazzaqbutt/

EE 328
Wave Propagation and Antennas

with

Dr. Naveed R. Butt

@

Jouf University

Introductions …

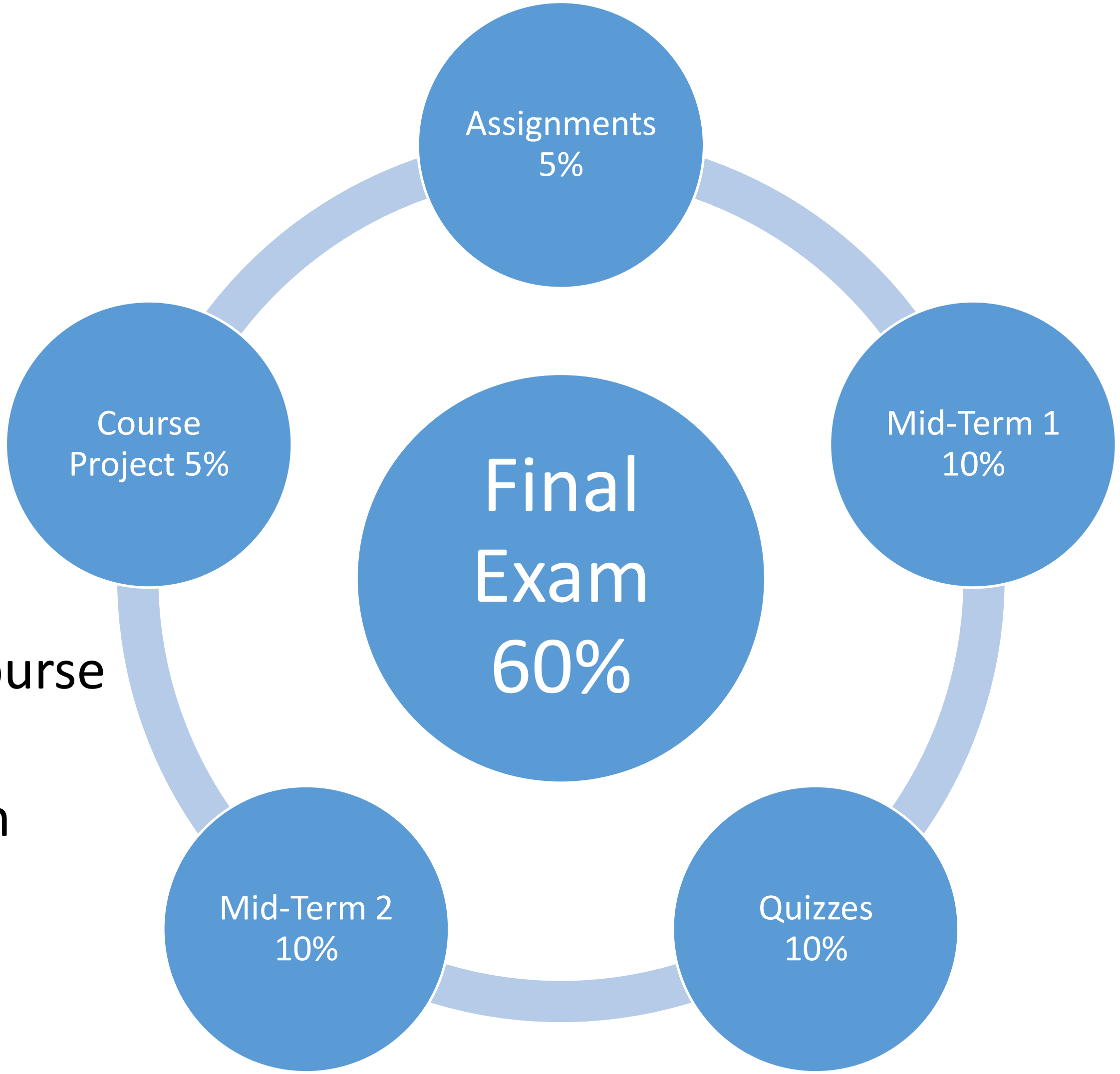
- Me
- You
- The Course

Important Business!!

- 75% attendance is mandatory!
- Textbooks
  - Constantine A. Balanis, “Antenna Theory : Analysis and Design”, Wiley, 4th Edition, 2016.
  - Christopher Haslett, “Essentials of Radio Wave Propagation”, Cambridge University Press, 2008.
- Contact
  - nbutt@ju.edu.sa
  - office: 1140

Learning Plan

- **Lectures**
  - Help discover and grasp new concepts
- **Quizzes**
  - Help prepare/revise each week's concepts
  - Keep you from lagging behind in course
- **Assignments**
  - Learn to solve involved problems based on course
- **Course Project**
  - Helps learn independent work & presentation
  - Prepares for final year project
- **Exams**
  - Help prepare entire course material



What are some of our common daily activities?

What are some of our common daily activities?

What are some of our common daily activities?

- Eating
- Drinking
- Sleeping
- ….
- …..
- Communicating!!

“Communicating” is an important human activity!!

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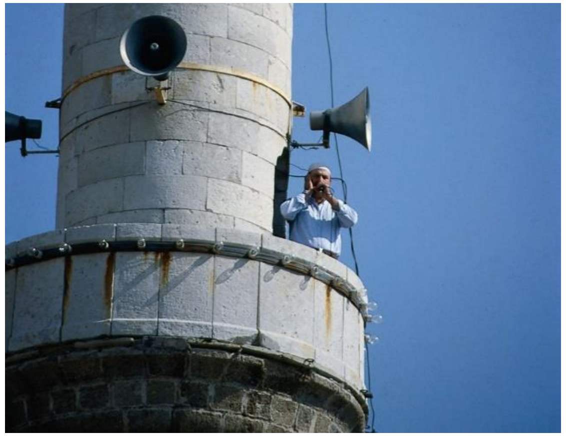
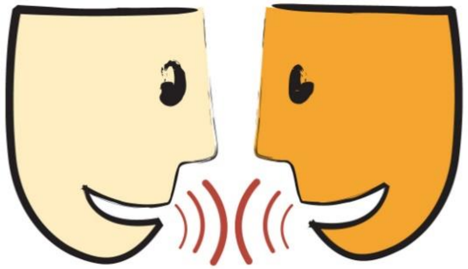
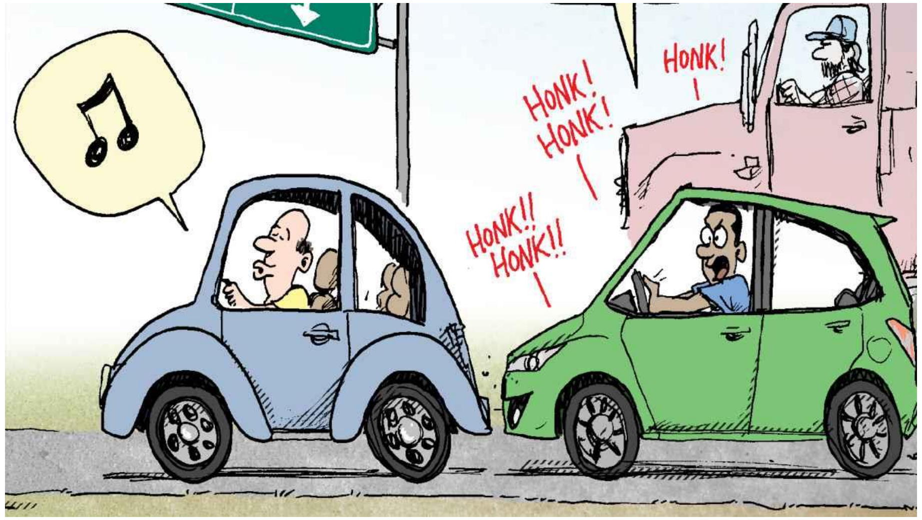




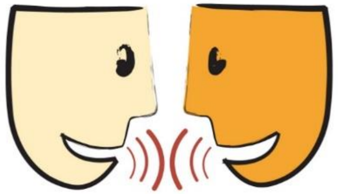


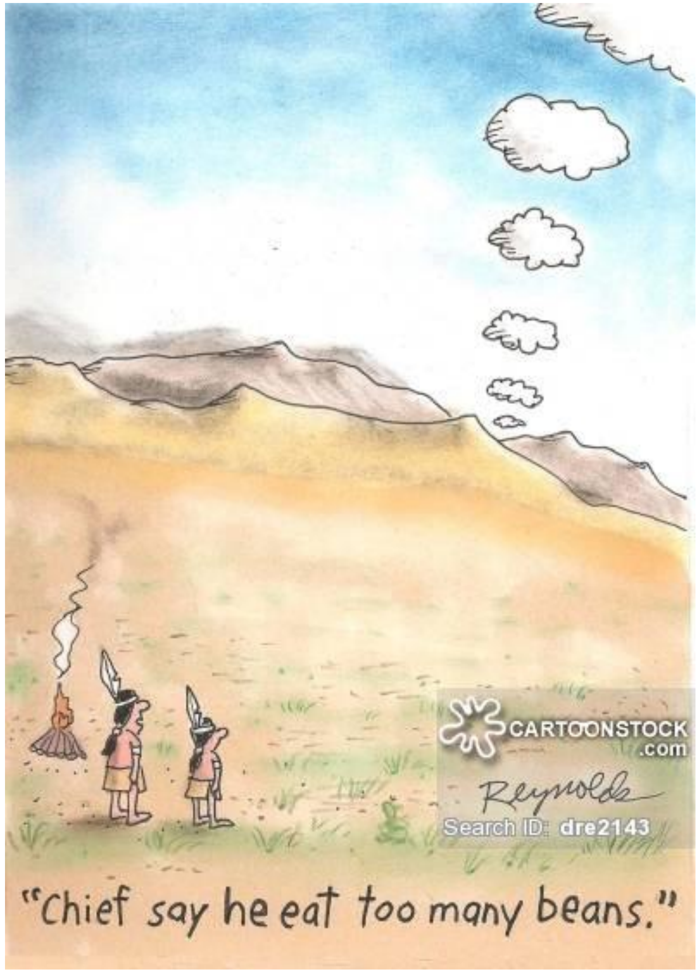
“Communicating” is an important human activity!!

- Instances of your communications today
  - Prayer call
  - Greeting family
  - Car honking
  - Car indicators
  - Waving to a friend
  - …
  - ….
  - Listening to this lecture…

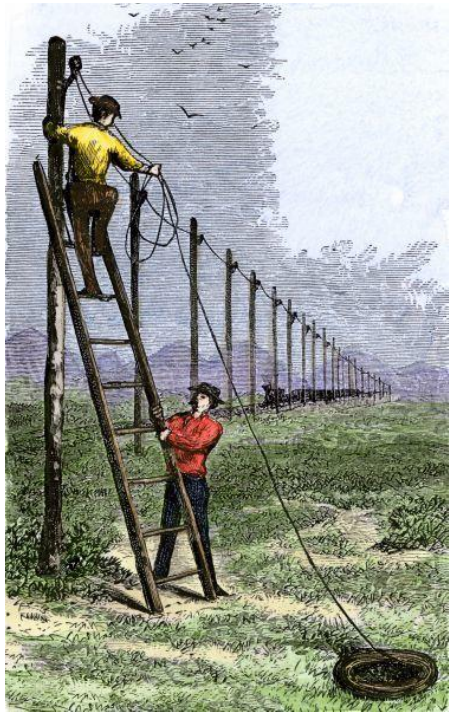
How have humans been communicating?

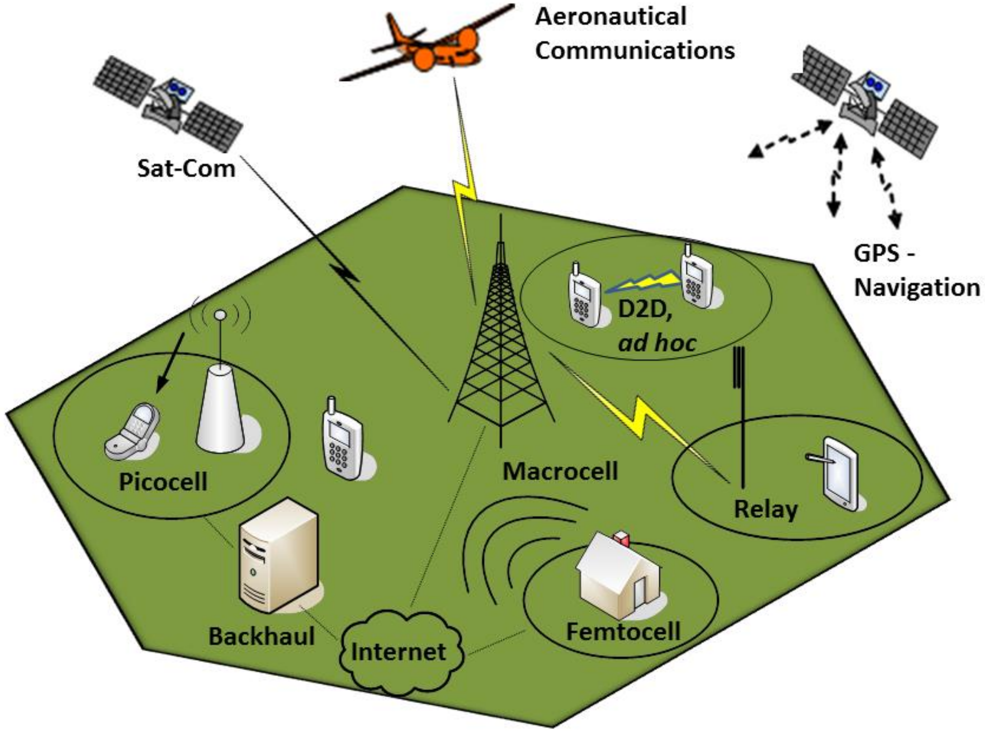
How have humans been communicating?





How have humans been communicating?





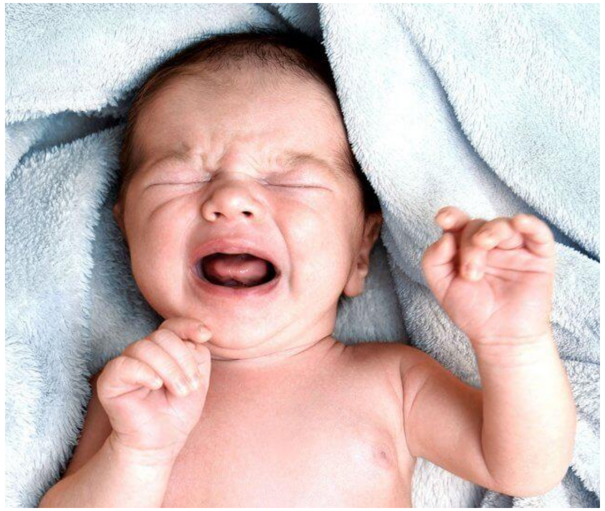
How have humans been communicating?

- Gestures (hand signs etc.)
- Verbal (talking)
- Drawing (cave walls, earth)
- Writing (clay, leaves, paper)
- Smoke/Gong Signals
- Mail (on foot, horses, ships, trains, planes, smaller vehicles)
- Electric Current (telegraph, landline telephone)
- Electromagnetic Waves (wireless communications)

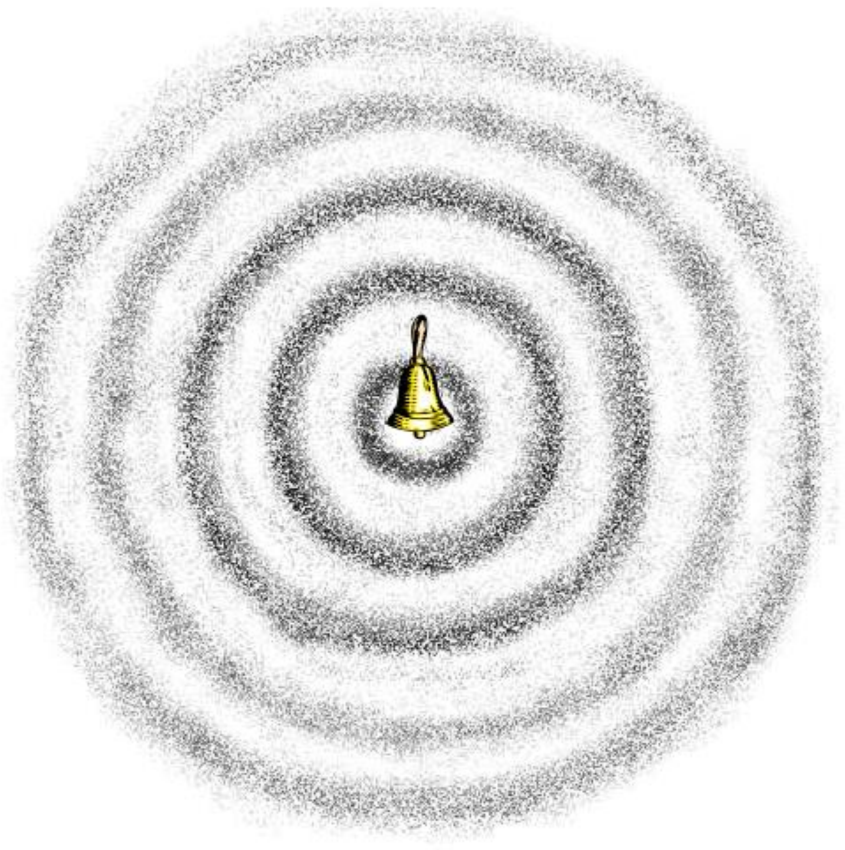
This Course

Let us talk about sound …

- From our very birth we start using sound waves for communication!

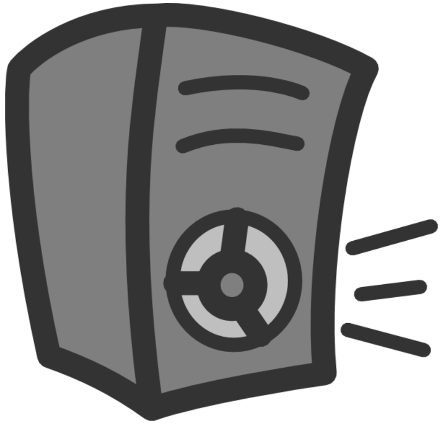


Sound waves travel through air around us, carrying some "information"

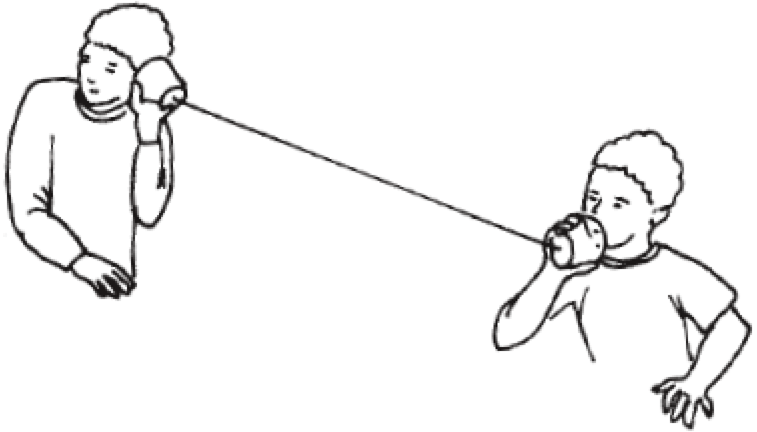


Dinner time!!

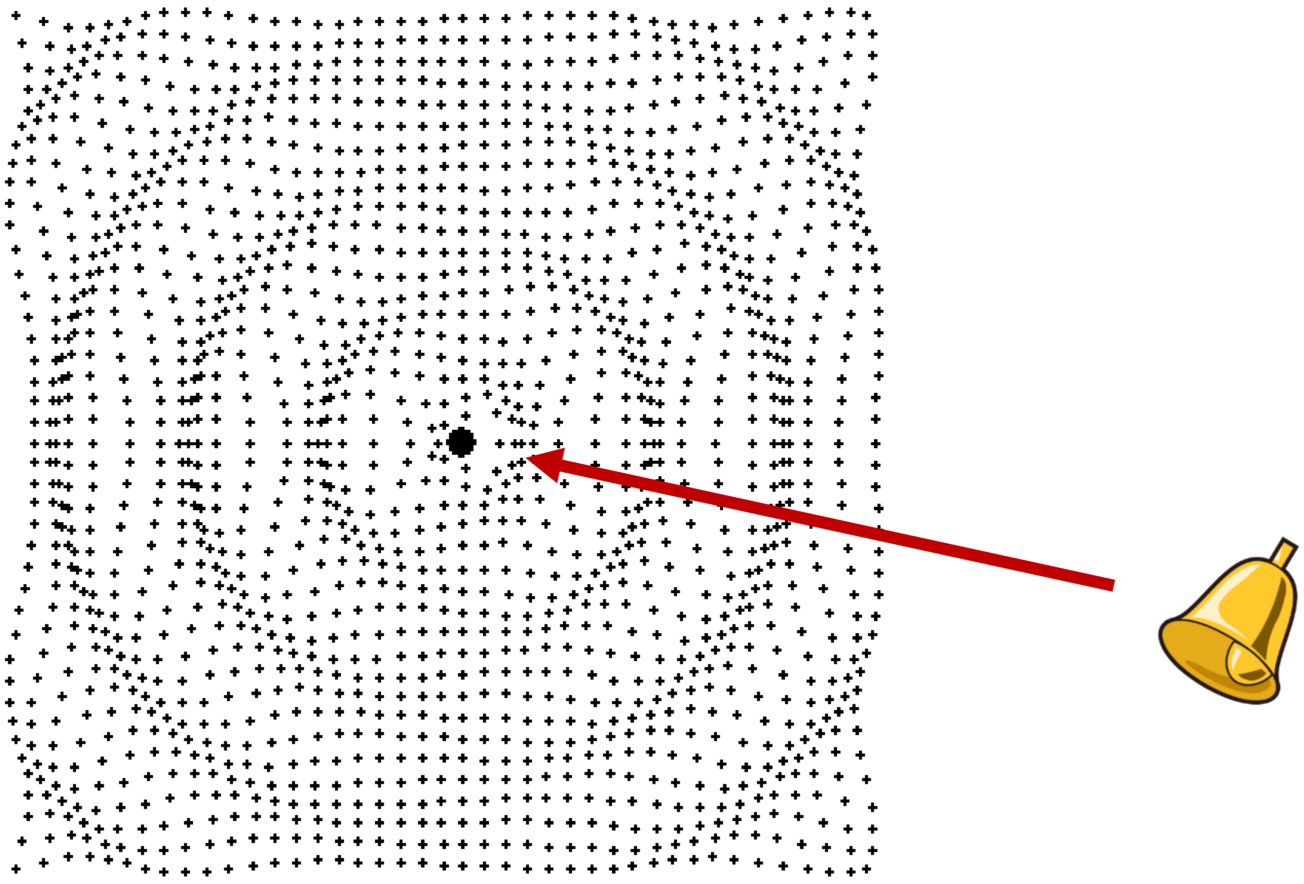
We use many kinds of sound-wave makers to produce different sounds



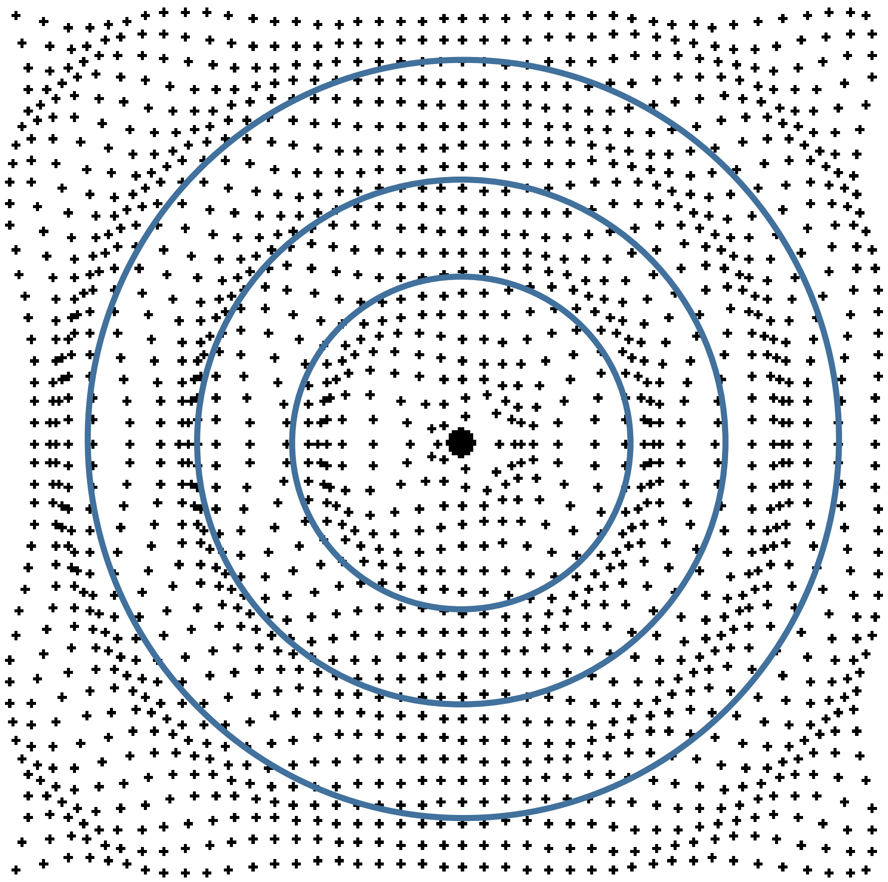
We also often "direct" sound waves to travel in certain directions



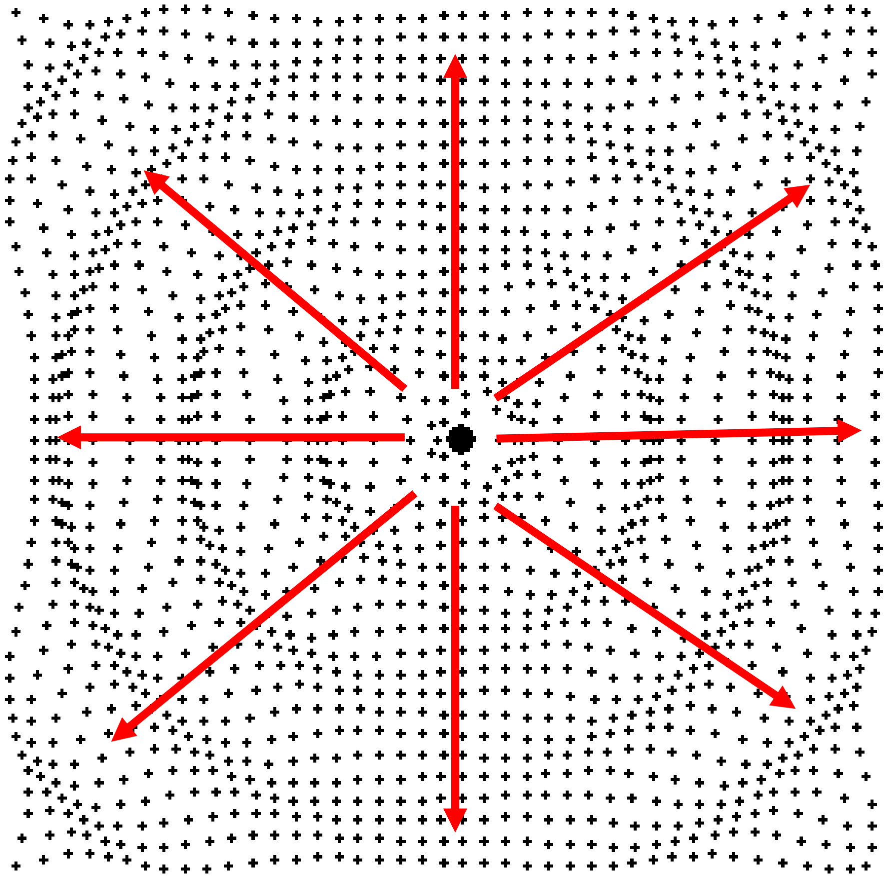
How do we visualize sound waves?



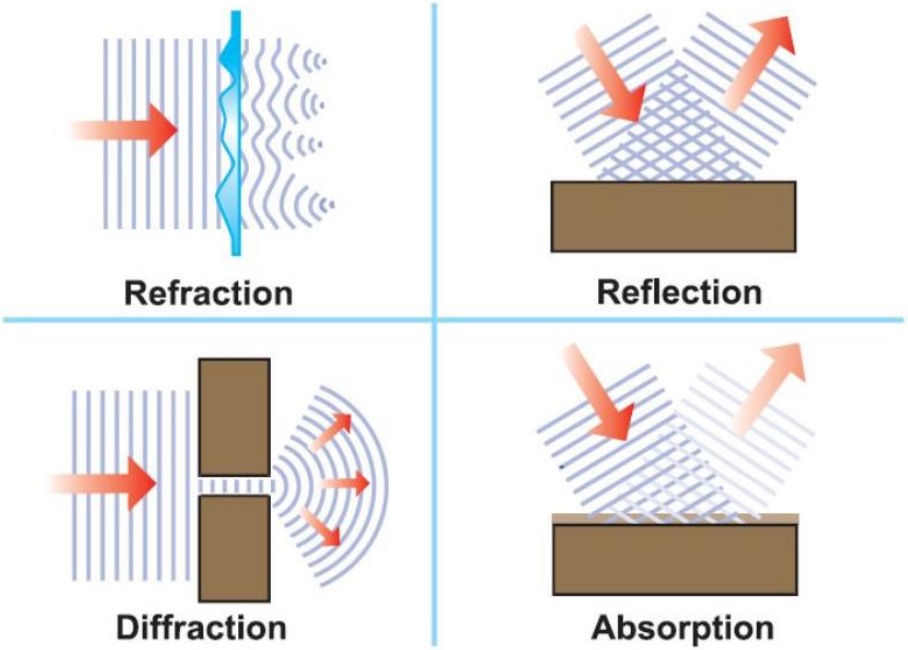
We can view the waves as "wavefronts"

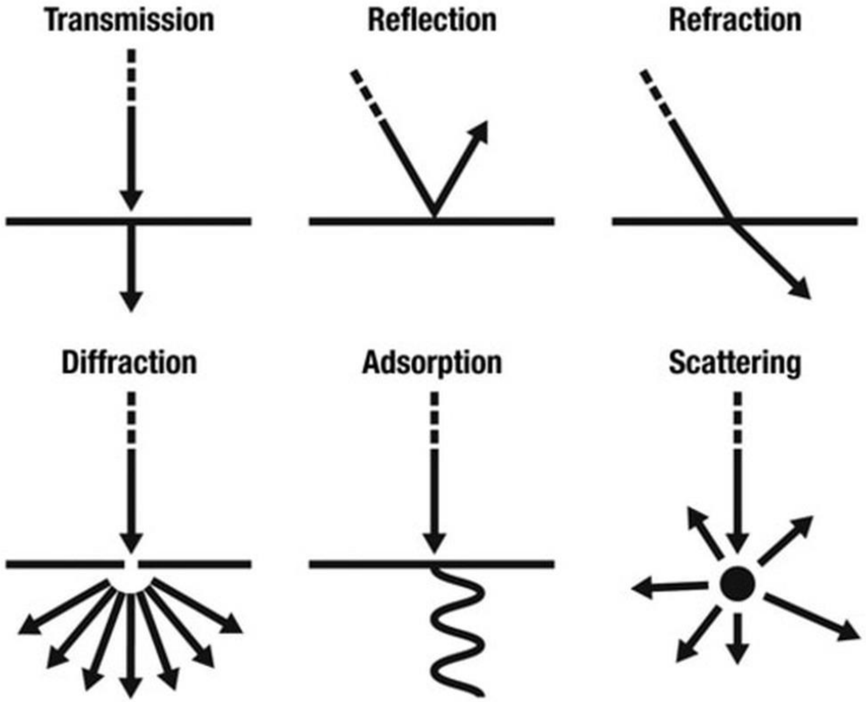


… or as rays (arrows) showing direction of travel!

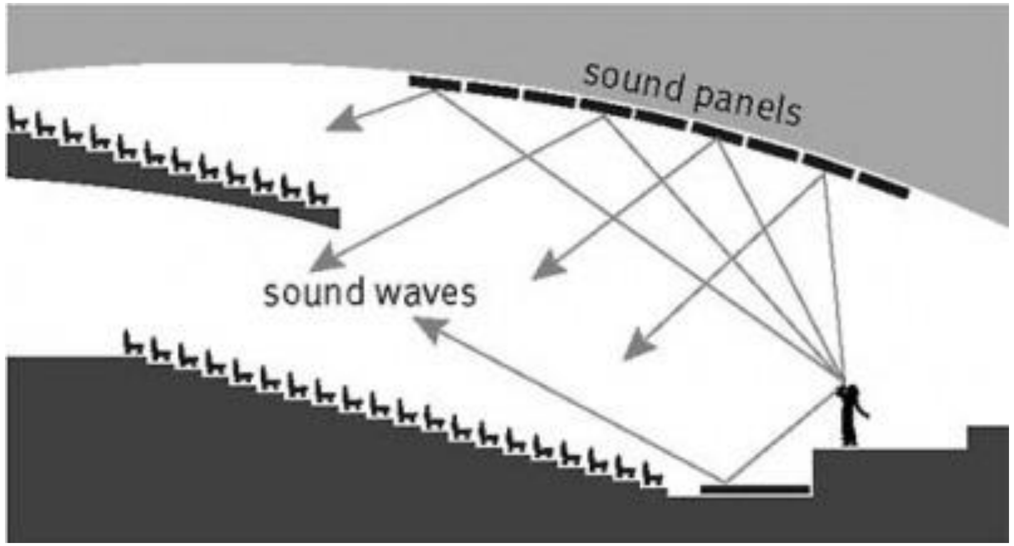


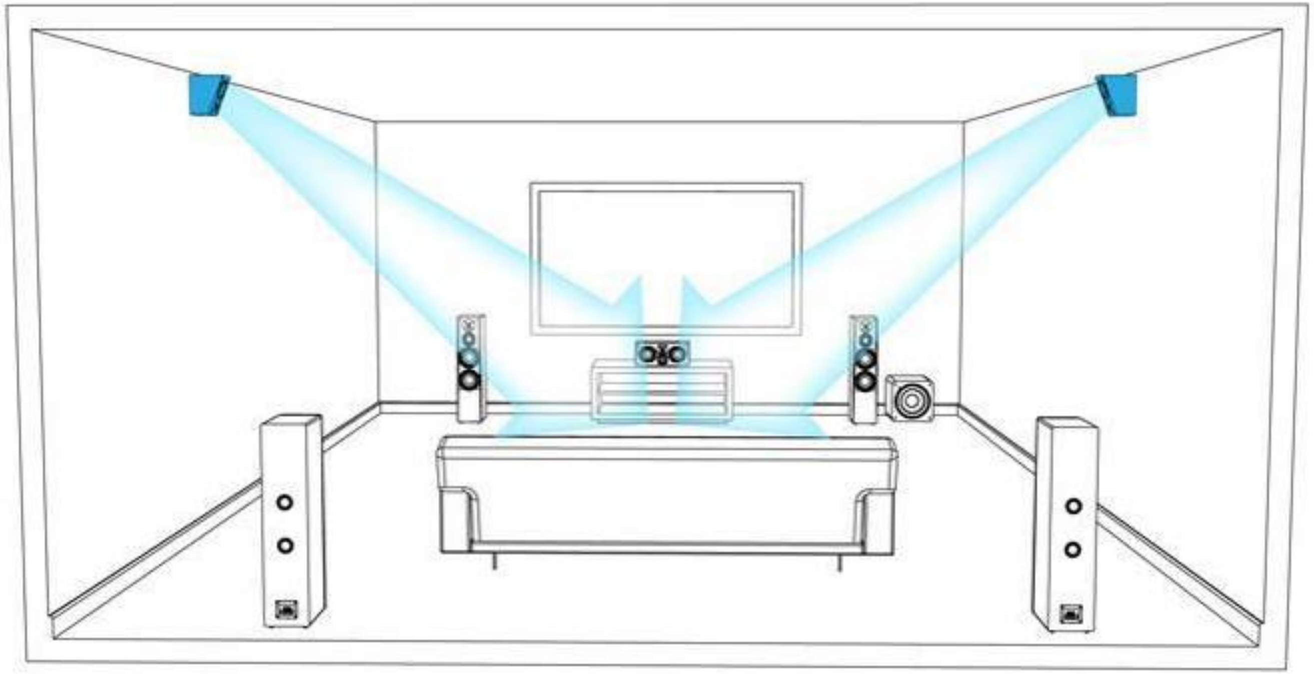
Using these, we can examine how sound travels and interacts





We can use understanding of the waves to design better sound-based [communication] "systems"





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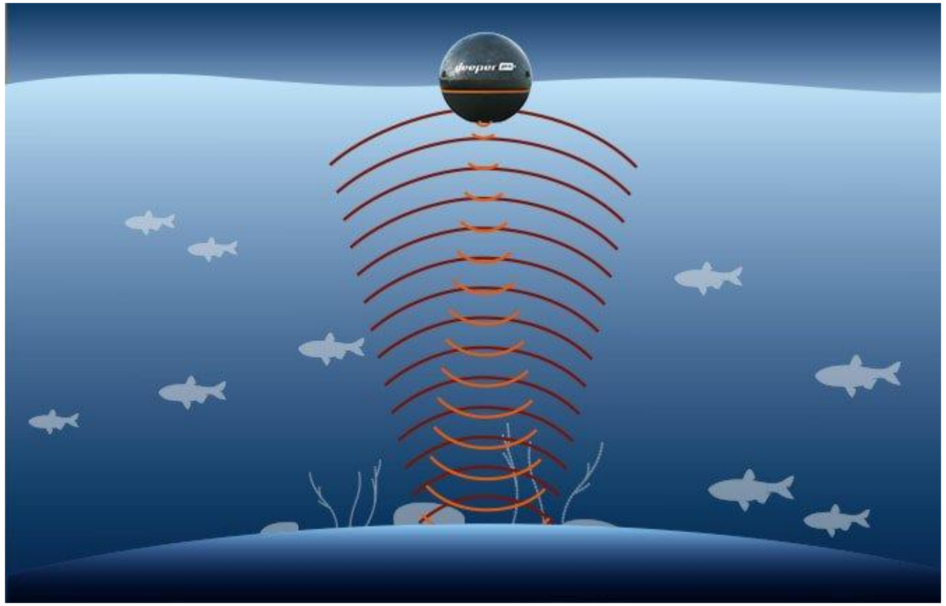
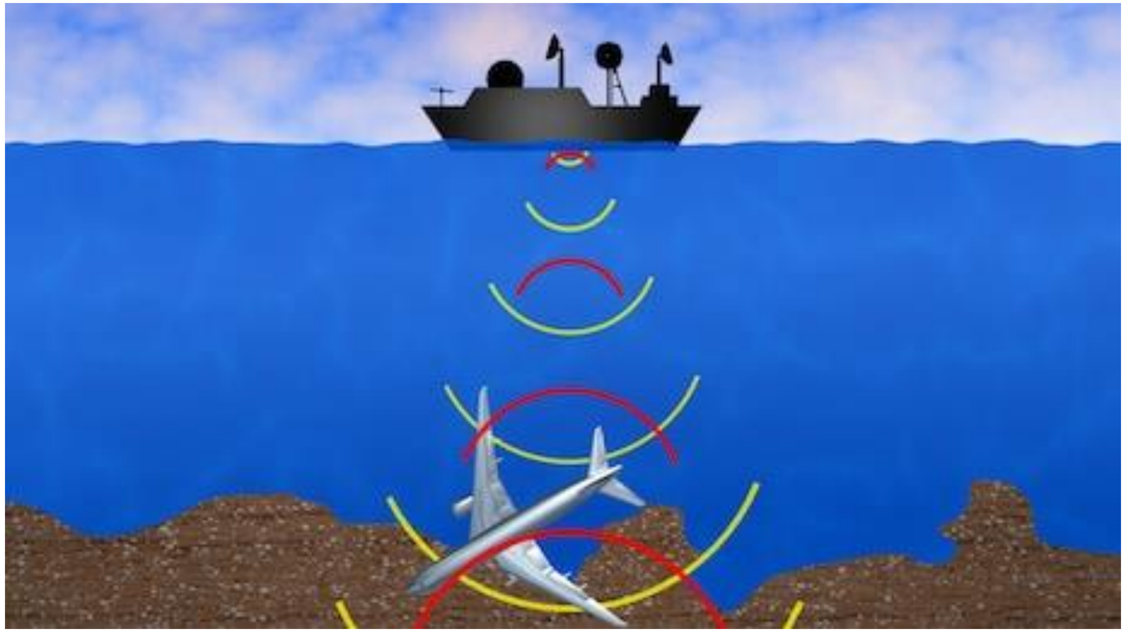
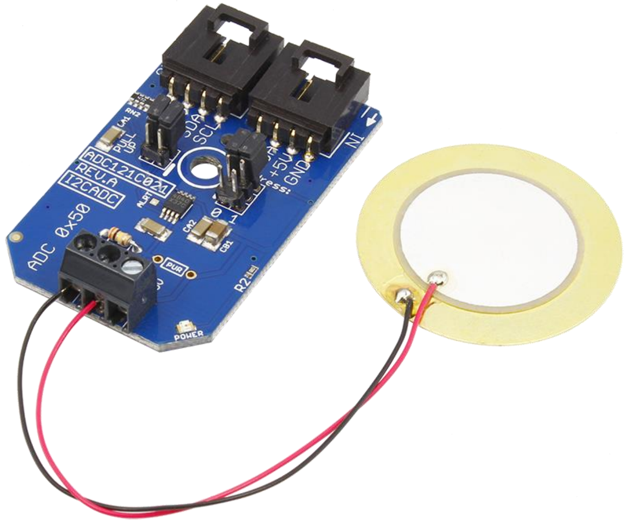


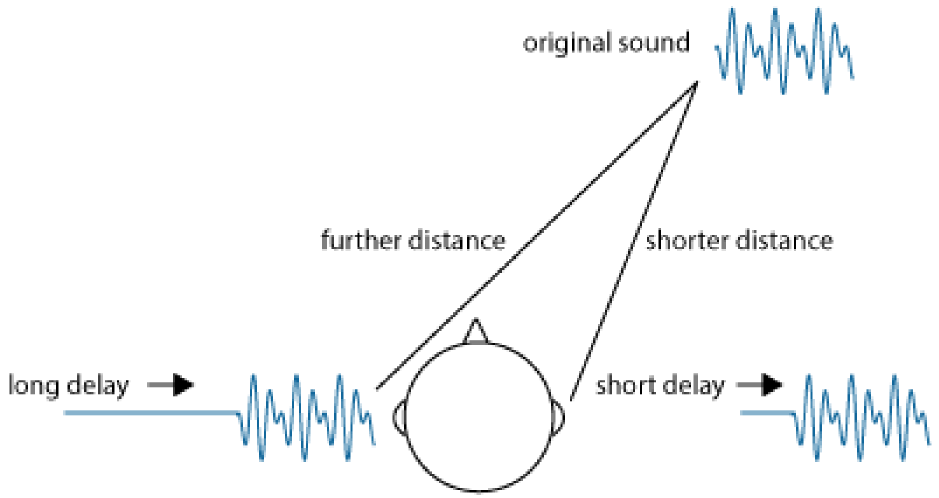


We use many kinds of "receivers" to collect sound waves



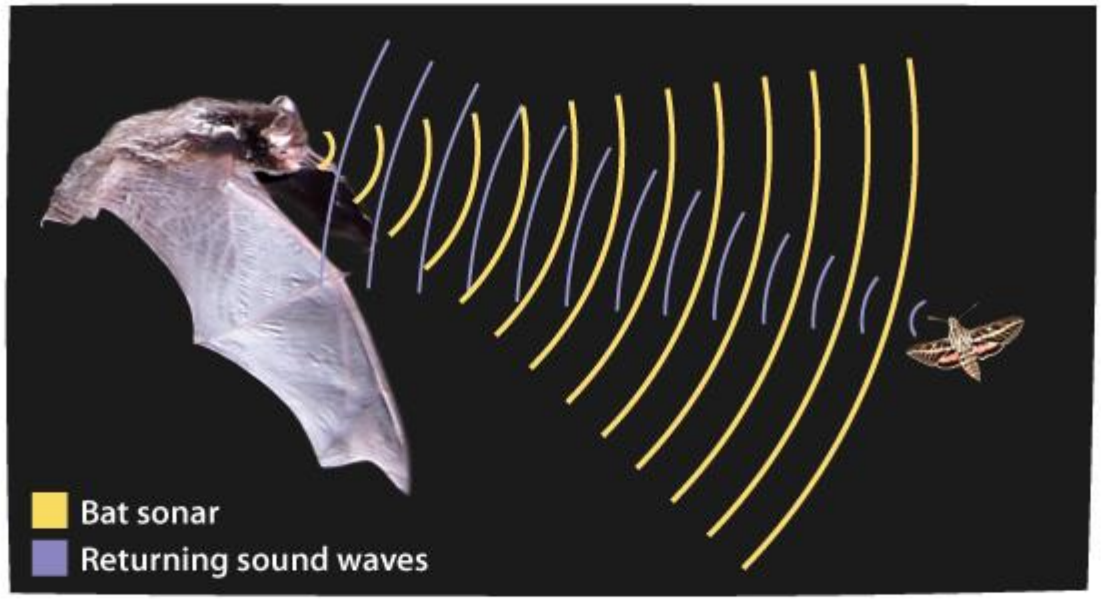
Some times we use more than one receiver …

- E.g., our two ears
  - Using multiple receivers can help us extract more information!
    - Our brain can tell if the source of sound is to the right or left by judging if the sound reached the right ear first or the left.



Interesting fact

- Bats use sound waves as “radar” system.



Very useful when you are hunting in the dark!

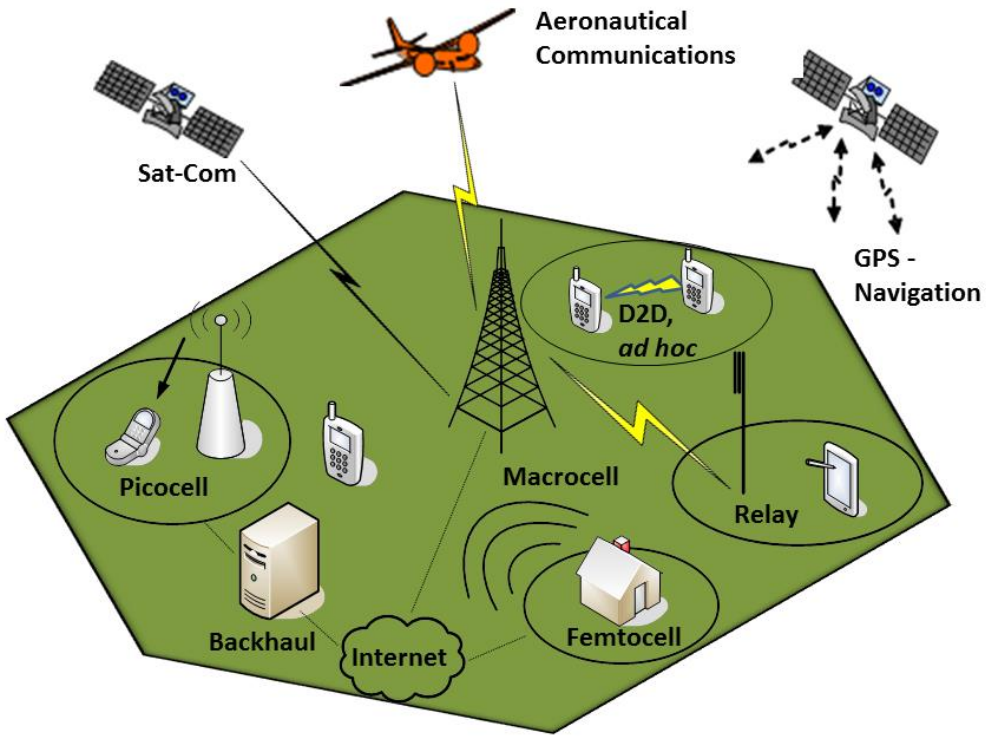
From “Sound Waves” to “Electromagnetic Waves”

- We have seen that
  - We often use sound waves as **carriers of information**
  - We use many kinds of **generators** to produce sound waves
  - We use many kinds of **sensors** to receive sound waves
  - We can visualize sound waves both as **wavefronts** and as **rays**
  - An understanding of how sound waves travel leads to **many useful applications**
- The story is quite similar with electromagnetic waves!

"Waves" used as carriers of information

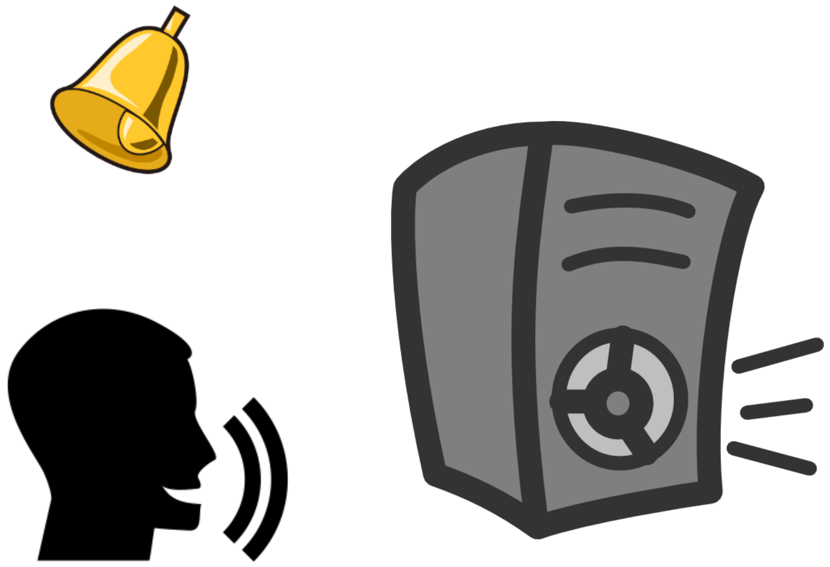
Sound

Electromagnetic



Many kinds of sources used to generate waves

Sound

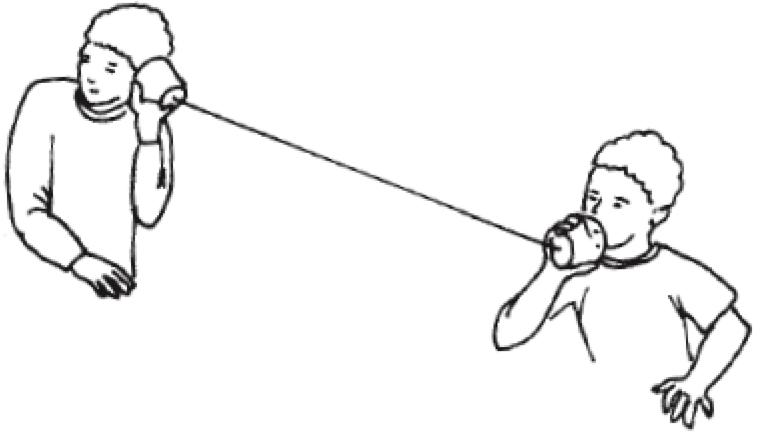


Electromagnetic

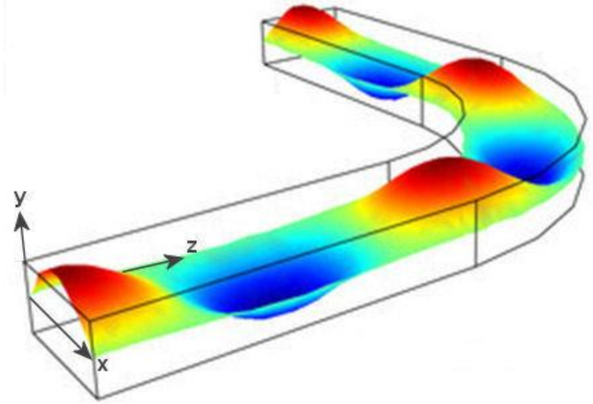
antennas

Waves directed in certain ways

Sound



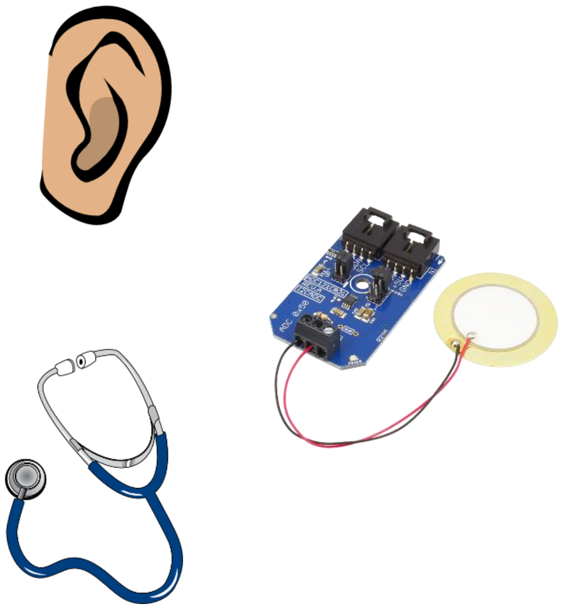
Electromagnetic



waveguides

Many kinds of receivers used

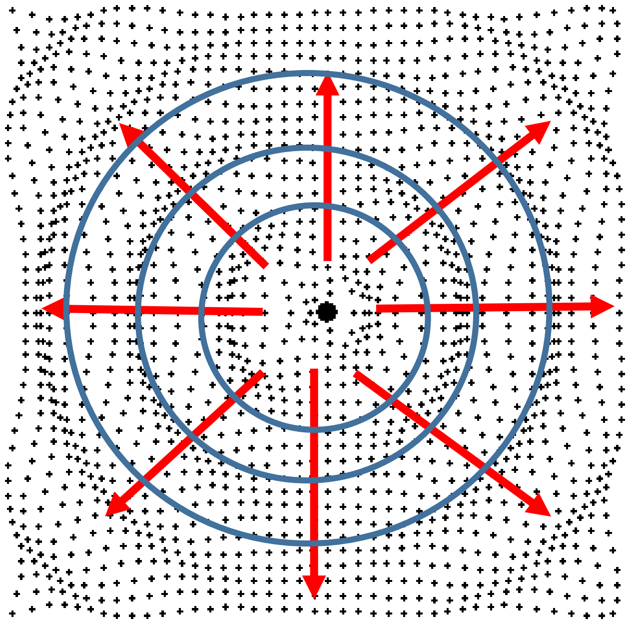
Sound



Electromagnetic

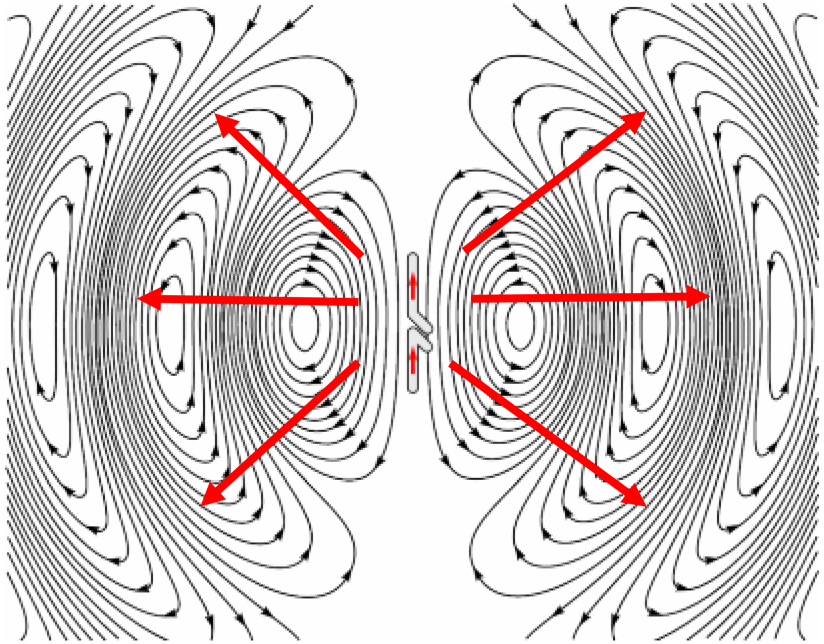
Visualized as wavefronts and as rays

Sound



Physical vibration of bell

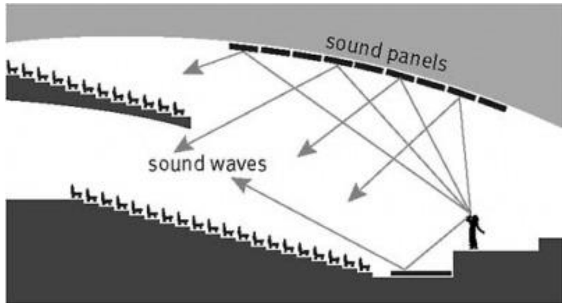
Electromagnetic

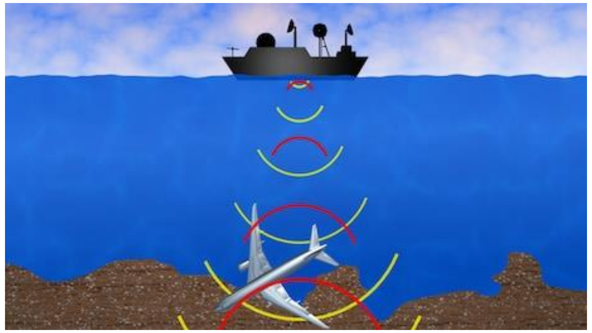


Alternating current in wires

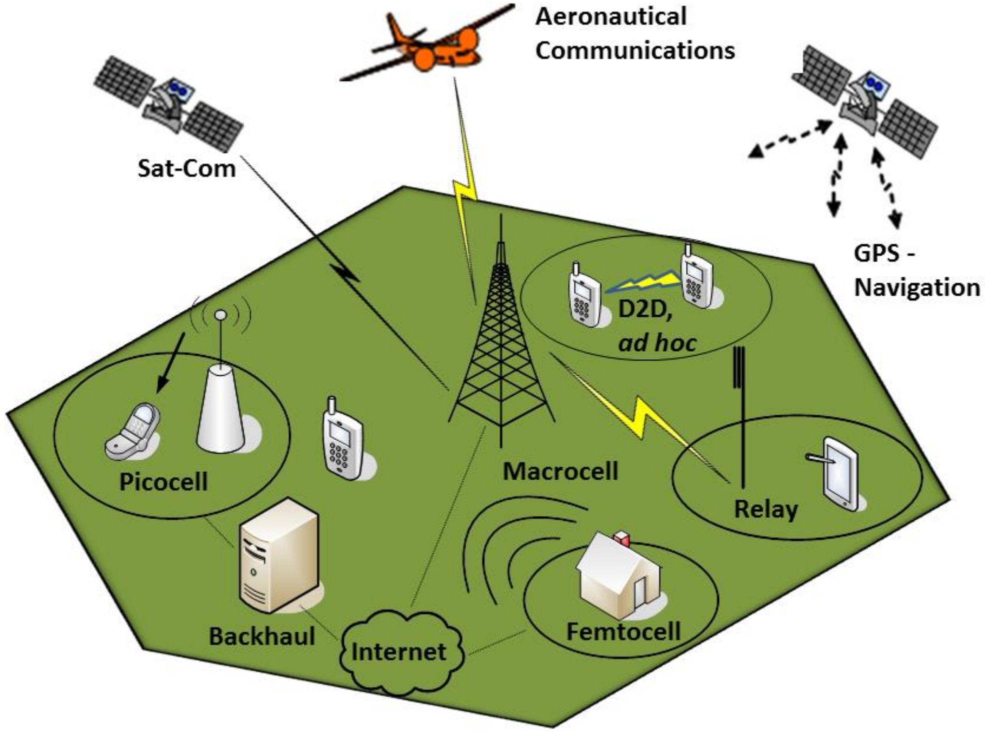
Understanding of waves leads to many useful applications

Sound





Electromagnetic



In this course we will see …

- What is meant by “**electromagnetic waves**” and “**antennas**”
- **How does energy radiate** from an antenna
- What are the different **characteristics of the radiated energy**
- What are the different **kinds of antennas** and their properties
- What are the various **mechanisms by which electromagnetic waves propagate**

Questions?? Thoughts??

EE 328
Wave Propagation and Antennas

with

Dr. Naveed R. Butt

@

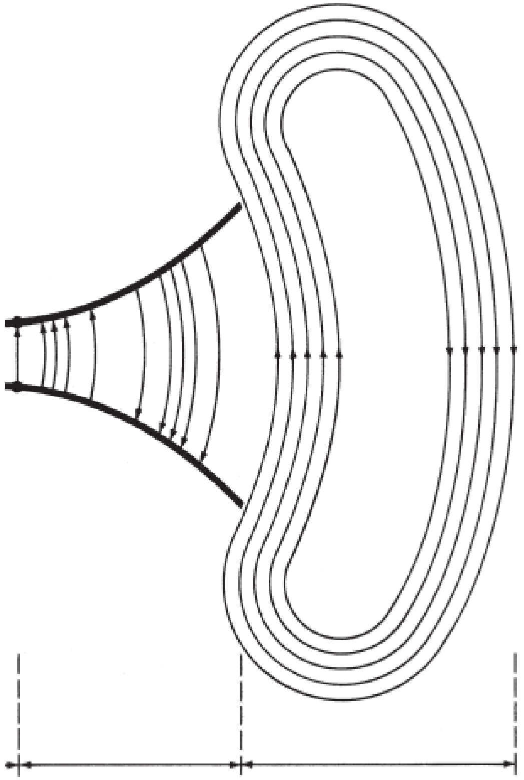
Jouf University

Hello Antenna!!

- What is an “antenna”?
  - What are “electromagnetic (EM) waves”?
- Why do antennas radiate EM waves?
- What are the main functions of an antenna?
- What is a good antenna system?
- What different types of antennas have we come up with so far?

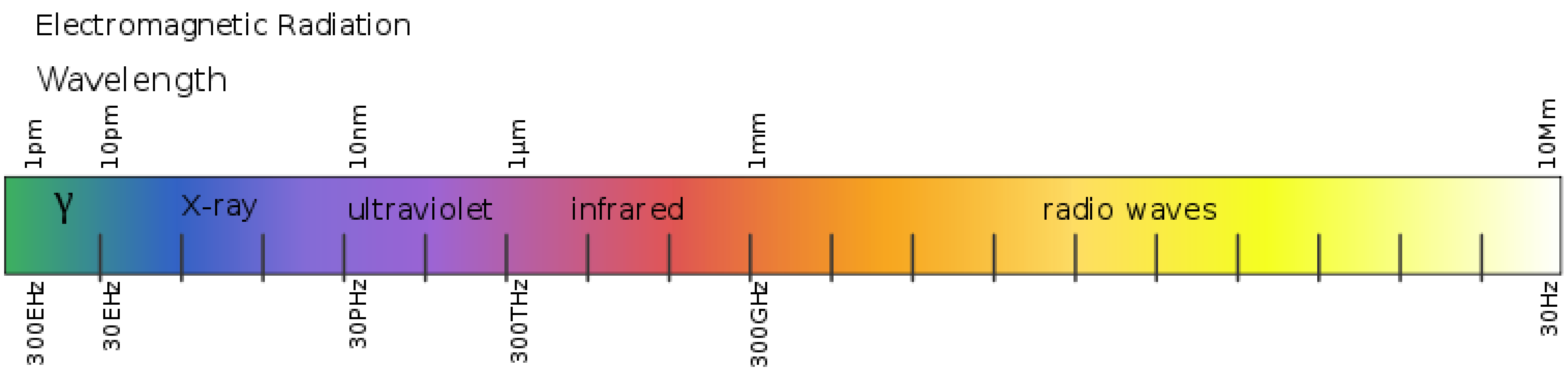
Antenna = something that radiates or receives radio waves

- Something?
  - Usually a metallic rod or strip
- Radiates?
  - Throws into free-space
- Receives?
  - Captures from free-space



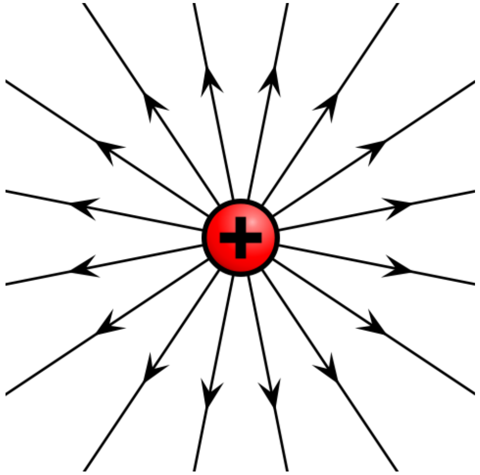
Antenna = something that radiates or receives radio waves

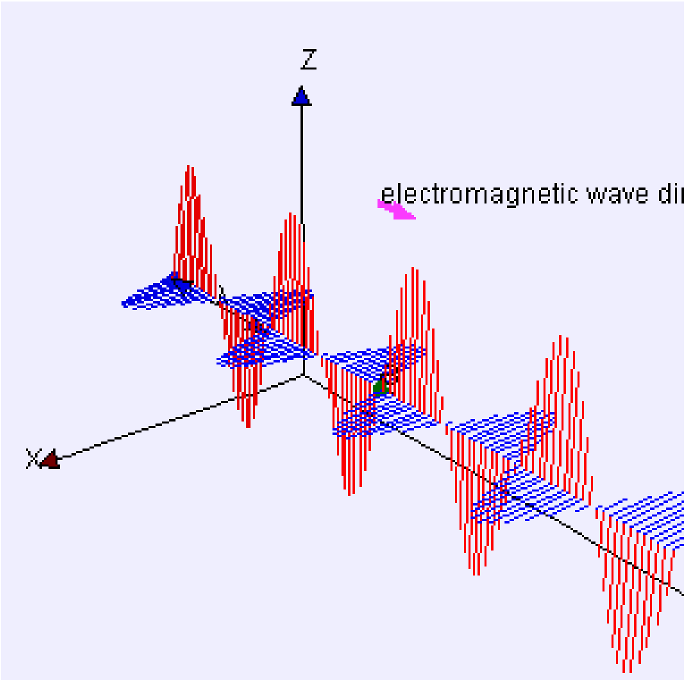
- Radio waves?
  - **Electromagnetic waves** with frequencies in the range 30Hz to 300GHz



Electromagnetic waves = oscillating electric and magnetic fields

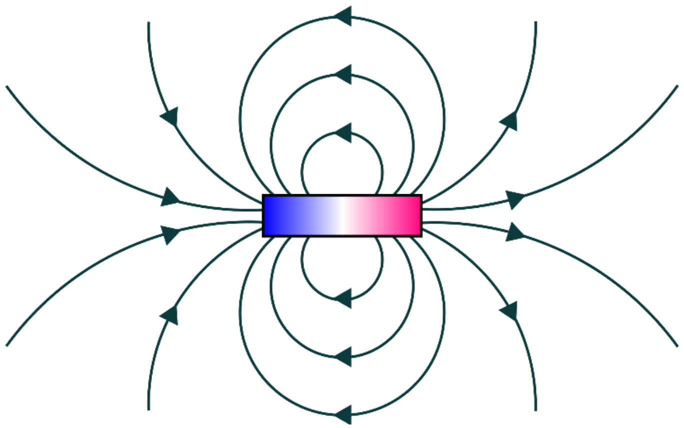
- Electric Field?
  - The field around a charge that affects other charges
  - Can also be generated by a time-varying magnetic field

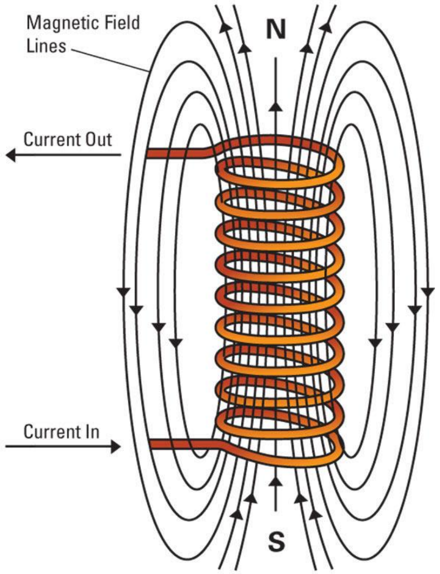


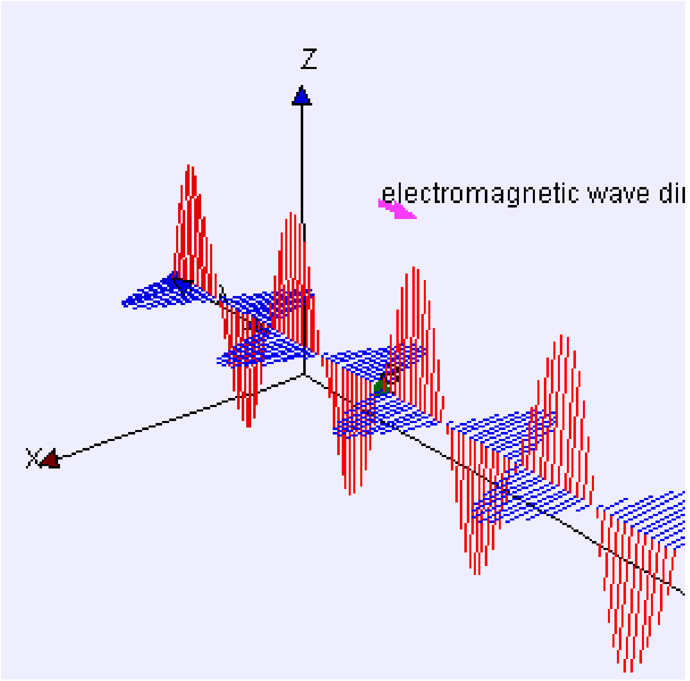


Electromagnetic waves = oscillating electric and magnetic fields

- Magnetic Field?
  - The field that affects magnetic materials and moving charges
  - Can be generated by a magnetized material, by moving electric charges, or by a time-varying electric field

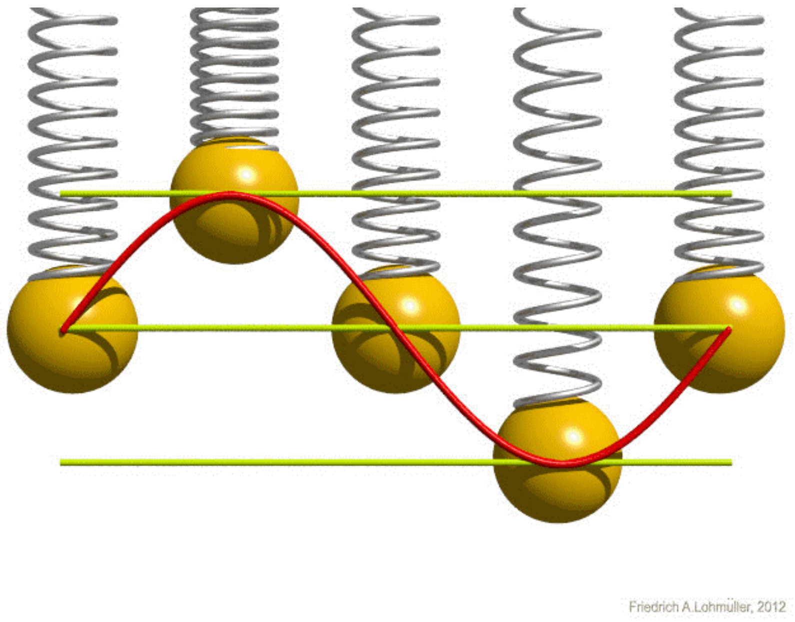






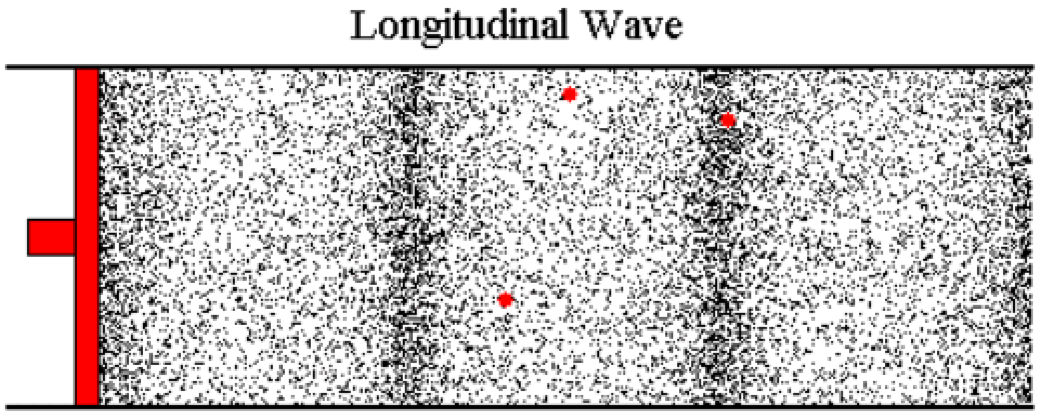
Electromagnetic waves = oscillating electric and magnetic fields

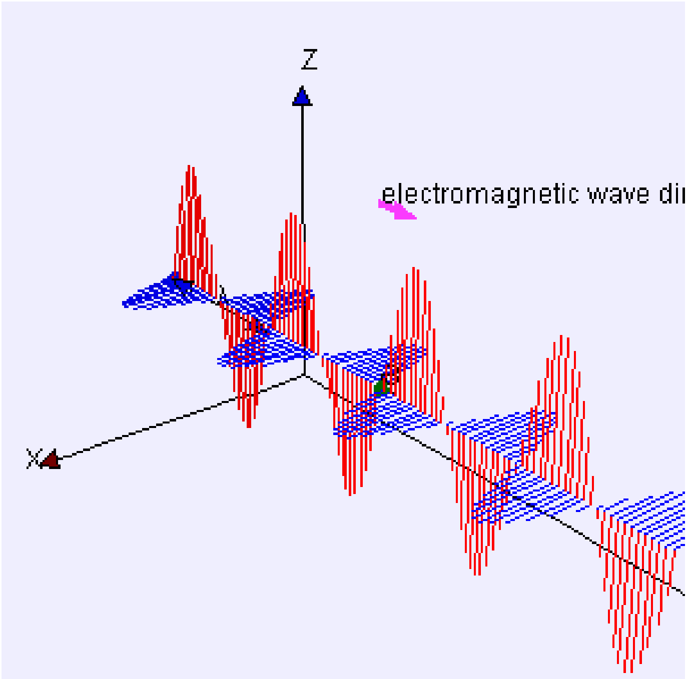
- Oscillation?
  - A change that repeats (in time)
- Wave?
  - Oscillation of some medium or field



Electromagnetic waves = oscillating electric and magnetic fields

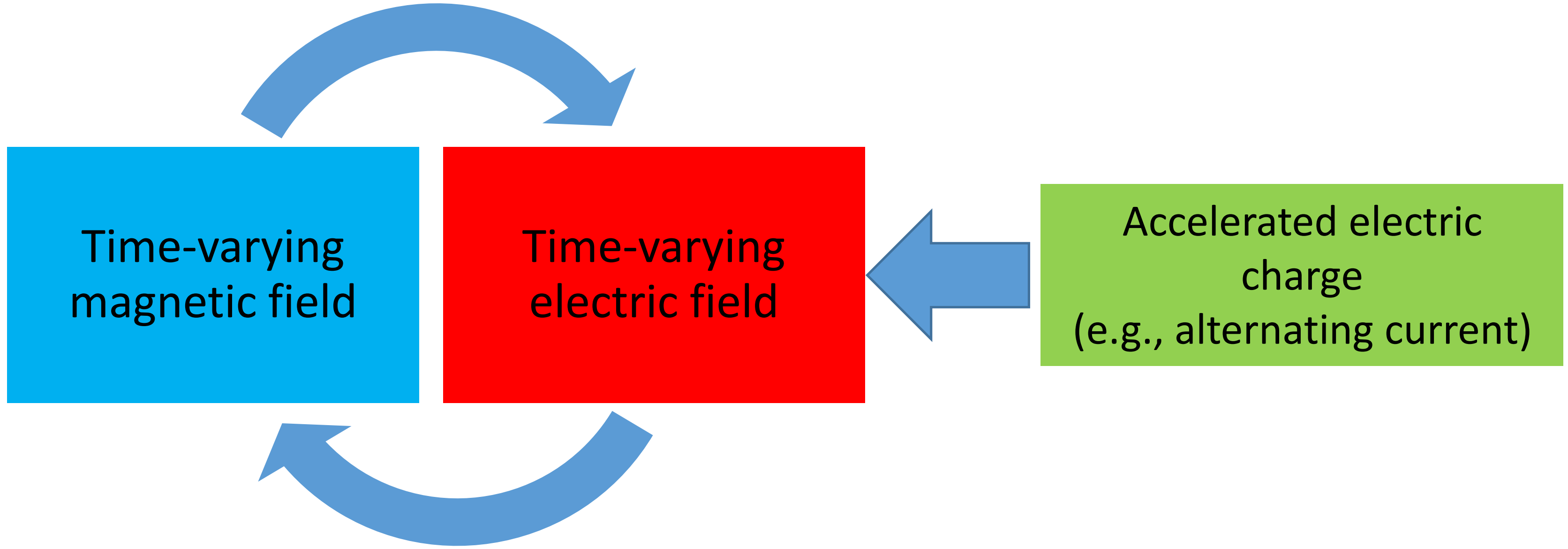
- Wave = oscillation of some medium or field, e.g.,
  - Water waves = repeated up and down motion of water molecules
  - Sound waves = repeated compression and decompression layers of air etc.
  - Electromagnetic waves = repeated increase and decrease of electric and magnetic fields





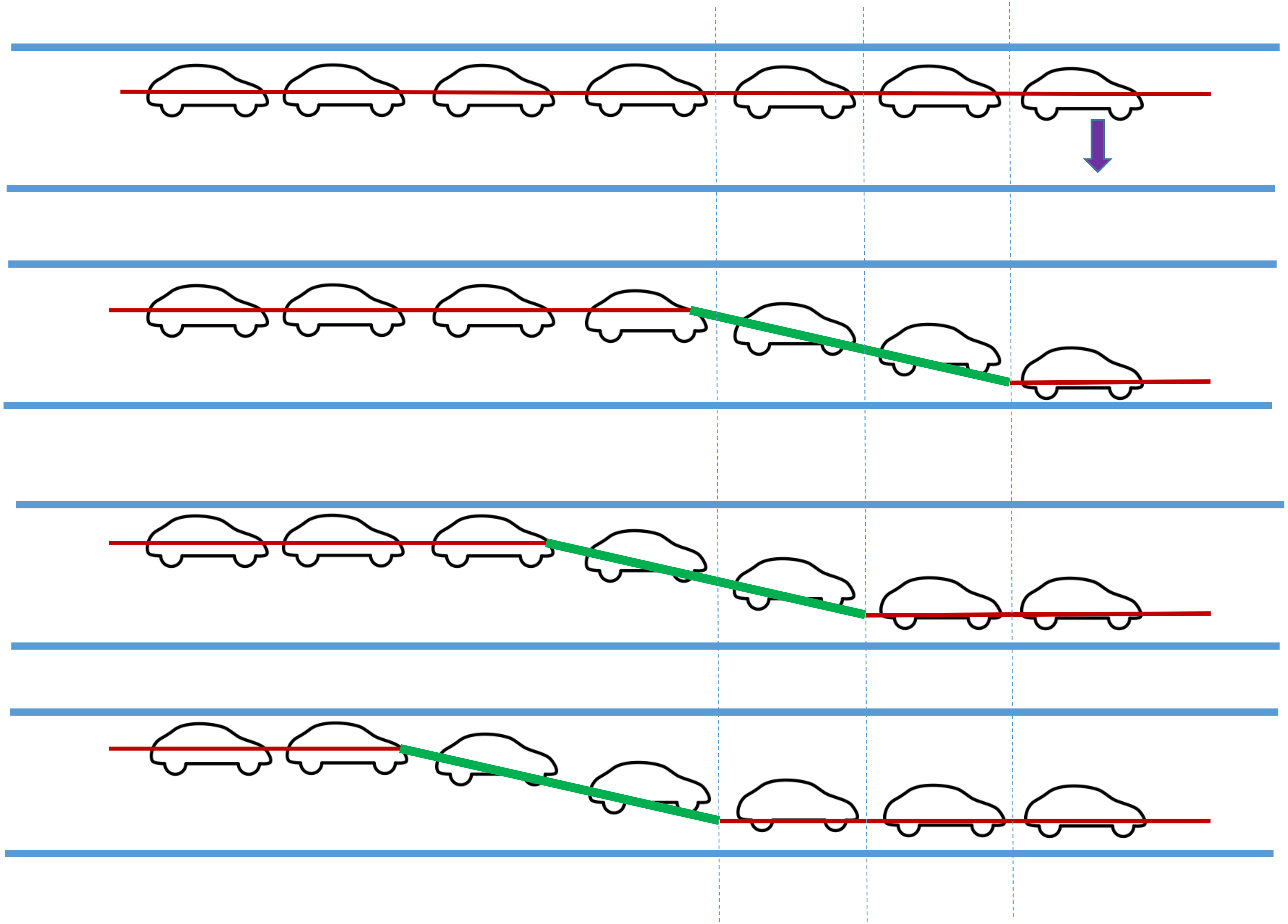
Why do antennas radiate EM waves?

- Things humans discovered about electric and magnetic fields
  - A time-varying electric field generates a time-varying magnetic field
  - A time-varying magnetic field generates a time-varying electric field
  - Accelerated electric charges generate time-varying electric fields



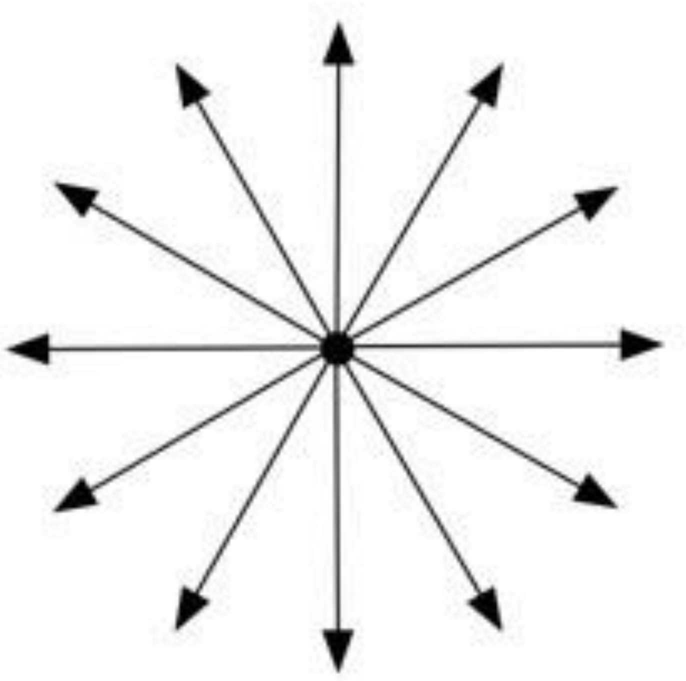
Why do accelerated charges generate electromagnetic waves?

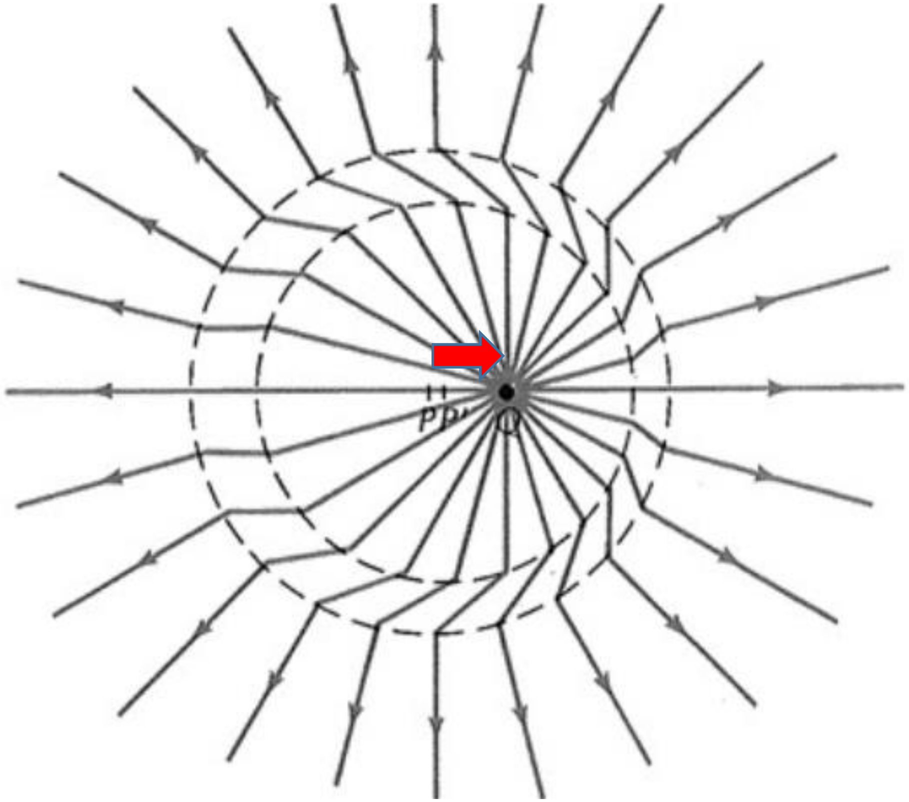
- Let us go for a drive!
  - Assume that for some strange reason you and your friends have decided to follow exactly the movements of the car in front.



A “change” will travel from right to left (green line)

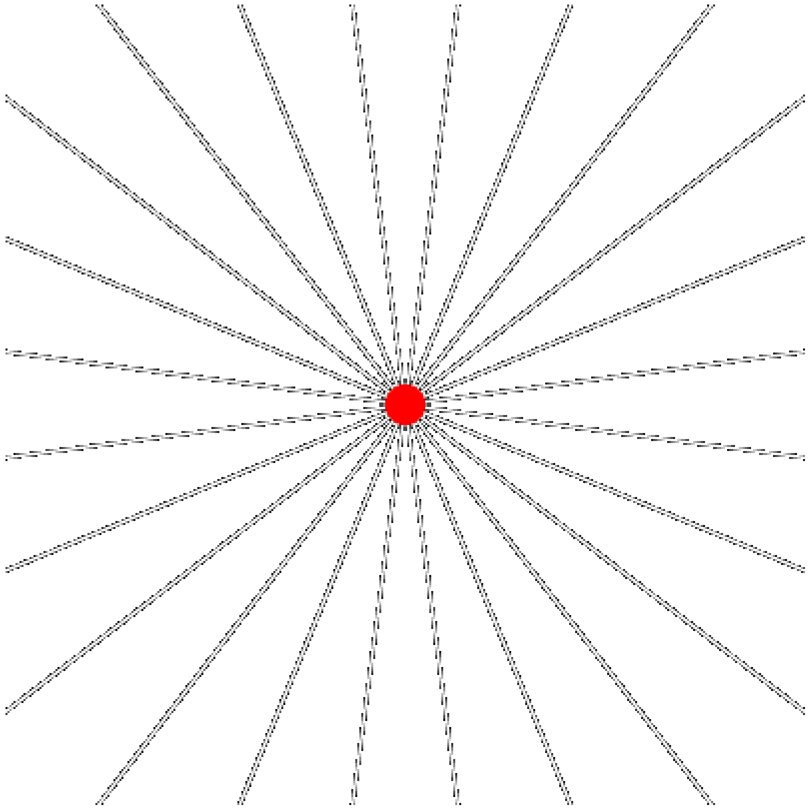
Electric field lines like to keep the charge in the middle!





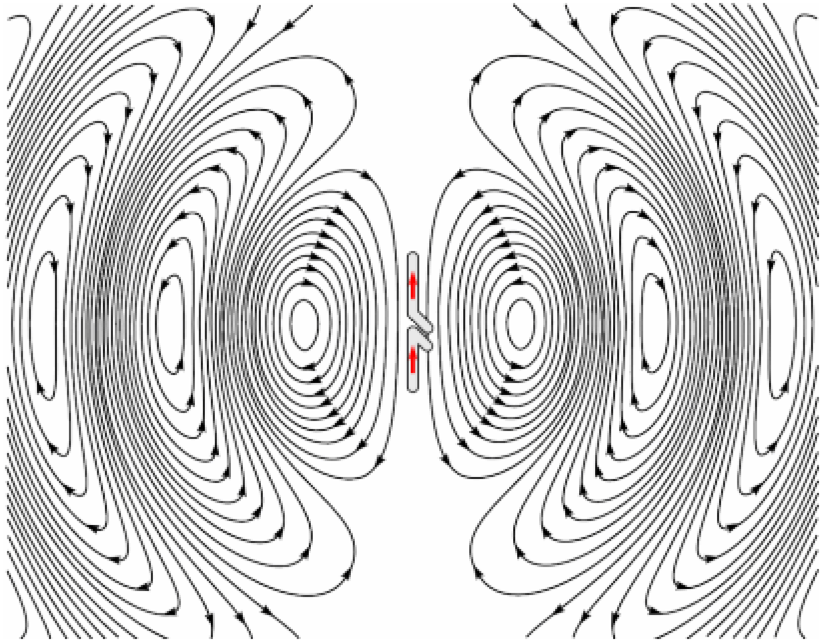
If the charge suddenly changes its position, the field lines adjust themselves to keep it in the middle. As they do this, a "ripple" travels through space around the charge.

Electric field lines like to keep the charge in the middle!



So, why do antennas radiate EM waves?

- We have seen in previous slides that
  - Accelerated electric charges can be used to generate EM waves
- ***Antennas use acceleration of charges (e.g., by alternating current levels) to generate EM waves***



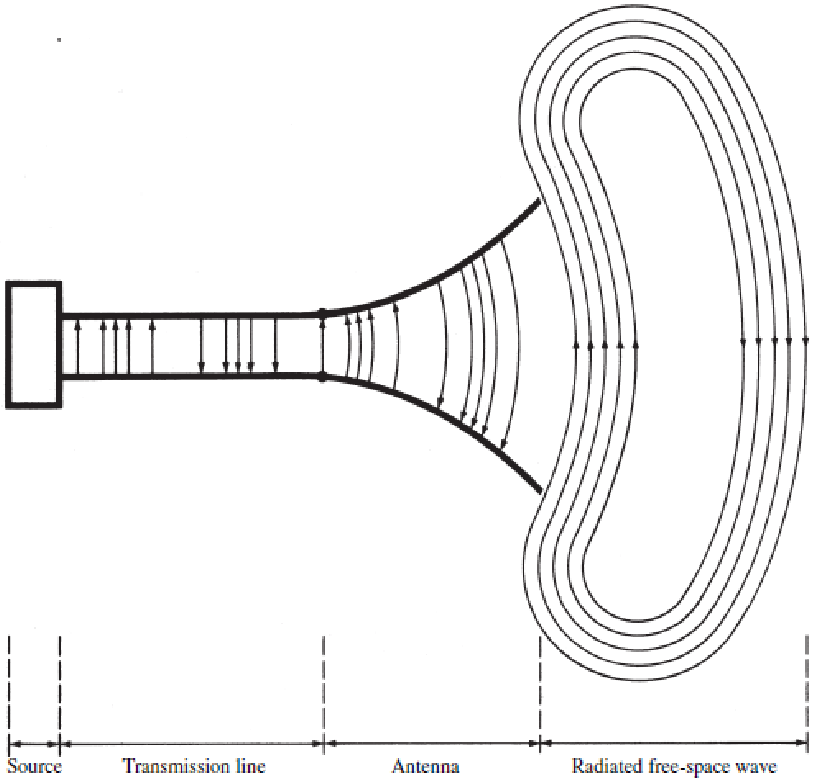
Alternating current in wires

Two main functions of an Antenna

1. To radiate or receive radio waves
2. To act as a ***directional device*** – i.e., to radiate energy strongly to certain directions (or receive radiation energy more keenly from certain directions)

A "good" antenna system

- ***A good antenna system converts most of the energy provided by the source into radiated energy*.**



A “good” antenna system

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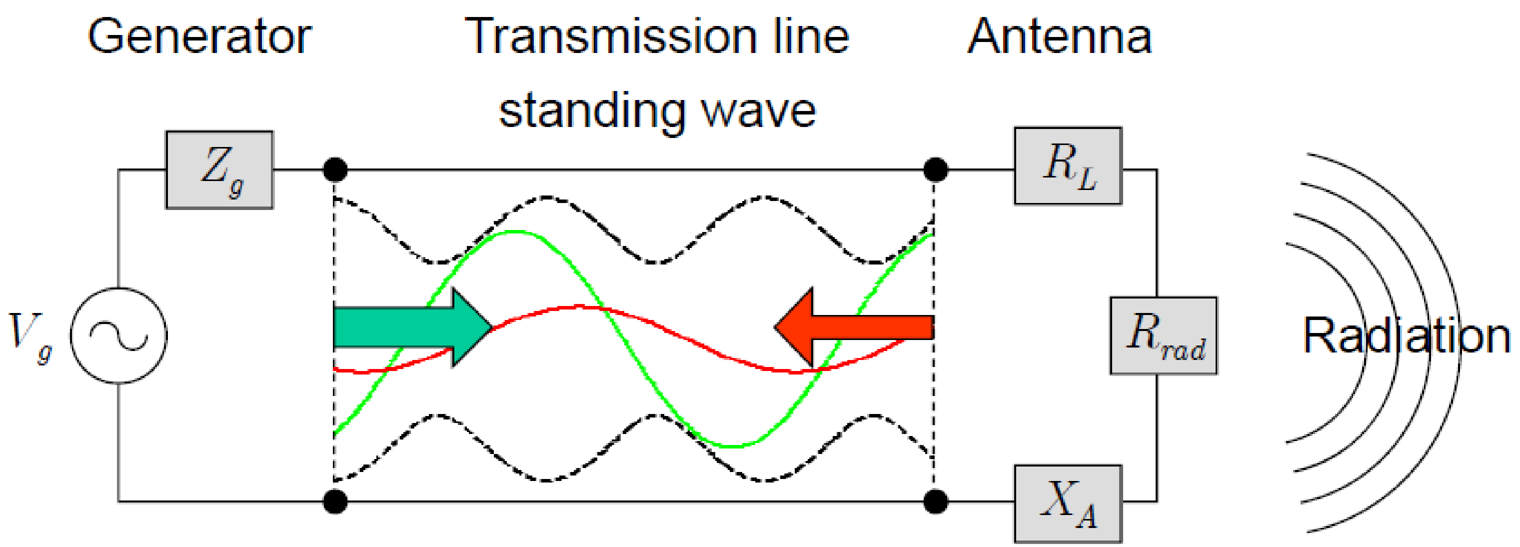
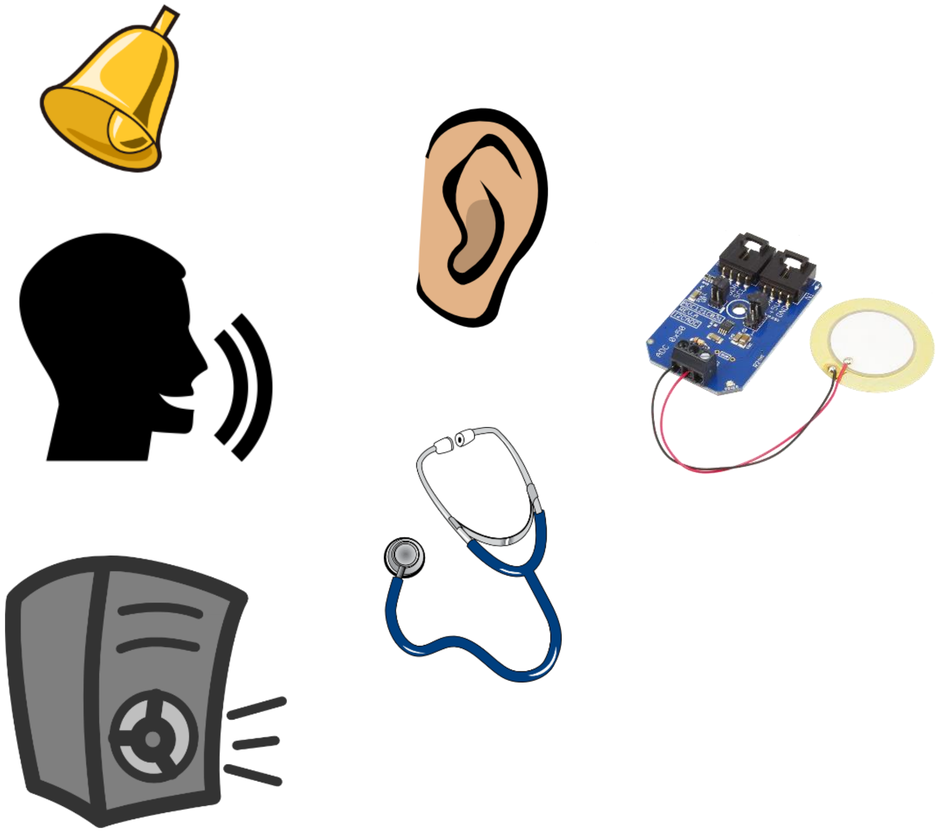


A “good” antenna system

- Some of the problems (leading to energy loss) an antenna system can face
  - Losses in transmission line
  - Losses in antenna
  - Standing waves due to impedance mismatch between antenna and line
- A good antenna system avoids these by
  - Selecting low-loss lines
  - Reducing conduction and dielectric losses in antenna
  - Matching impedance of the antenna to the characteristic impedance of the line

Different kinds of antenna in use

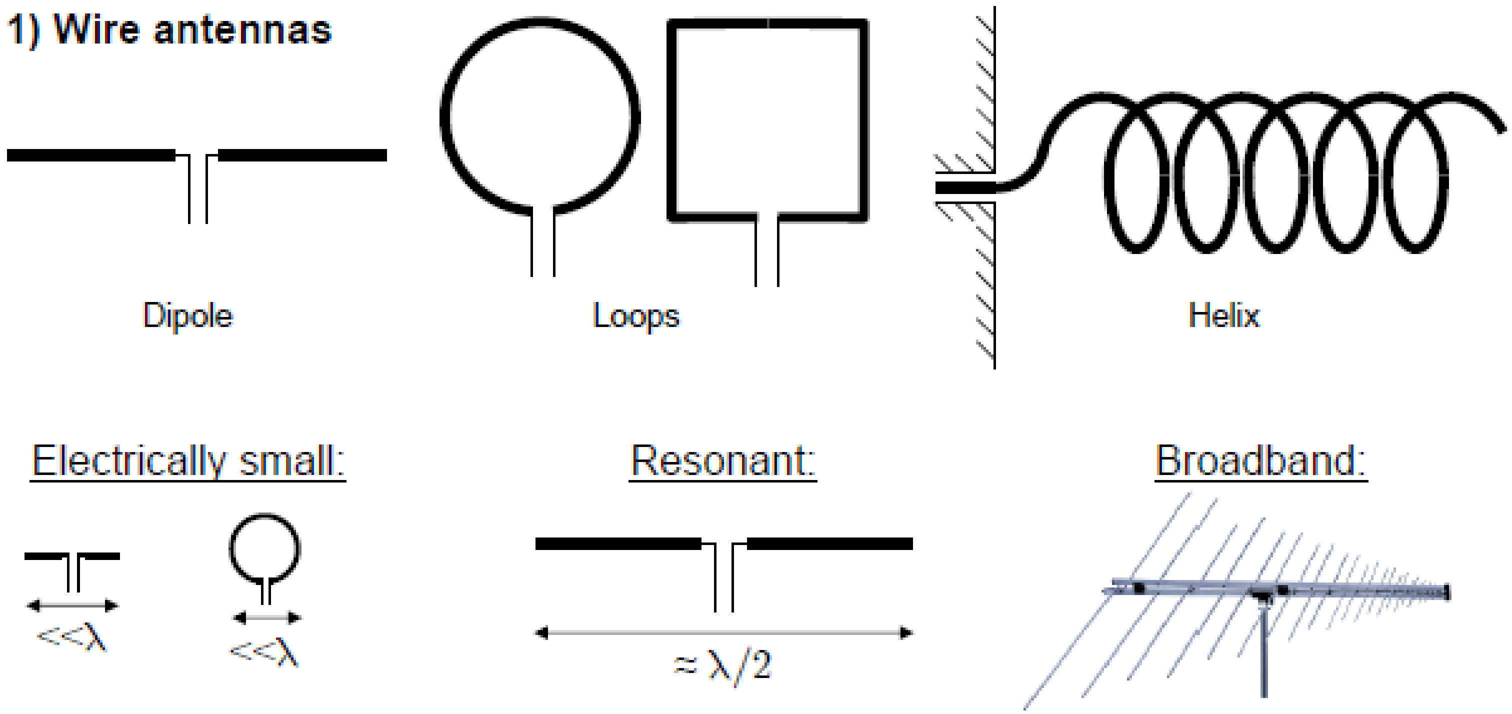
Sound waves



Radio waves

antennas

Different kinds of antenna in use



- Very low directivity
- Low radiation efficiency
- Low input resistance
- High input reactance

- Real input impedance
- Narrow bandwidth

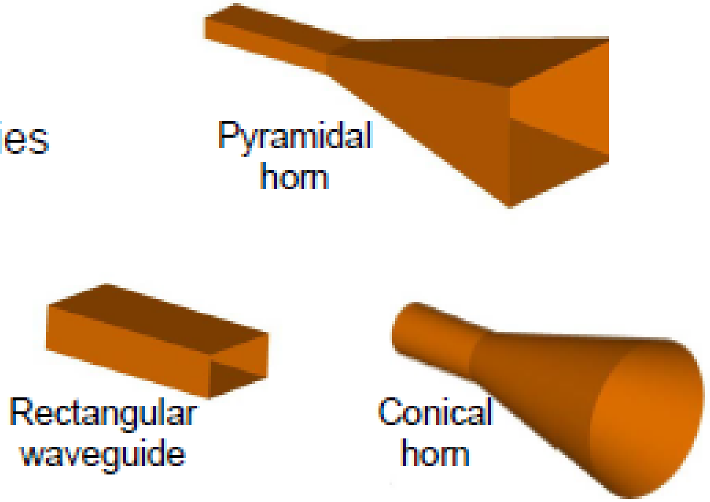
- Bandwidth >2:1
- Active regions
- Example:
  Log periodic dipole array

Mostly used for low frequencies

Different kinds of antenna in use

2) Aperture antennas

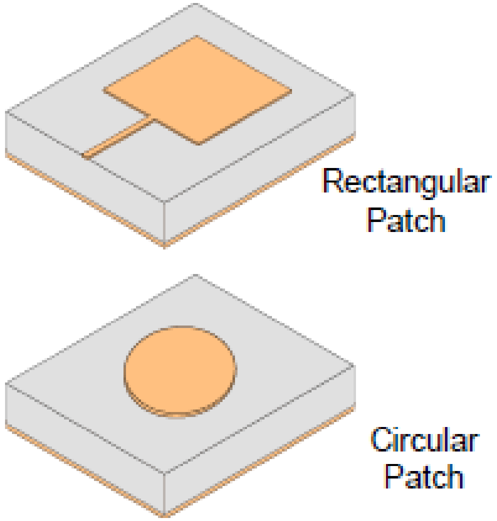
- Microwave and millimeter wave frequencies
- Good directional properties
- Dimensions: Several wavelengths
- Moderate bandwidth
- Compatible with waveguide technology



Different kinds of antenna in use

3) Microstrip antennas

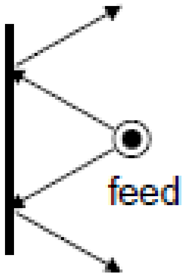
- Since 1970's
- Microwave and millimeter wave frequencies
- Low cost, low profile
- Compatible with printed circuit board technology
- Very versatile
- Resonant structure (small bandwidth – however, increased research activity for UWB applications)
- Low efficiency



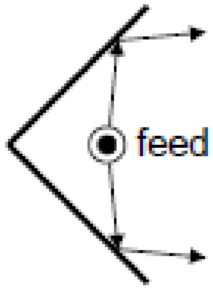
Different kinds of antenna in use

4) Reflector antennas

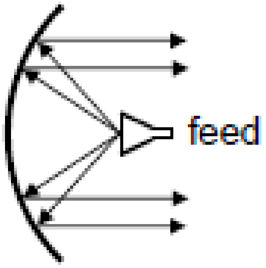
- Very good directivity (high gain)
- Long distance communication



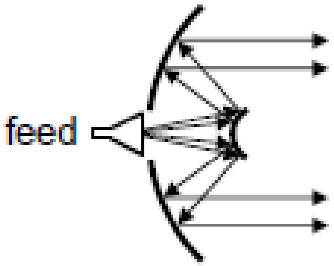
Plane sheet



Comer reflector



Parabolic reflector
(front feed)

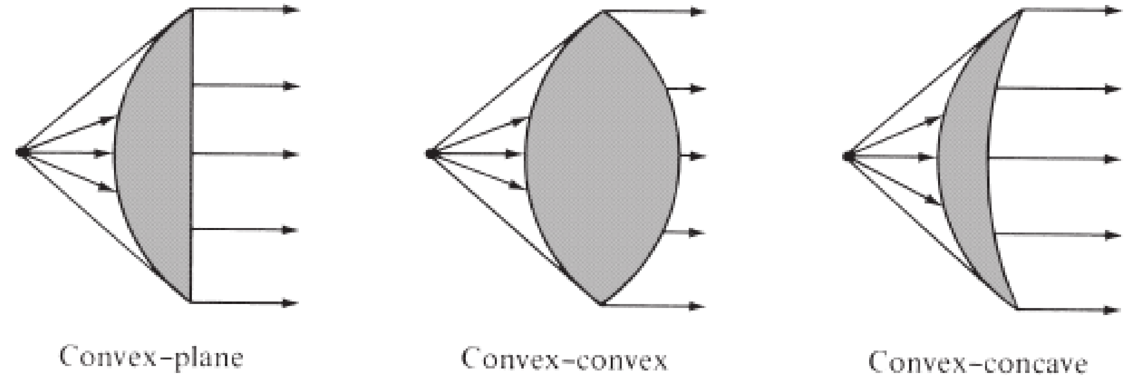


Parabolic reflector and
hyperbolic subreflector
(Cassegrain feed)

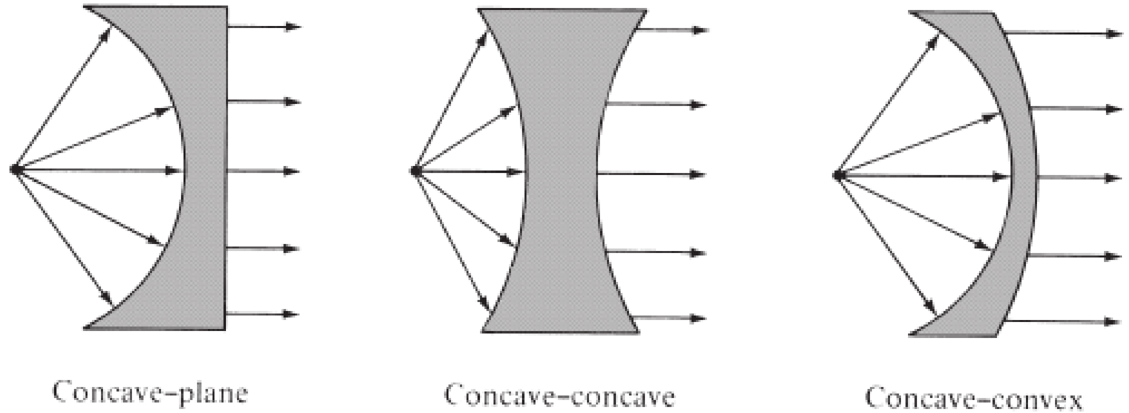
Different kinds of antenna in use

5) Lens antennas

- Primarily used to collimate or focus radiation
- High frequency applications



(a) Lens antennas with index of refraction $n > 1$

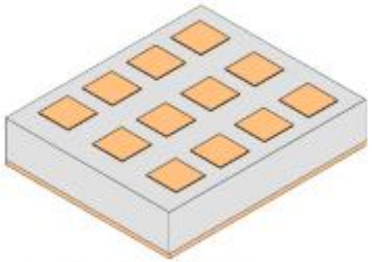


(b) Lens antennas with index of refraction $n < 1$

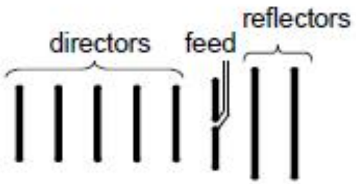
Different kinds of antenna in use

6) Array antennas

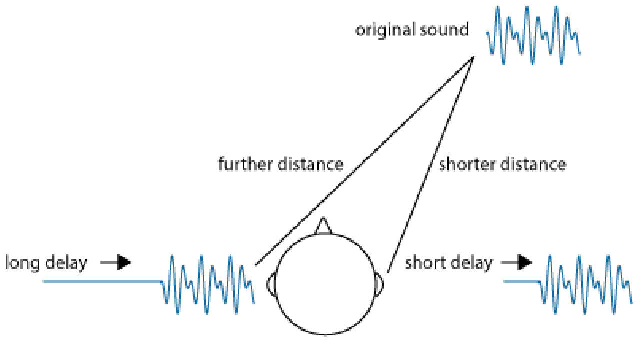
- Use the cooperative behavior of several antennas
- Constructive or destructive interferences depending on direction
- Permit to obtain the desired radiation characteristics (beam forming)
- Permit electronic scanning of beams in space (phased arrays)



Microstrip patch array

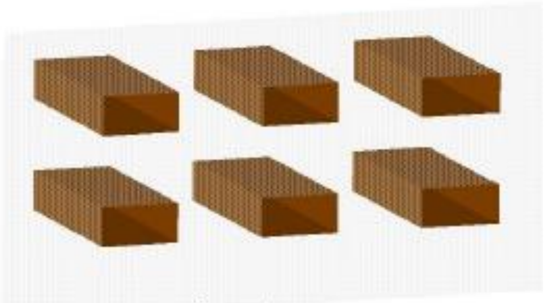


Yagi-Uda antenna



Sound-wave comparison

Reflector antenna array



Aperture array

Questions?? Thoughts??

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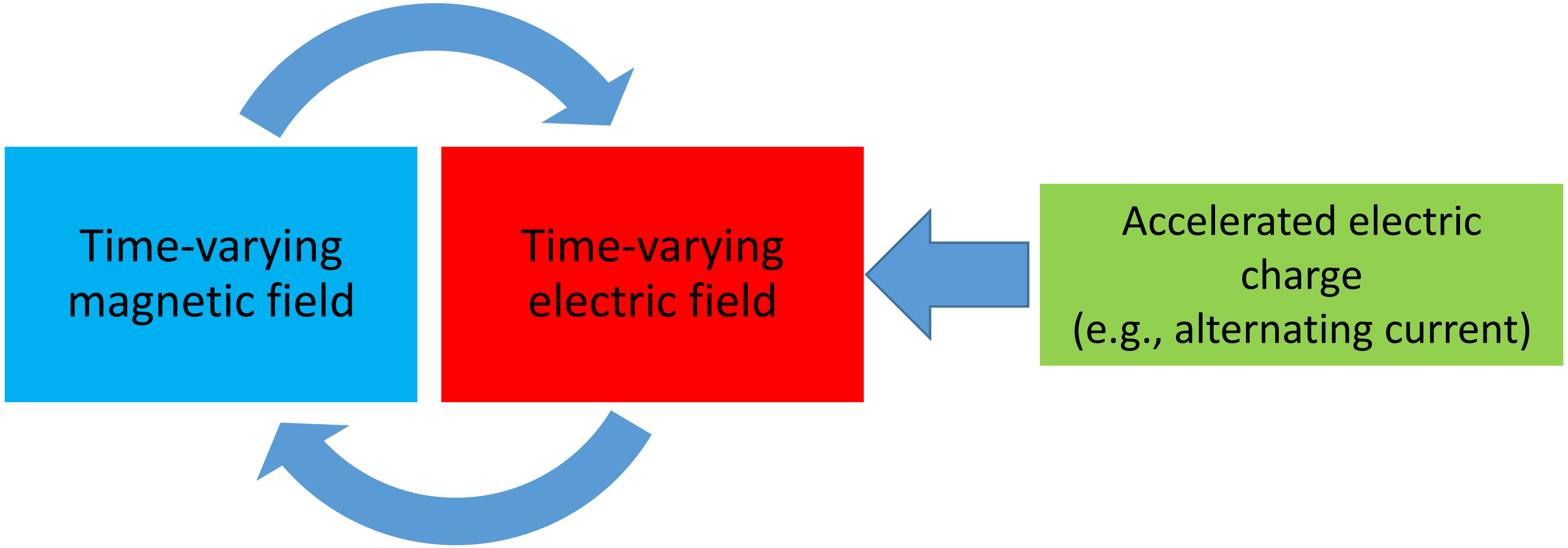
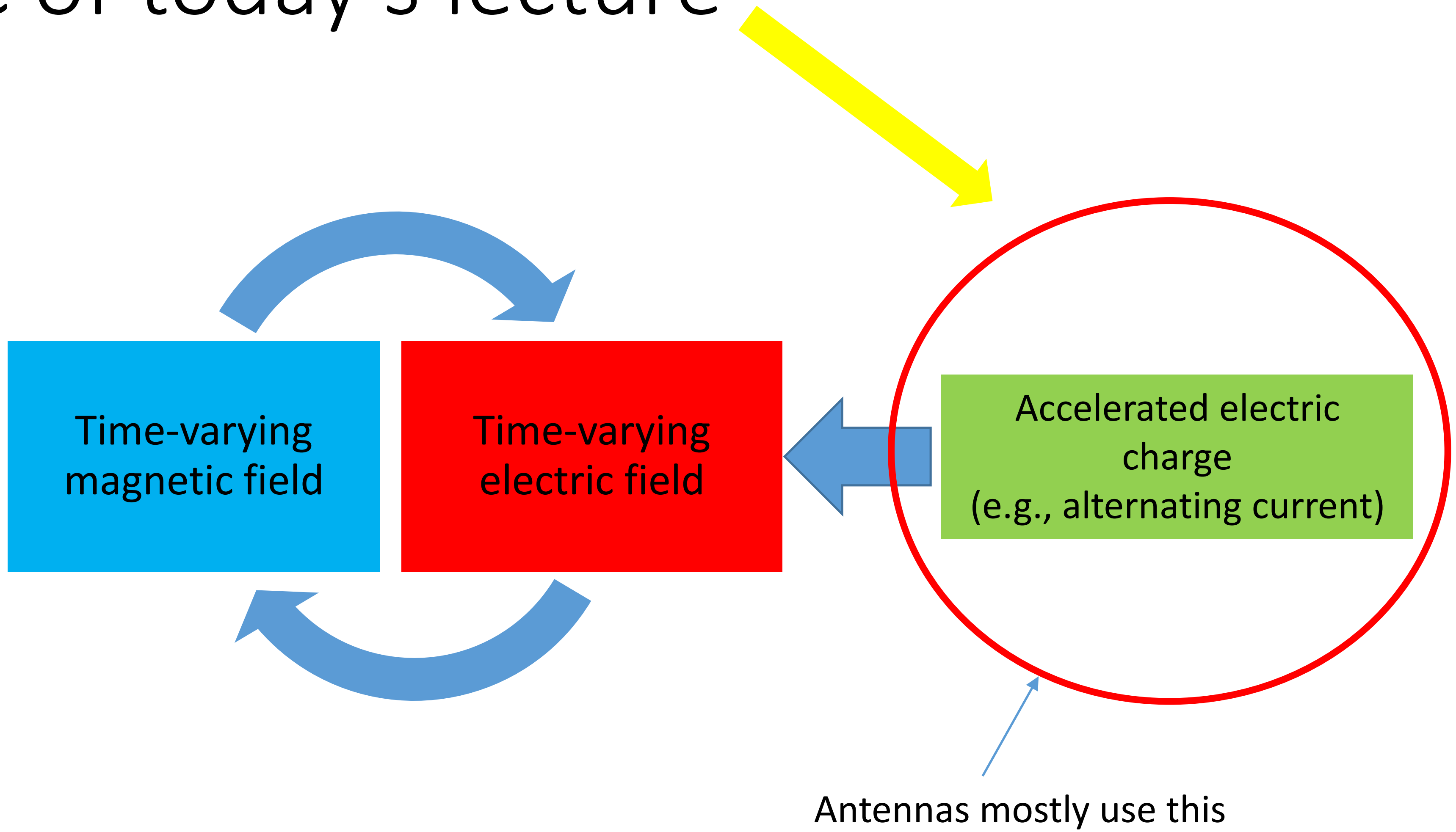
Why do antennas radiate EM waves?



Topic of today's lecture

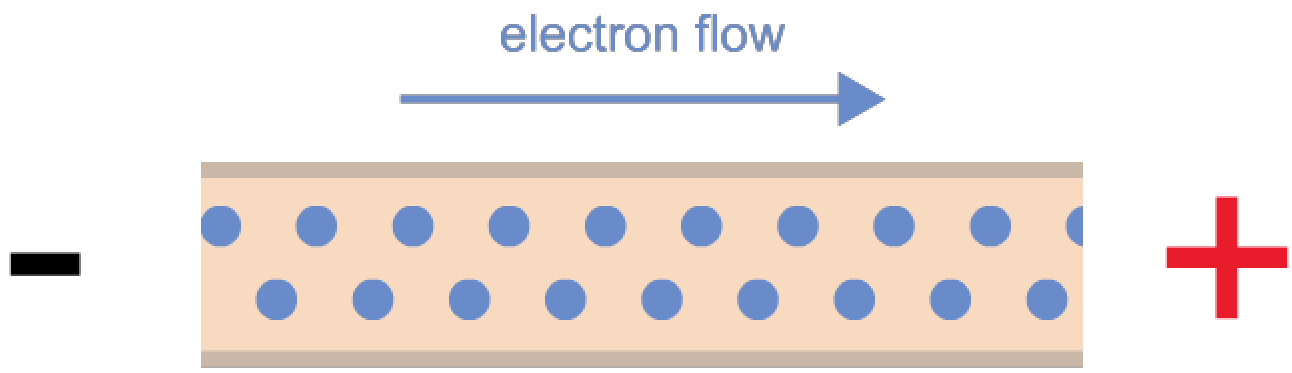


Charges, currents, wires, and waves!

- How **time-varying current** and **acceleration of charges** in a **thin wire** are related?
  - What is current? What is acceleration?
- How a **two-wire antenna** radiates EM waves?

Current = Flow

- Electric current = flow of electric charges



Acceleration = rate of **change** of **speed** and/or **direction**

- E.g., when you are driving, any change in your speed or direction (or both) will mean "acceleration".
  - Quicker change = higher acceleration!

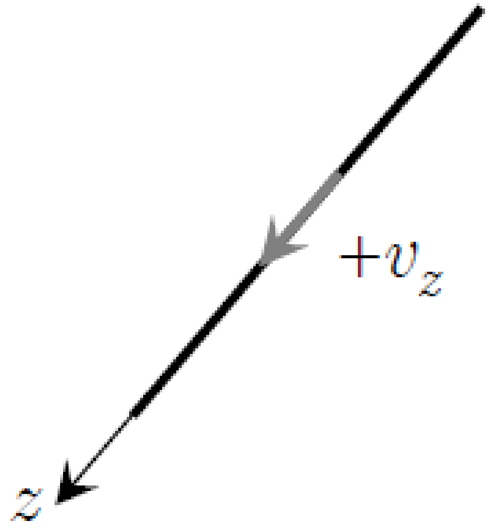
Current in a thin wire …

Velocity (speed and direction) of the charges flowing in the thin wire (*Unit: m/s)*

$$I_z = q_l v_z$$

Current flowing in the wire along direction *z* *(Unit: Amperes)*

Charge distributed per unit length of the thin wire (*Unit: Coulomb/m)*



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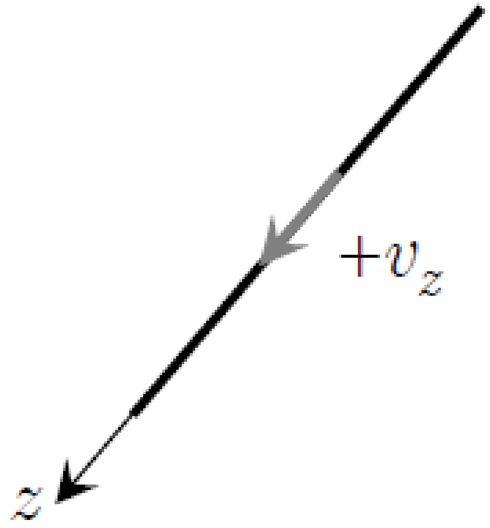


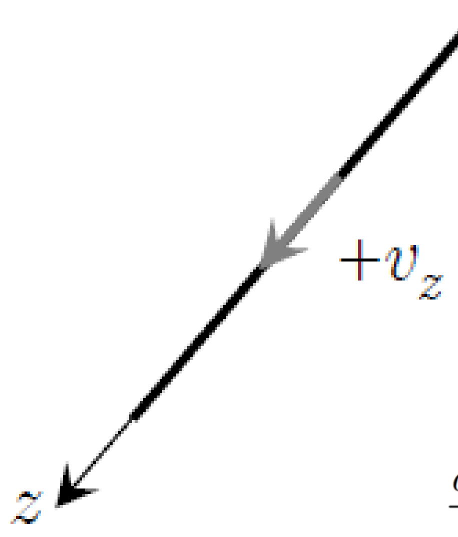
Practice: how much is the current in the wire if the charge velocity is zero?

Time-varying current in a thin wire …

Accelerated charges

$$\frac{dI_z}{dt} = q_l \frac{dv_z}{dt} = q_l \cdot a_z$$

Time-varying current in a thin wire



acceleration:

$$\frac{dv_z}{dt} = a_z \quad (\mathrm{m/s^2})$$

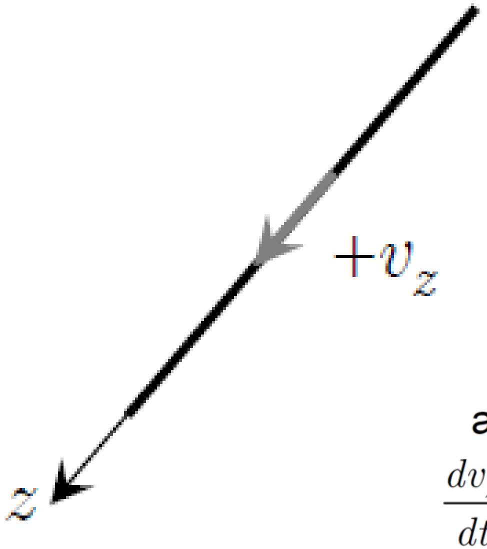
Relation between time-varying current and acceleration of charges in a thin wire

Accelerated charges

Length of the wire

$$l\frac{dI_z}{dt} = lq_l\frac{dv_z}{dt} = lq_l a_z$$

Time-varying current in a thin wire



acceleration:

$$\frac{dv_z}{dt} = a_z \quad (\mathrm{m/s^2})$$

Now we have a mechanism for creating radiation!

$$l\frac{dI_z}{dt} = lq_l\frac{dv_z}{dt} = lq_l a_z$$

To create radiation with a thin wire

- Cause charges to accelerate, or
- Vary the current in the wire

Example

- A thin wire has a time-varying current of 10 Amperes/second, which accelerates the charges in it to 5 m/s^2. How much charge is there in two meters of the wire?

given $\dfrac{dI_z}{dt} =$ 10 A/s a_z = 5 m/s^2

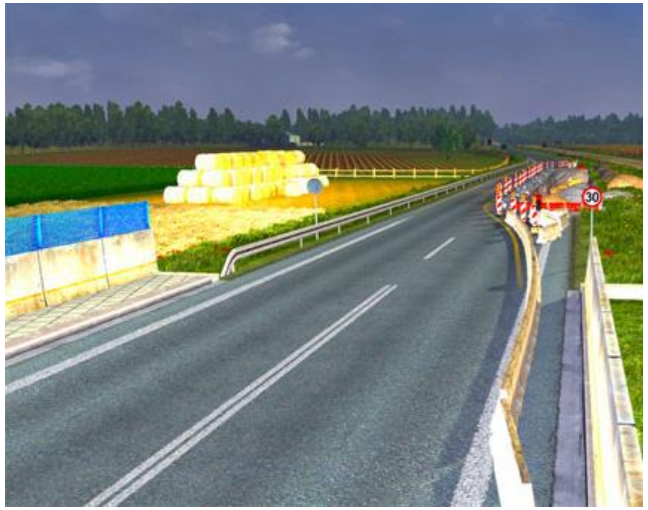
Use $\dfrac{dI_z}{dt} = q_l \dfrac{dv_z}{dt} = q_l \cdot a_z$ = ?

Charge density = 10/5 = 2 Coulomb/m
Charge in 2 meters of wire = (charge density)*2 = 4 Coulomb

How can we accelerate charges in a thin wire?

- Let us go for a drive!
  - remember: acceleration = change in speed or direction (or both)
  - When you are driving what changes in the road cause you to change your speed or direction?

Curves or Bends in the Road

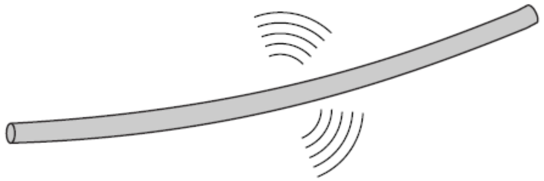


Road Narrowing

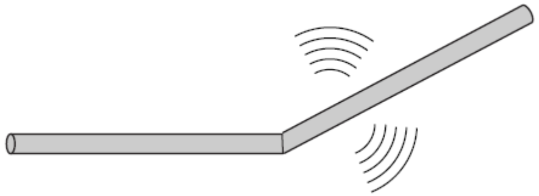
Dead End

How can we accelerate charges in a thin wire?

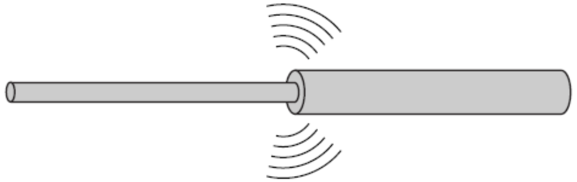
- We can cause charges in a thin wire to accelerate (slow down or speed up or to change their direction) by playing with the wire shape



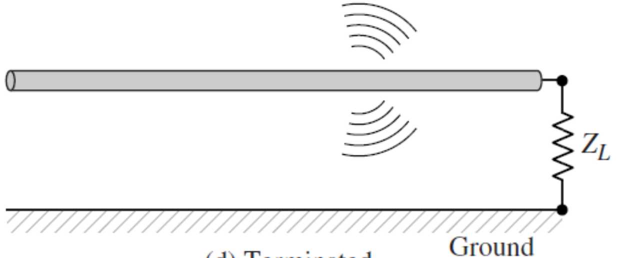
(a) Curved



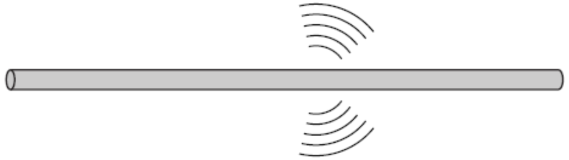
(b) Bent



(c) Discontinuous

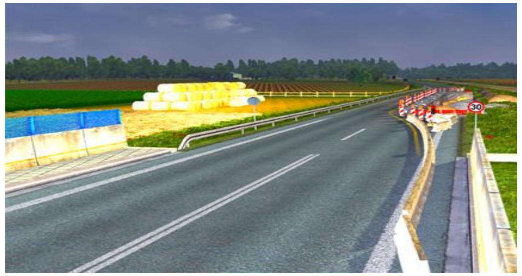


(d) Terminated

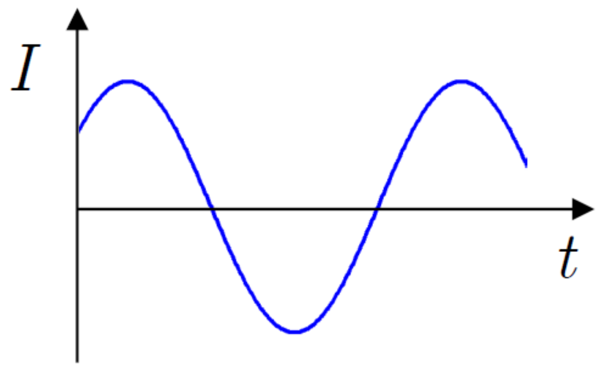


(e) Truncated

Each of these will cause the wire to radiate EM waves, thus making it an “antenna”



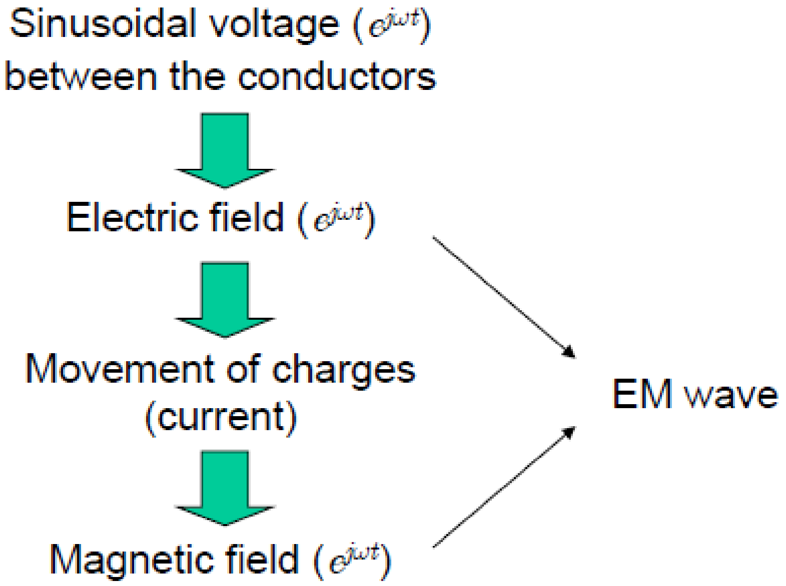
Or, we could apply a time-varying current to the wire to make it radiate!

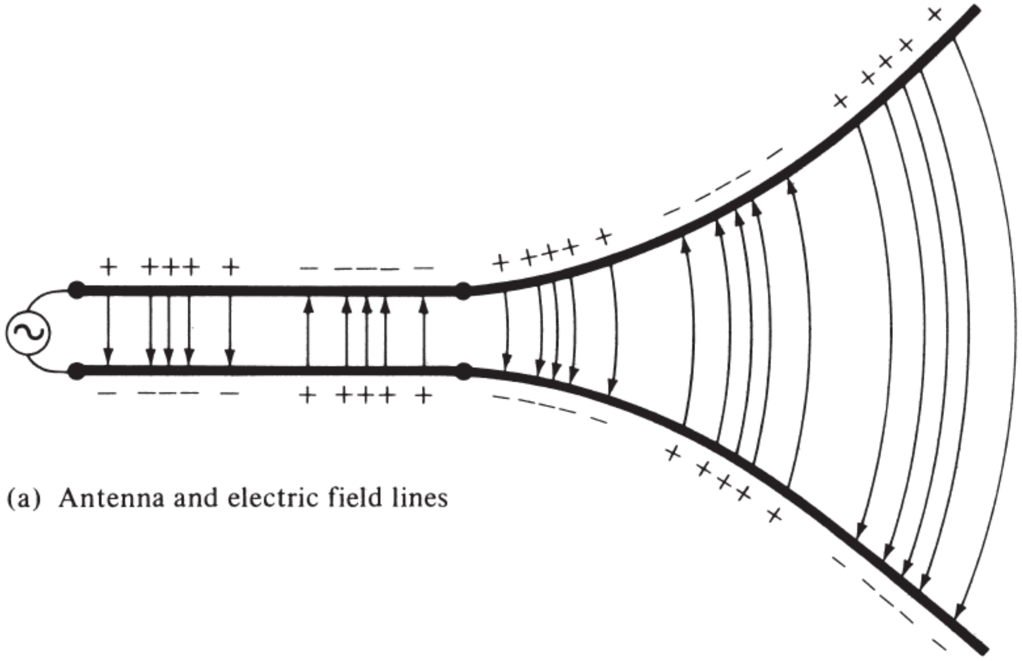


A periodic time-varying current causes charges to accelerate (i.e., charges are oscillating in a time-harmonic motion). *Then, even if the wire is straight, there will be radiation.*

From one wire to two …

- How does a two-wire antenna radiate?



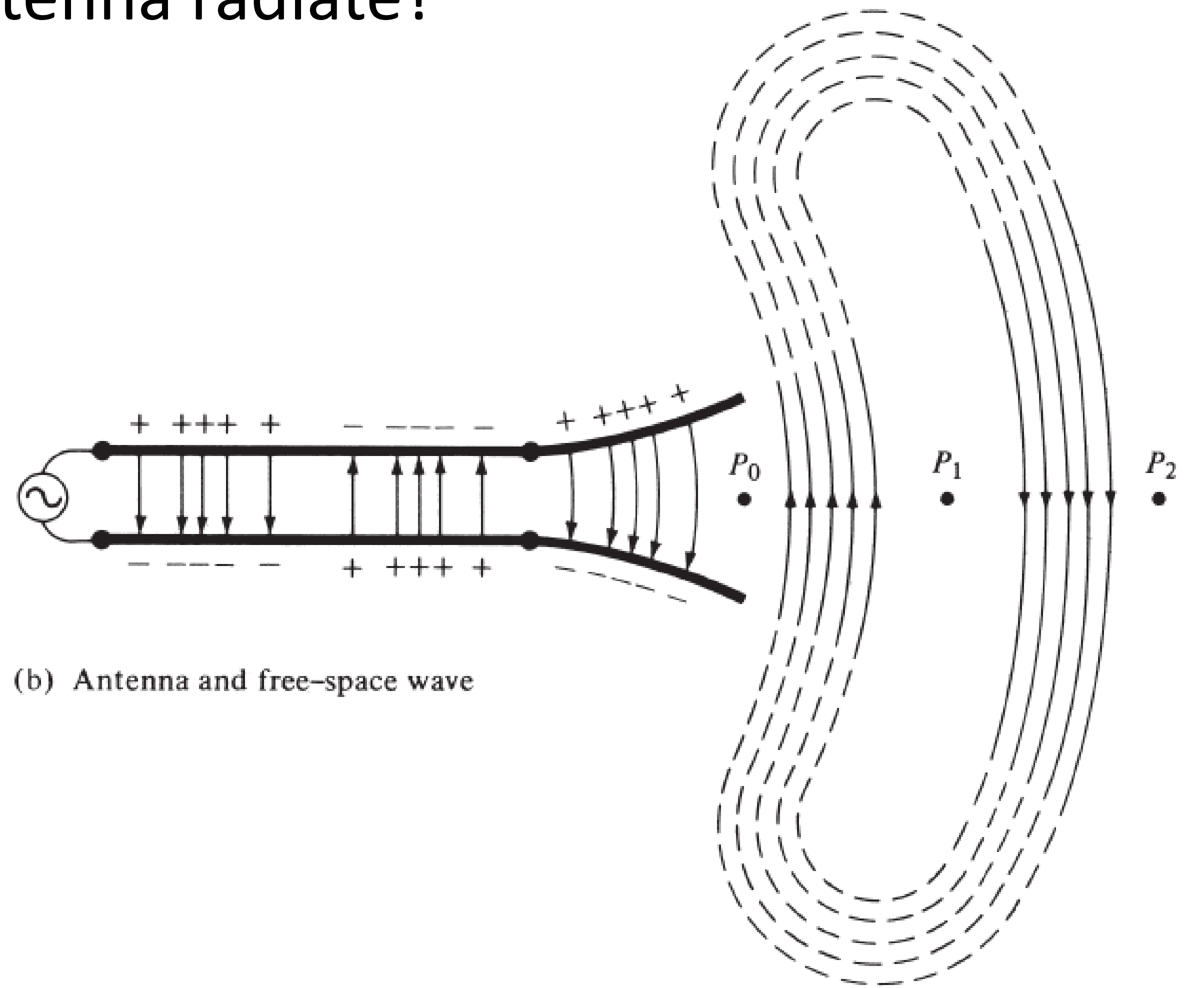


(a) Antenna and electric field lines

From one wire to two …

- How does a two-wire antenna radiate?

At the end of the wires, the electric and magnetic field lines continue travelling as a free-space wave.



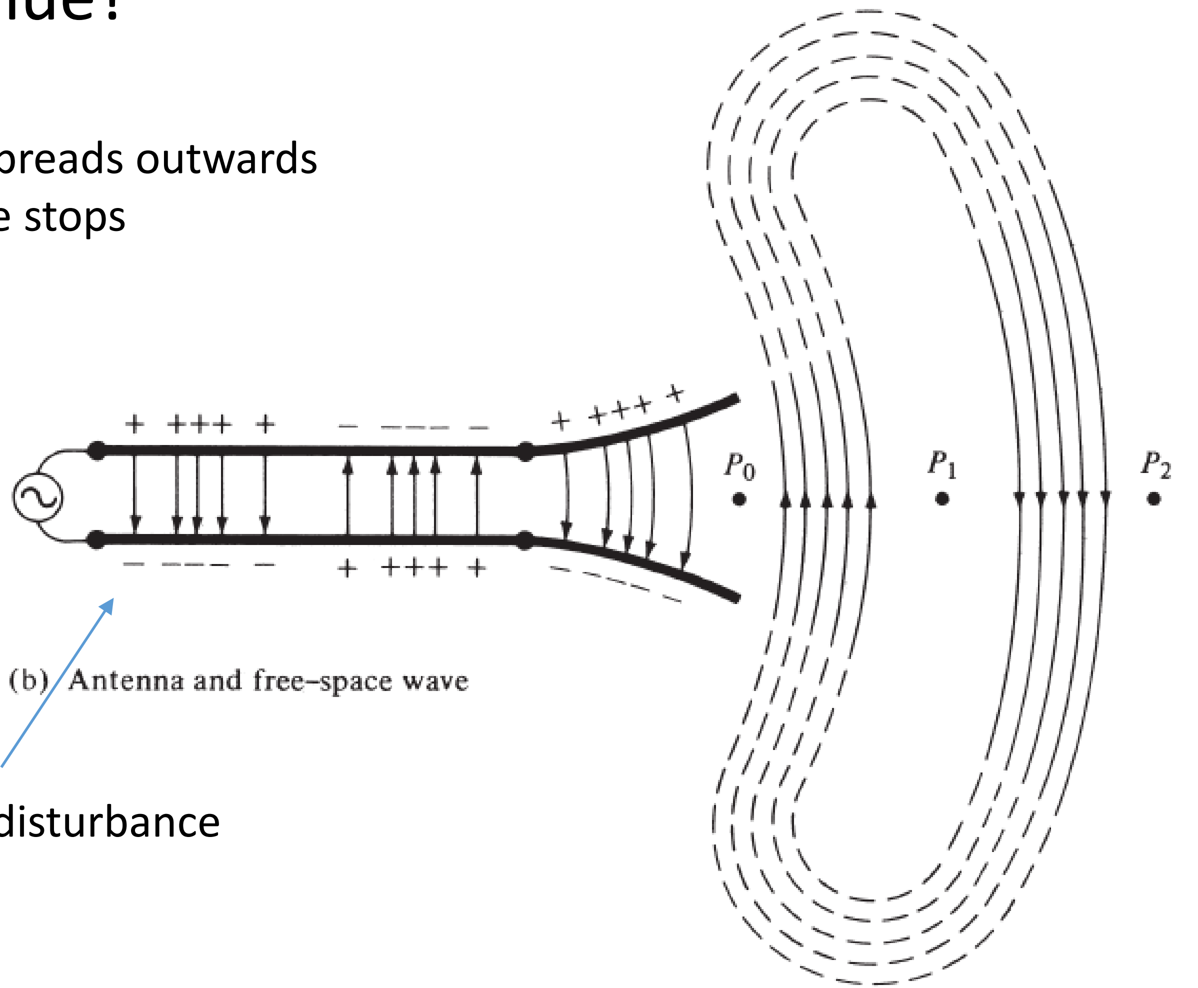
(b) Antenna and free–space wave

From one wire to two ...

- Why do the waves continue?

Disturbance continues into free-space

Analogy: water wave spreads outwards even when disturbance stops



(b) Antenna and free-space wave

Source of disturbance

Questions?? Thoughts??

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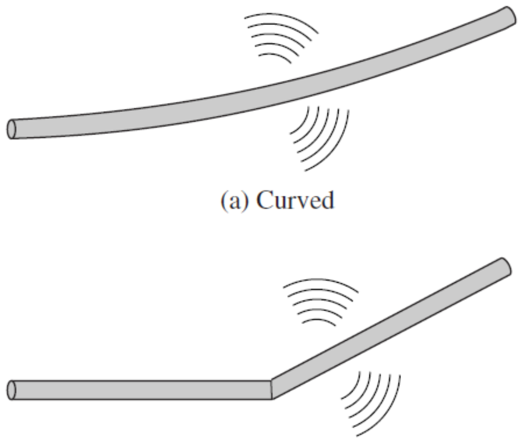
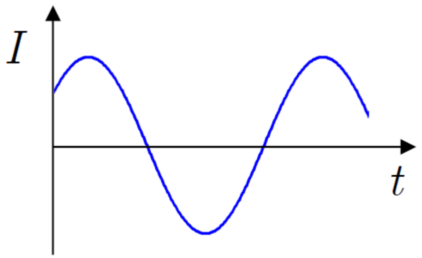
In the last lecture we saw that

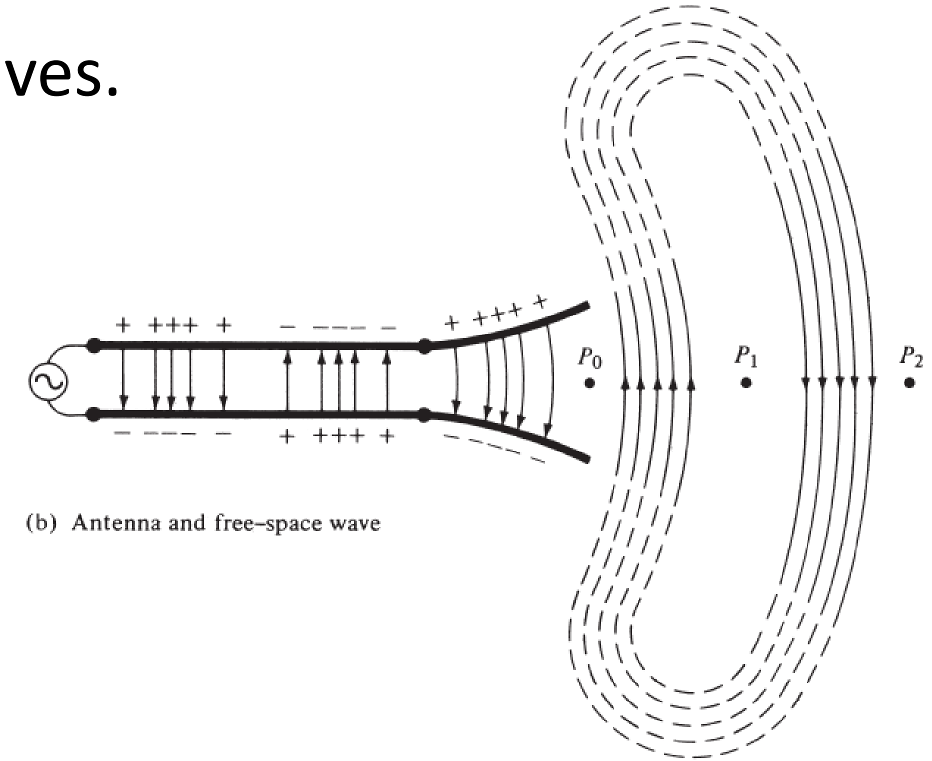
- How **time-varying current** and **acceleration of charges** in a **thin wire** are related.
- How a **two-wire antenna** radiates EM waves.



(a) Curved

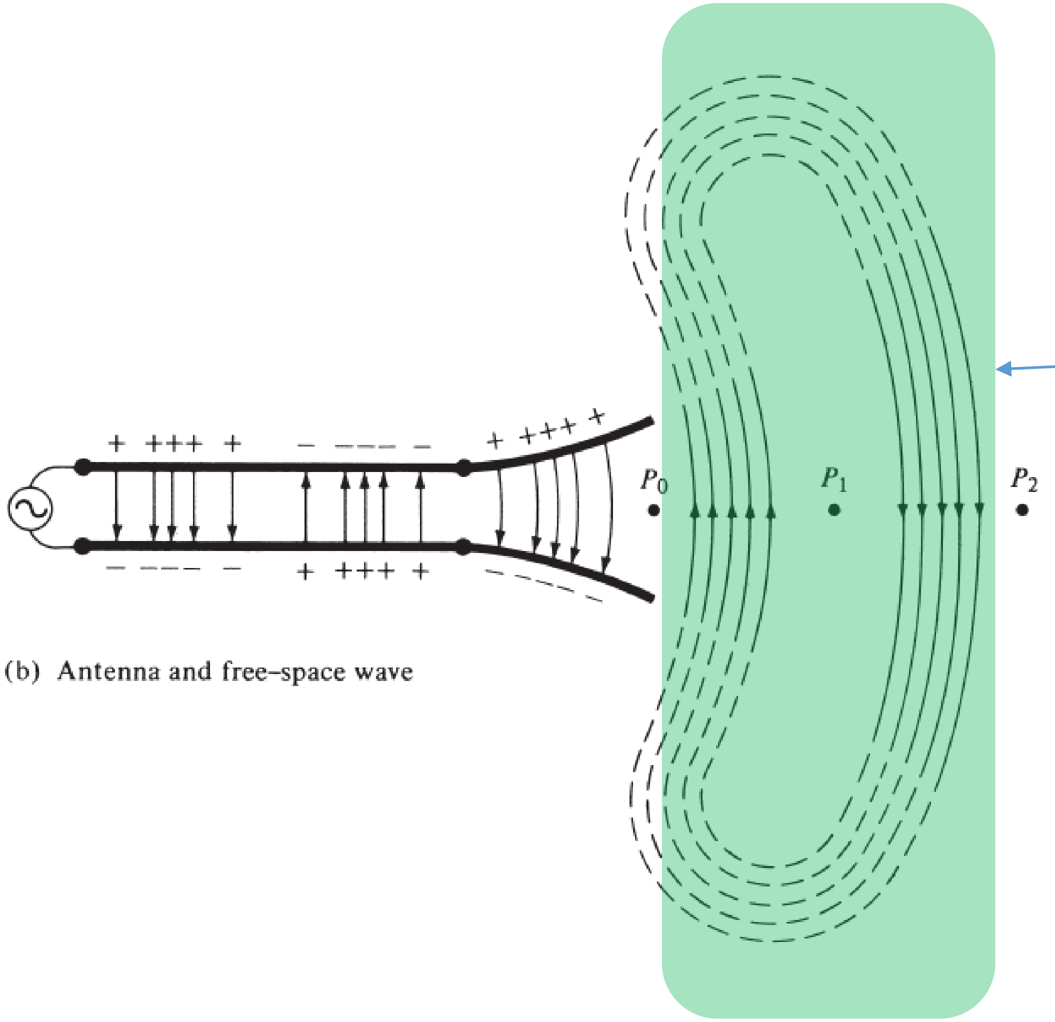
(b) Bent





(b) Antenna and free-space wave

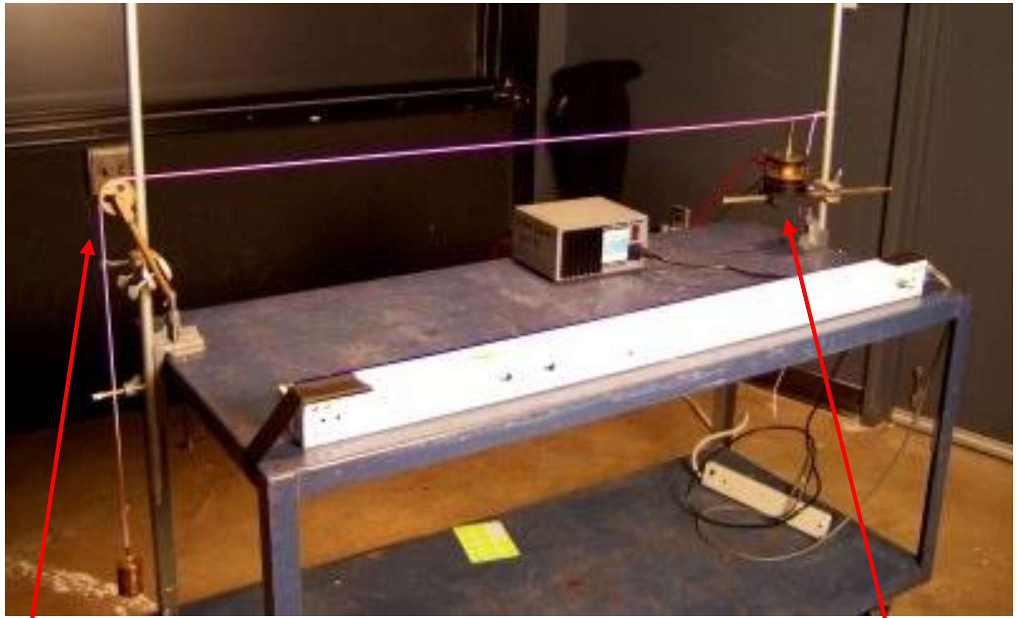
Next …

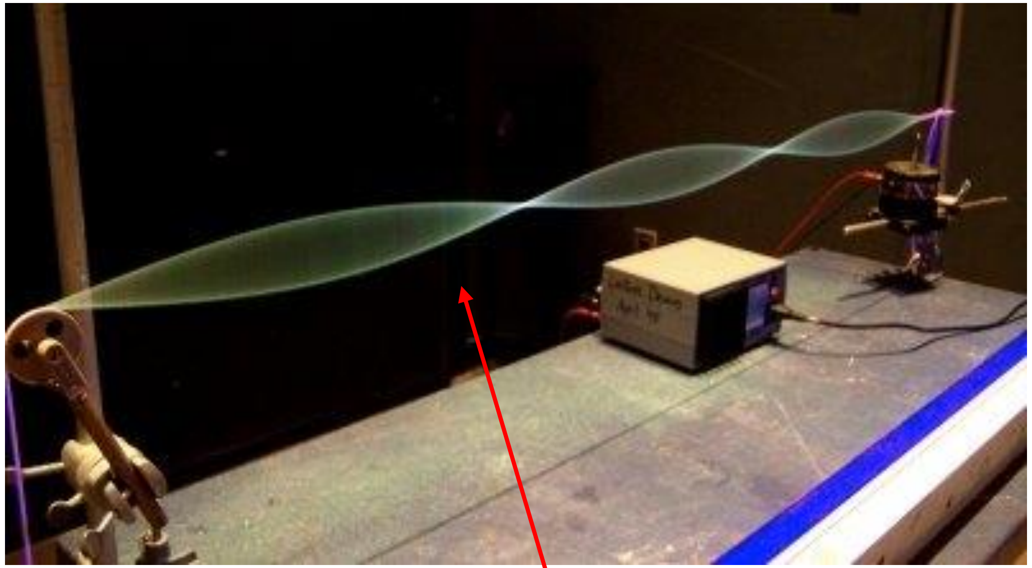


(b) Antenna and free–space wave

We will now focus on the radiated fields (strength, density, direction etc.)

Standing Waves



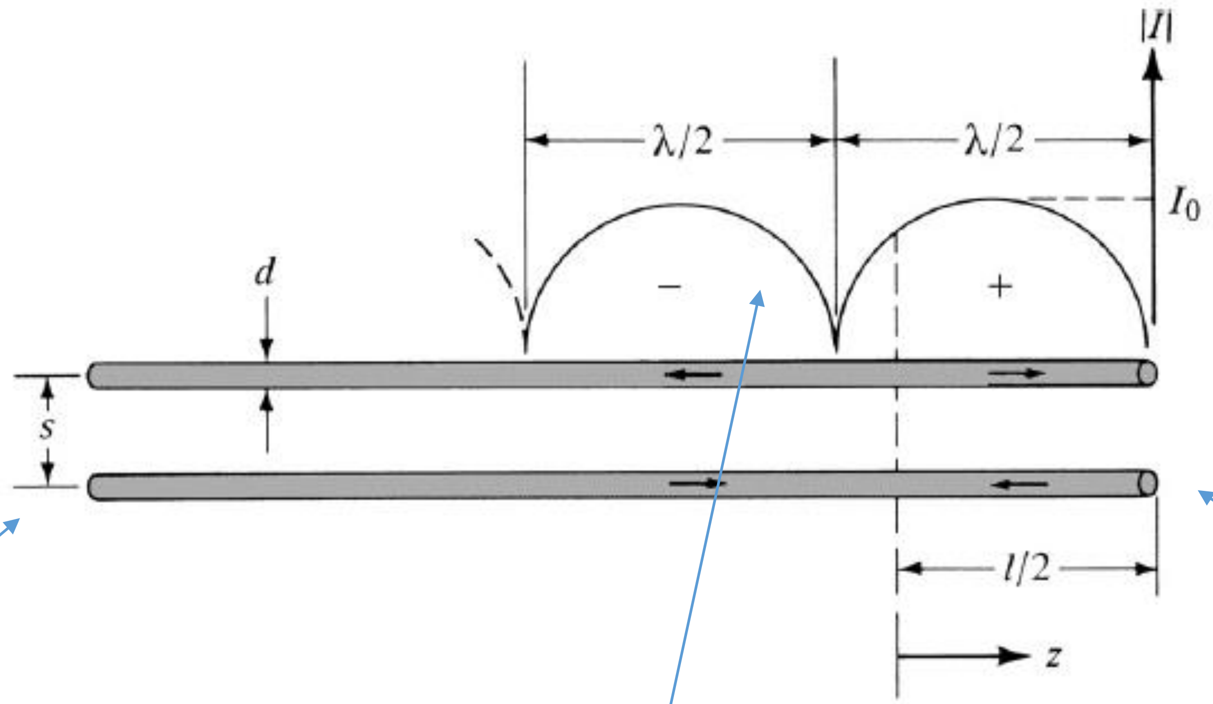


Tied or loaded end

Oscillating disturbance source

Result = standing waves in the string

Linear Dipole – Current Distribution and Radiation

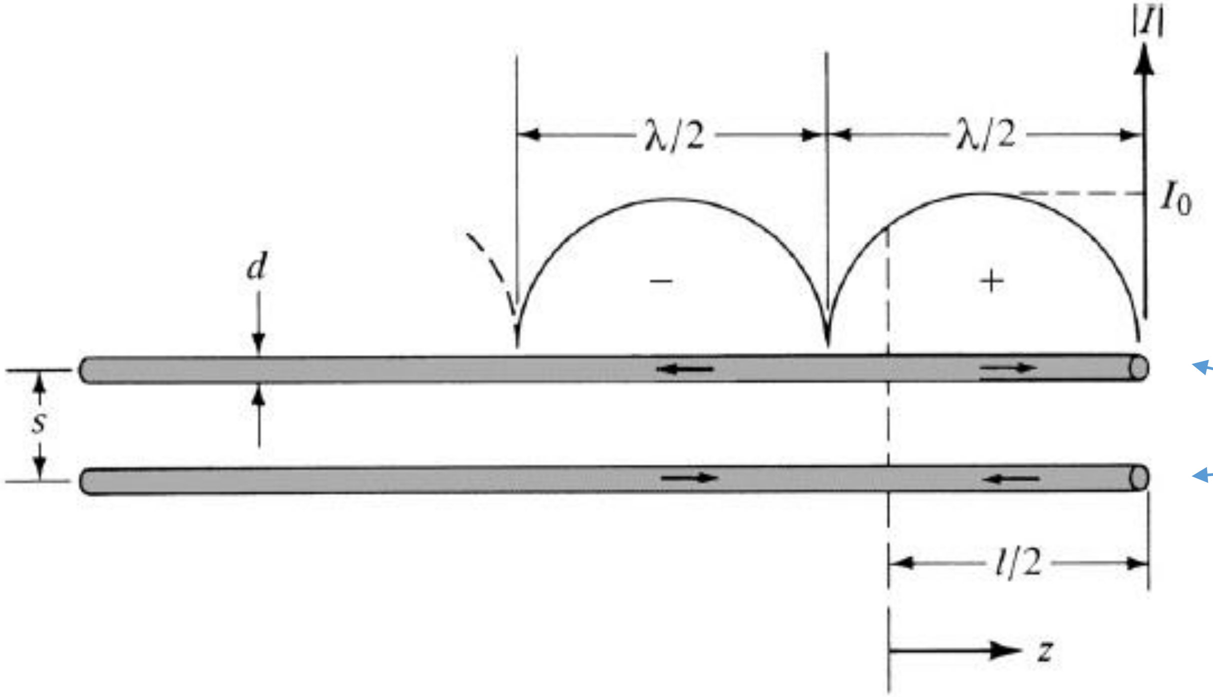


Alternating voltage (disturbance source) applied here

Together they create **standing waves** of current

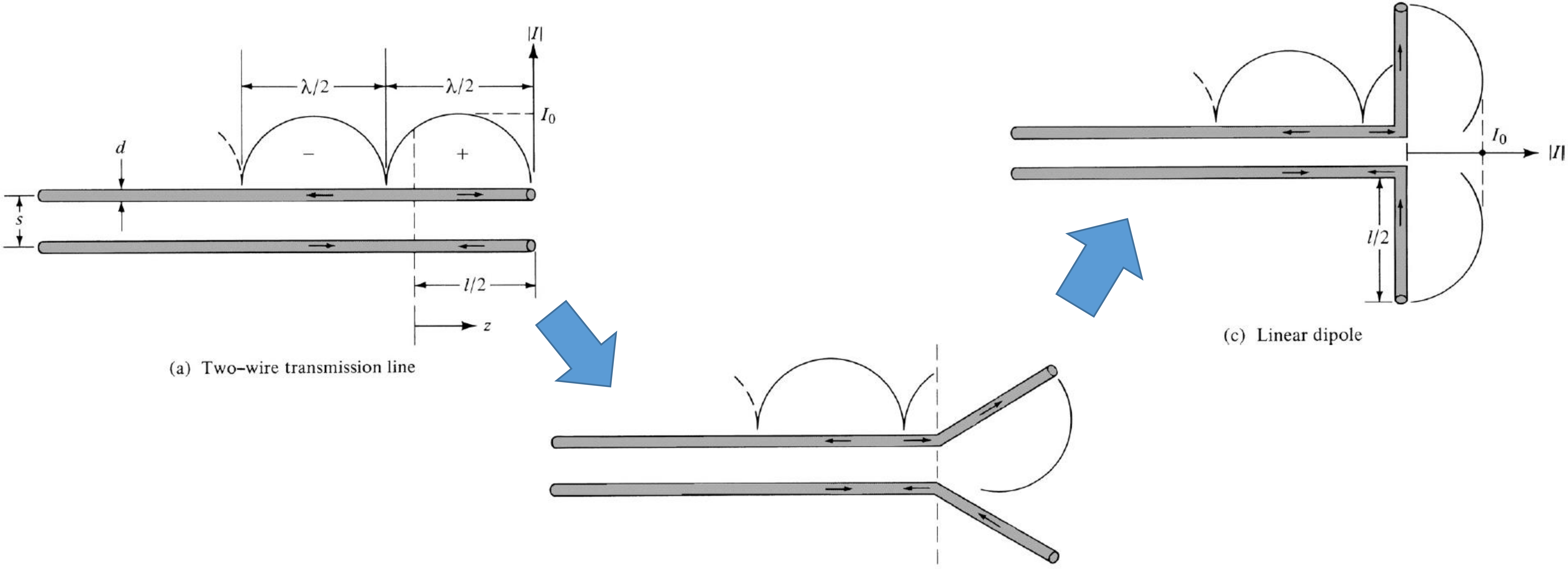
Dead-end causes charges to reflect back

Linear Dipole – Current Distribution and Radiation



What happens when we turn these ends outwards?

Linear Dipole – Current Distribution and Radiation

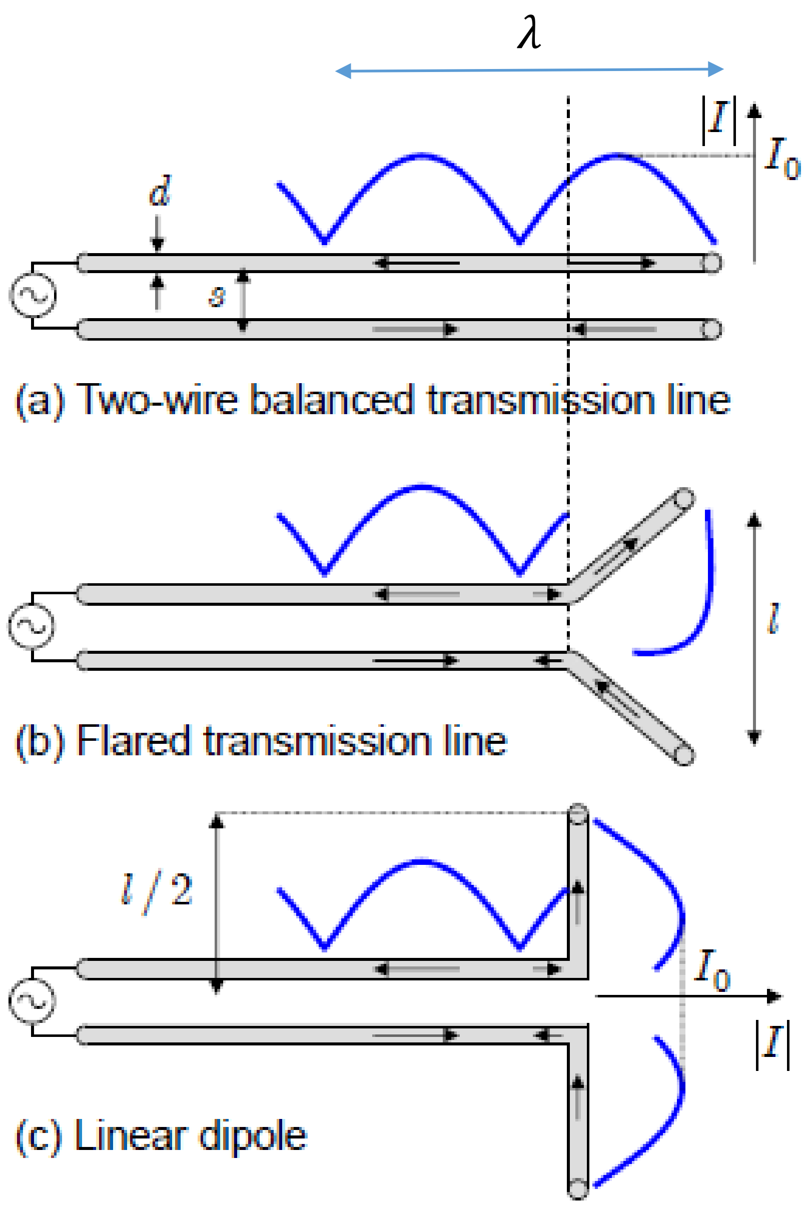


(a) Two–wire transmission line

(b) Flared transmission line

(c) Linear dipole

Summary

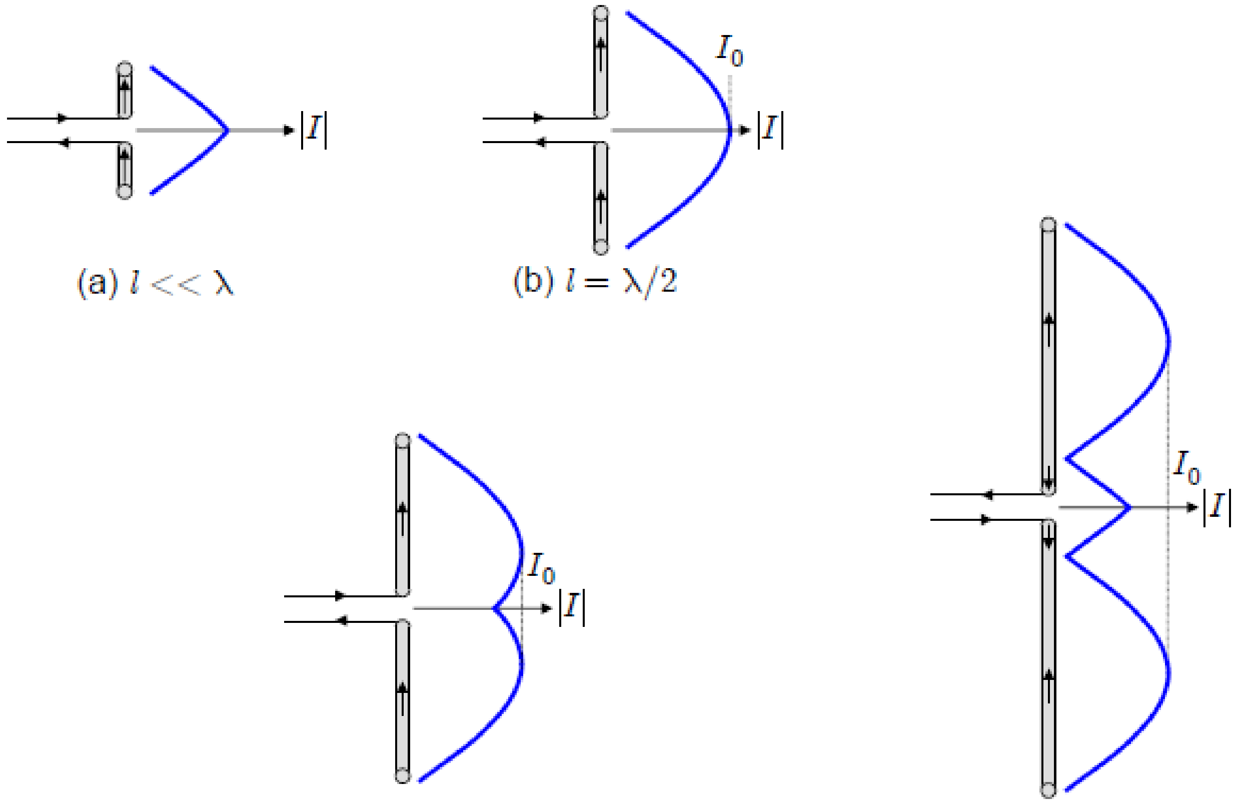


For a balanced line with $s \ll \lambda$, radiated fields from one wire are canceled by those from the other
→ No net power radiation

The two lines in the flared section are not necessarily close to each other $(l \not\ll \lambda)$
→ Net power radiation

Dipole radiates since both dipole arms reinforce each other toward most directions
$(l < \lambda)$

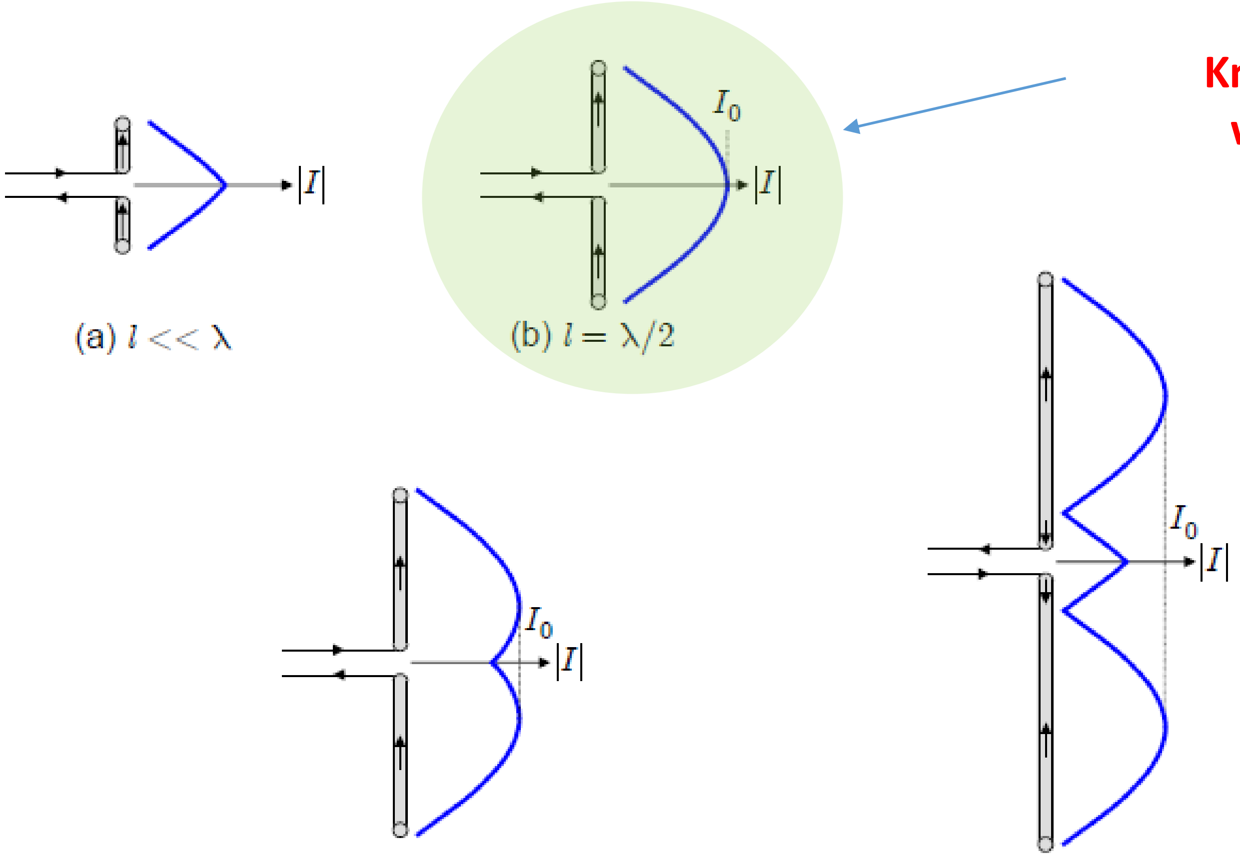
Linear Dipole – Current Distribution as a function of length



(a) $l << \lambda$ (b) $l = \lambda/2$

(c) $\lambda/2 < l < \lambda$ (d) $\lambda < l < 3\lambda/2$

Linear Dipole – Current Distribution as a function of length



(a) $l << \lambda$

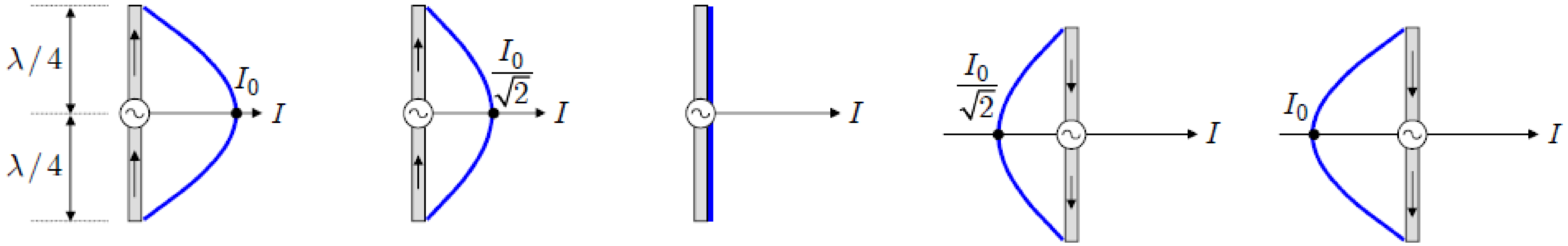
(b) $l = \lambda/2$

(c) $\lambda/2 < l < \lambda$

(d) $\lambda < l < 3\lambda/2$

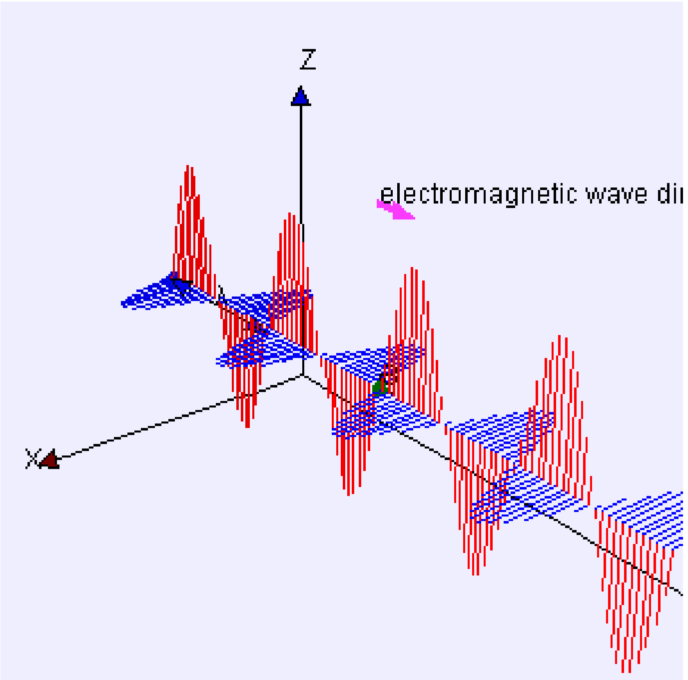
Known as a $\lambda/2$ wire antenna

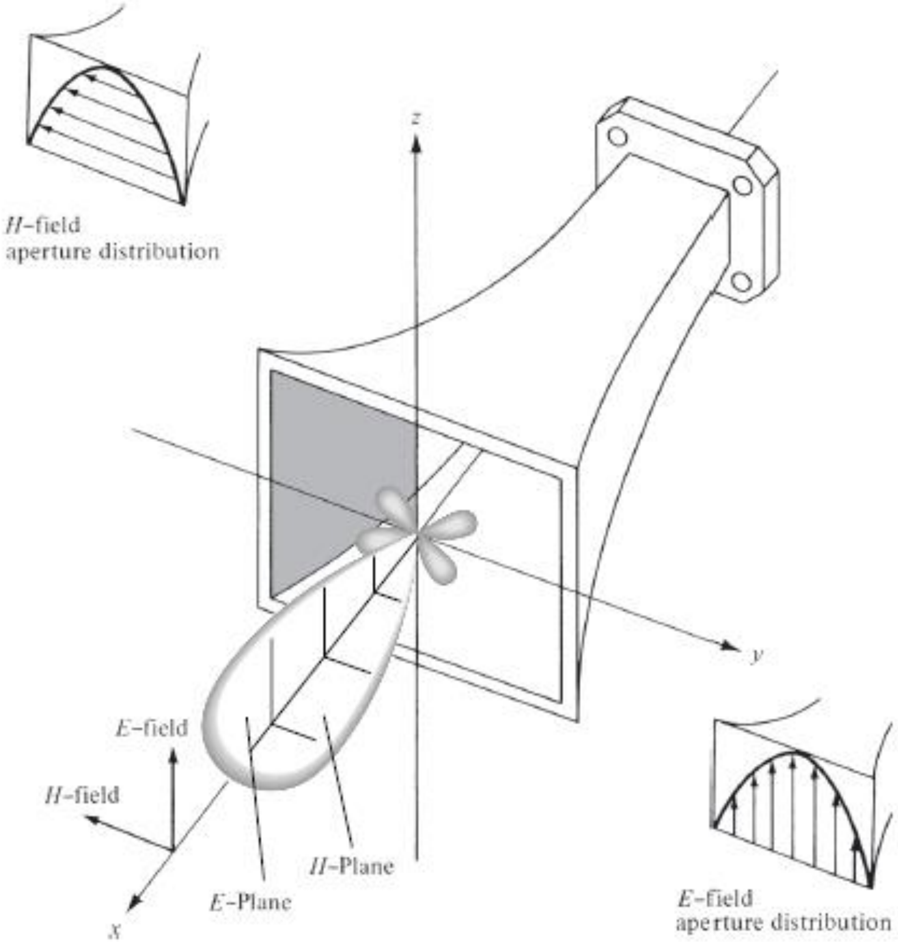
$\lambda/2$ Linear Dipole – Current Distribution with Time



(a) $t = 0$ (b) $t = T/8$ (c) $t = T/4$ (d) $t = 3T/8$ (e) $t = T/2$

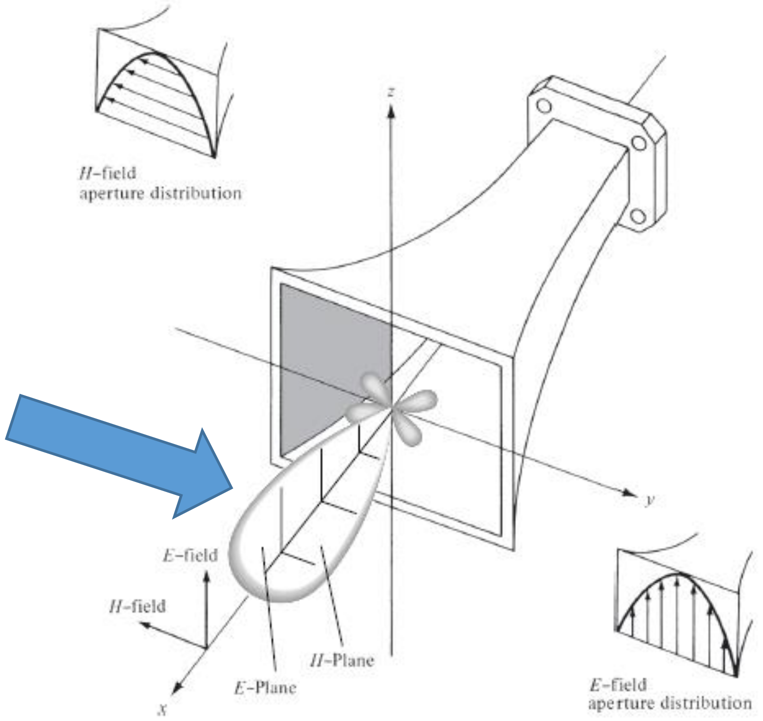
Radiation Pattern





Radiation Pattern

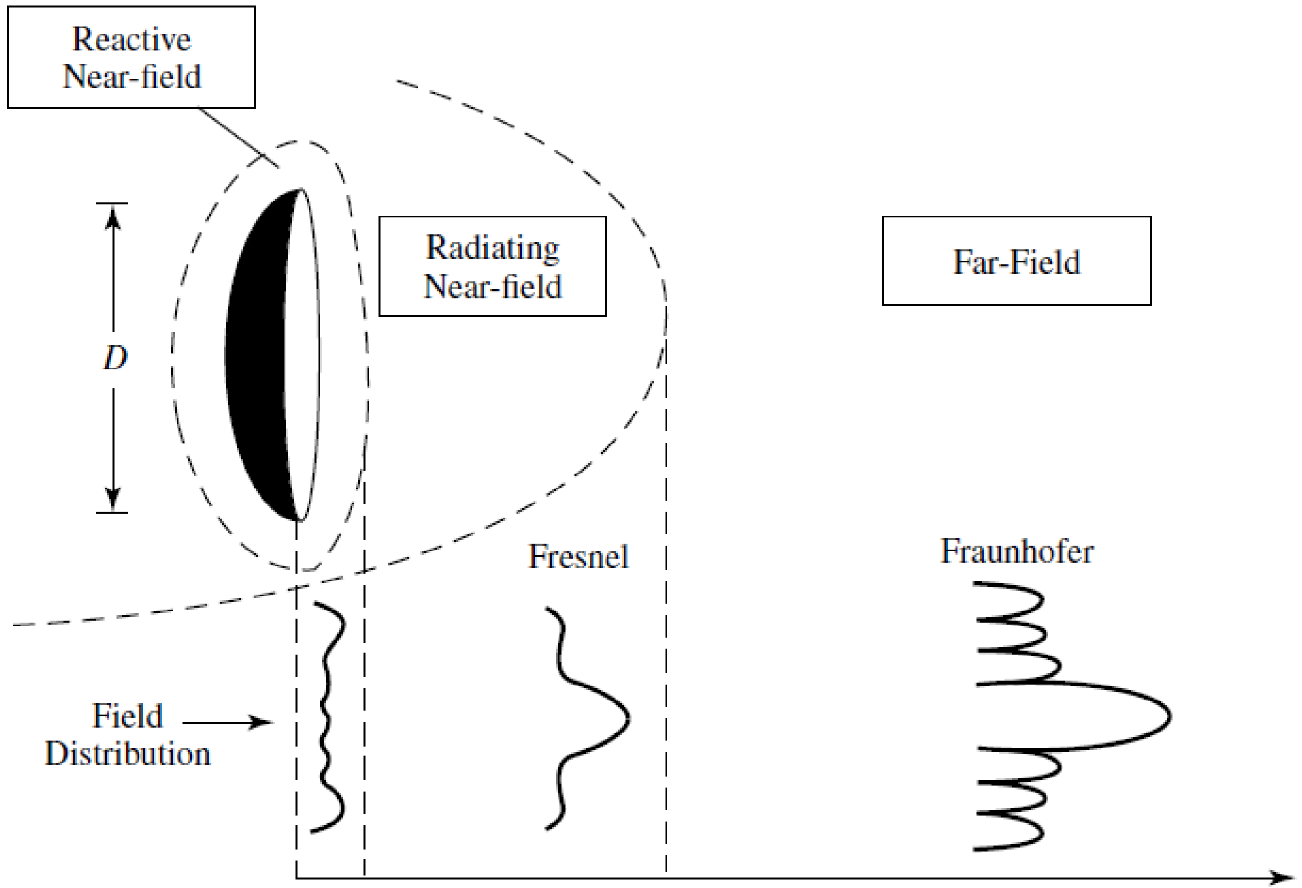
- Next, we focus more generally on the following
  - What different **shapes** (patterns) the radiated energy may take after it has left the antenna?
  - How we can **characterize** those shapes?
    - Direction, Intensity, etc.



Near and Far …

Flow behavior (and pattern) varies between regions

Antenna: Three Regions



Near and Far …

- We usually split the region around an antenna into **three sections** (based on distance from the antenna)
  - The field distribution in the three regions is usually different
  - We are mostly interested in the third region (called **Far-Field**)

How do we set the region boundaries?

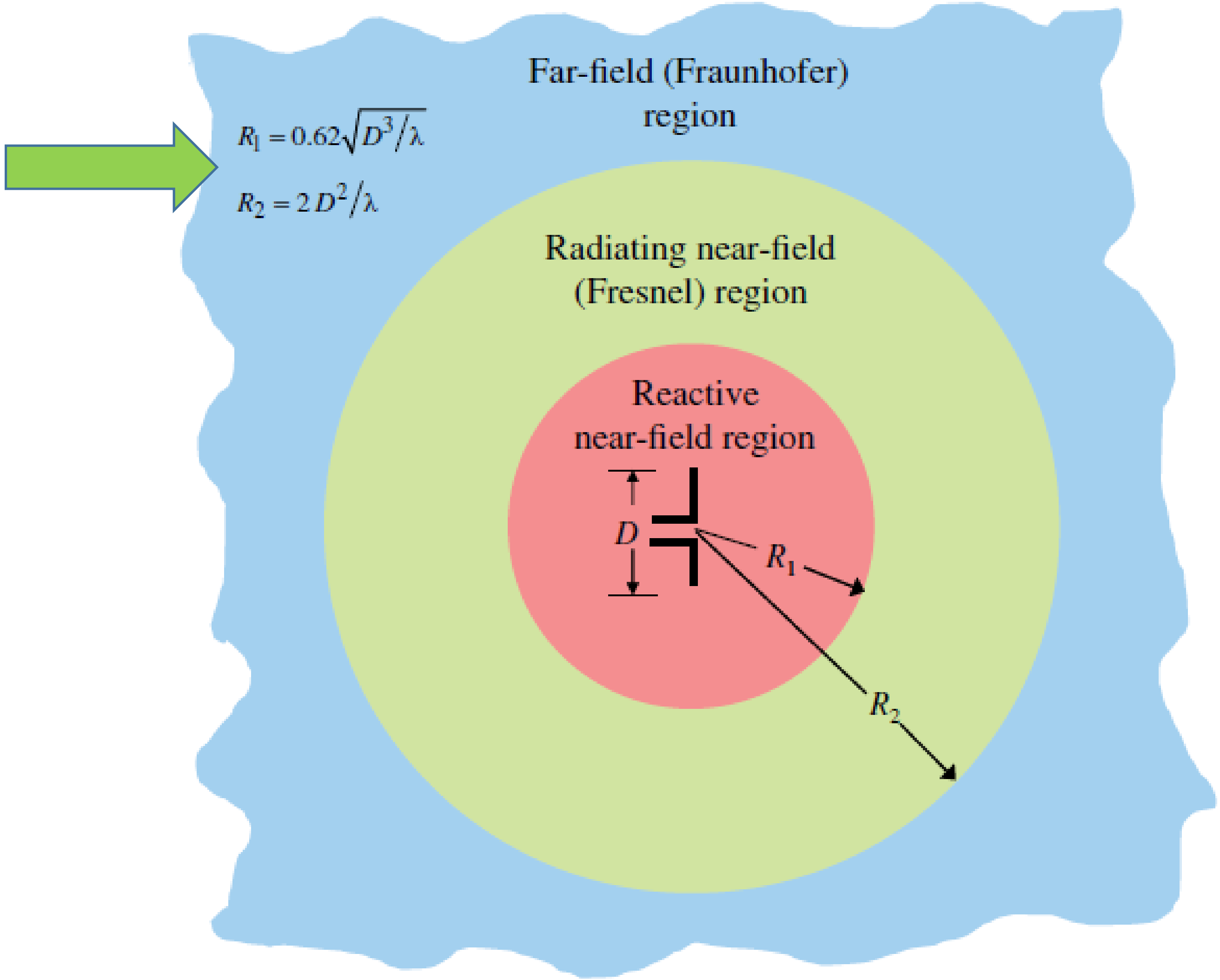


Figure 2.7 Field regions of an antenna.

Formal Definition

Radiation pattern (antenna pattern)

- Mathematical function or graphical representation of the radiation properties of an antenna.
- Can represent field strength or radiation intensity (see p. 12 ff).

Formal Definition

Radiation pattern (antenna pattern)

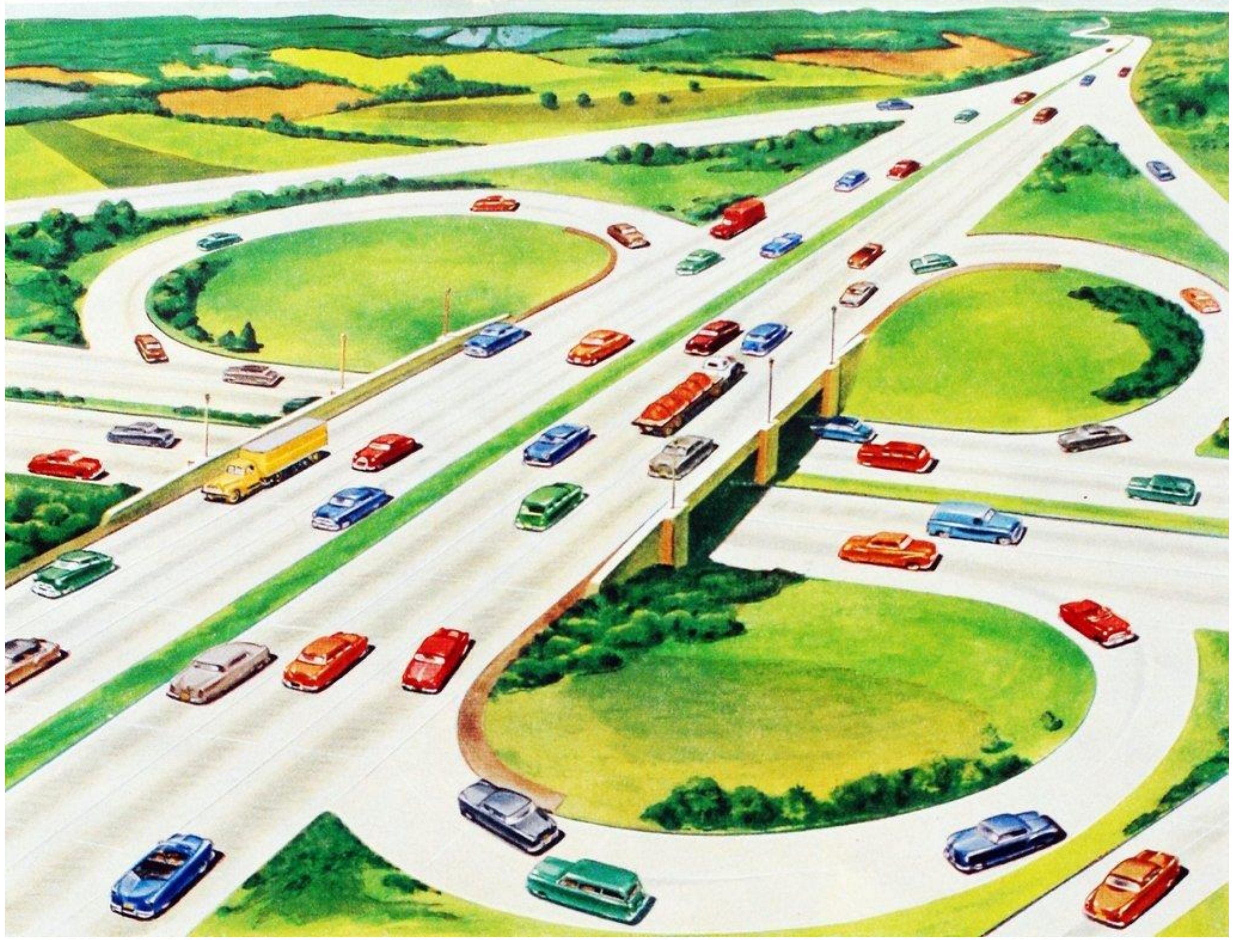
- Mathematical function or graphical representation of the radiation properties of an antenna.
- Can represent field strength or radiation intensity (see p. 12 ff).

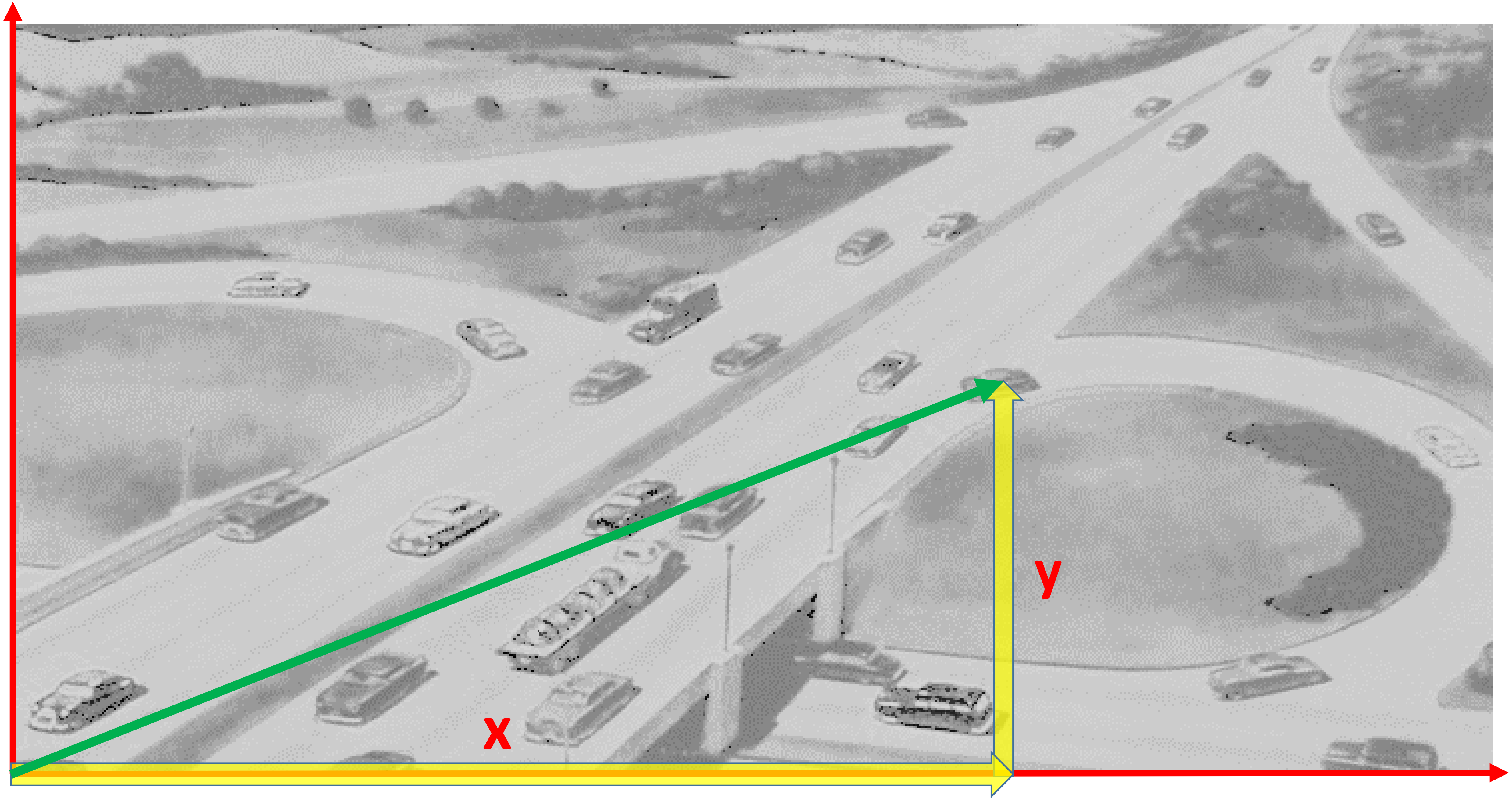
Next, we look at the following question:
Q. What can the mathematical notations and graphical depictions be?

Agreeing on a notation …

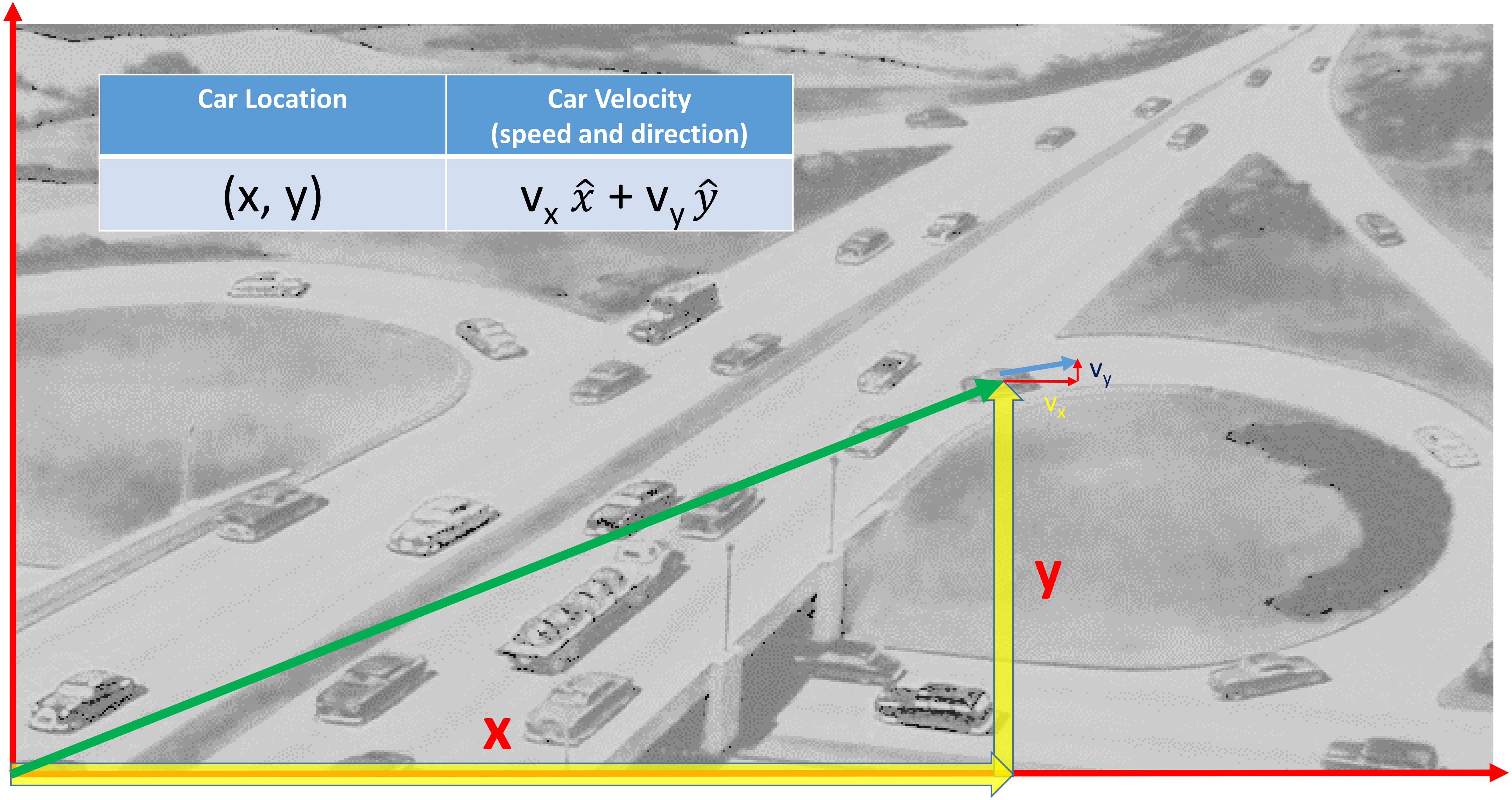
How to describe locations and velocities of the cars in the picture?

Good to agree on some common framework!

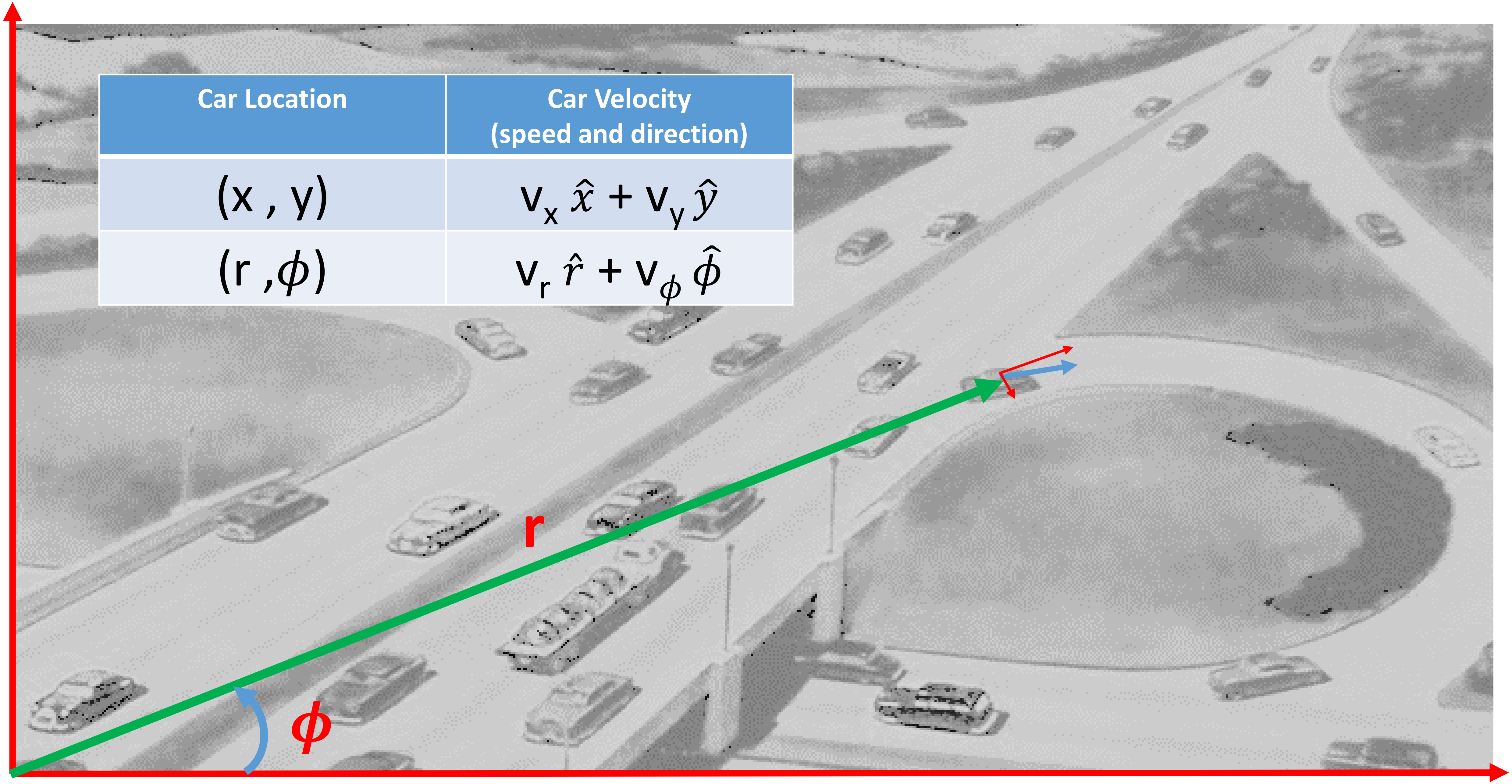



x
y

Car Location	Car Velocity (speed and direction)
(x, y)	$v_x \hat{x} + v_y \hat{y}$



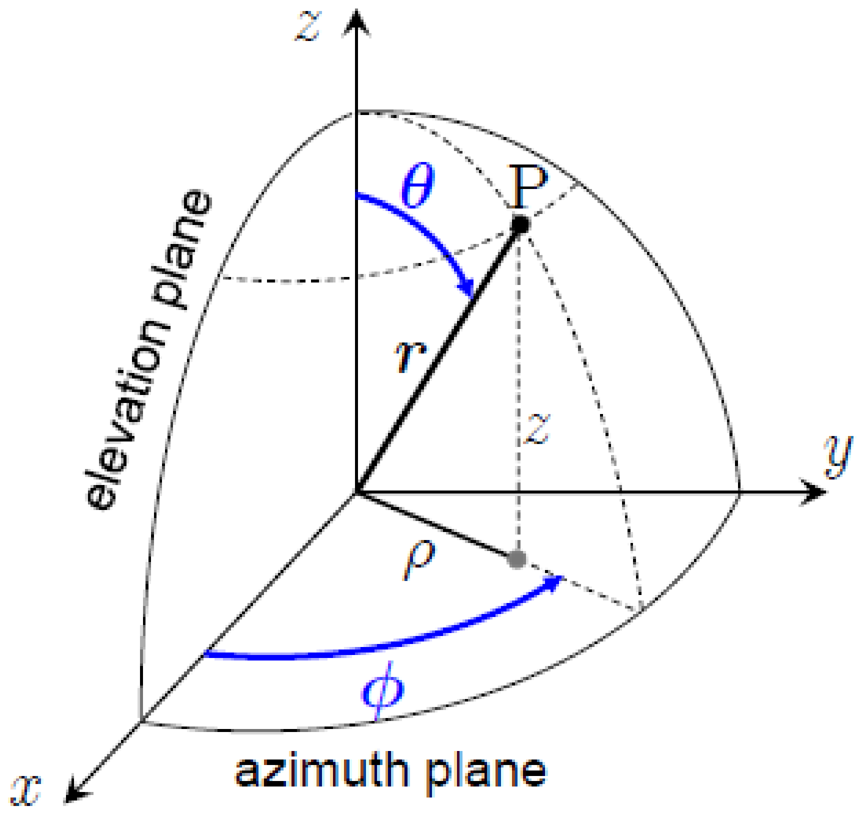
Car Location	Car Velocity (speed and direction)
$(x\ ,\ y)$	$v_x\,\hat{x} + v_y\,\hat{y}$
$(r\ ,\phi)$	$v_r\,\hat{r} + v_\phi\,\hat{\phi}$



For 3D Space: add direction $\boldsymbol{z}$ or angle $\boldsymbol{\theta}$

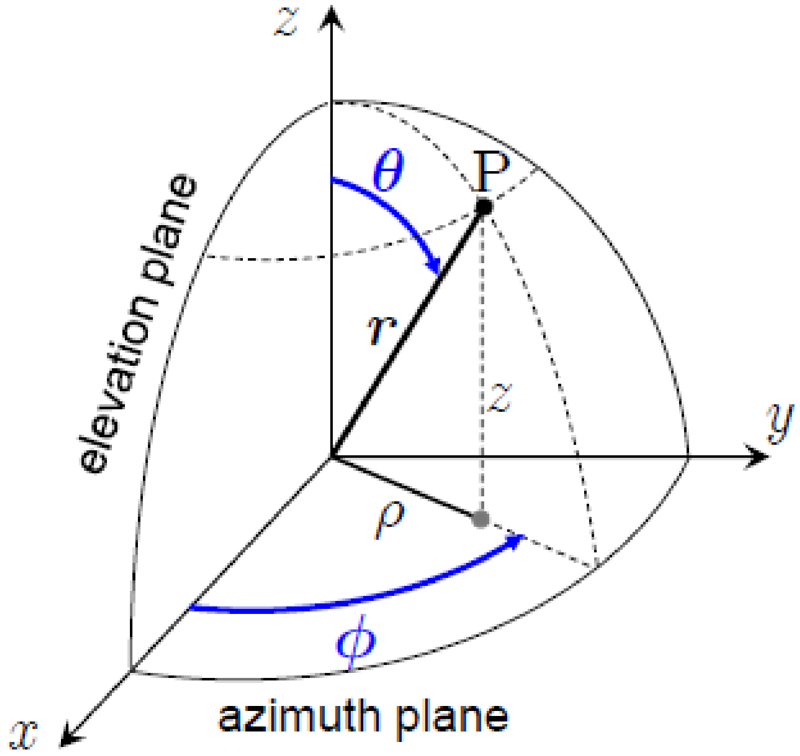
Cartesian Coordinates: $< x, y, z >$

Spherical Coordinates: $< r, \phi, \theta >$



Directions in space are described by two angles ϕ, θ $\begin{cases} \phi: & \text{azimuth} \\ 90^\circ - \theta: & \text{elevation} \end{cases}$

Relation between Cartesian and Spherical Coordinates



$$\boxed{\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned}}$$

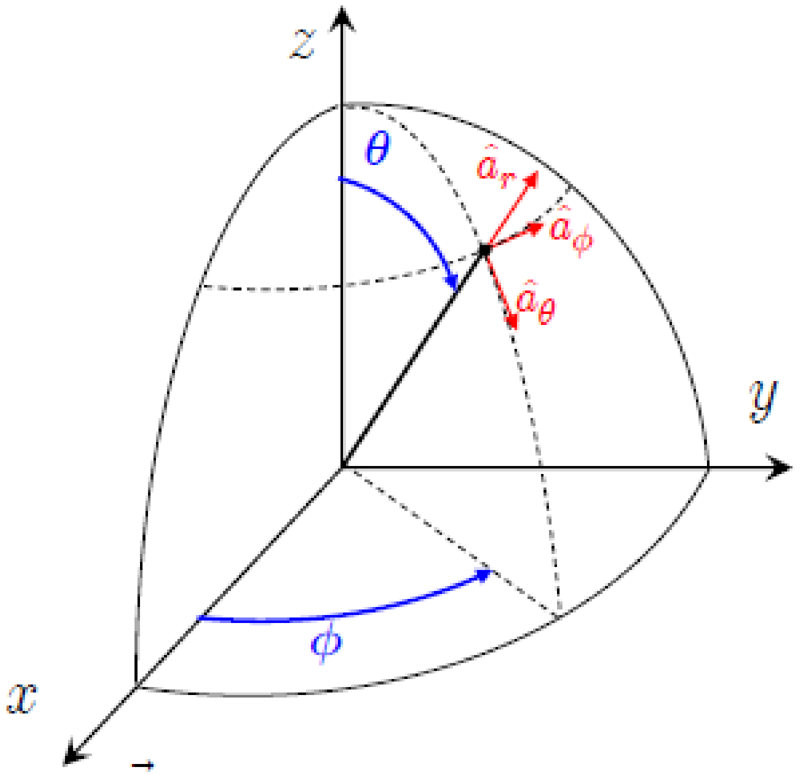
$$\rho = \sqrt{x^2 + y^2} = r\sin\theta \quad \begin{cases} x = \rho\cos\phi \\ y = \rho\sin\phi \end{cases}$$

$$\boxed{\begin{aligned} & r = \sqrt{x^2 + y^2 + z^2} \\ & \cos\theta = \frac{z}{r}, \qquad z = r\cos\theta \\ & \tan\phi = \frac{y}{x} \end{aligned}}$$

Five Points about Depiction and Notation

1. We will **use vectors** to represents fields in space
2. We may use **Cartesian** coordinates or **Spherical** Coordinates to represent the vectors in 3D

So, for example



$$\vec{E} = E_r \hat{a}_r + E_\theta \hat{a}_\theta + E_\phi \hat{a}_\phi$$

Five Points about Depiction and Notation

1. We will **use vectors** to represents fields in space
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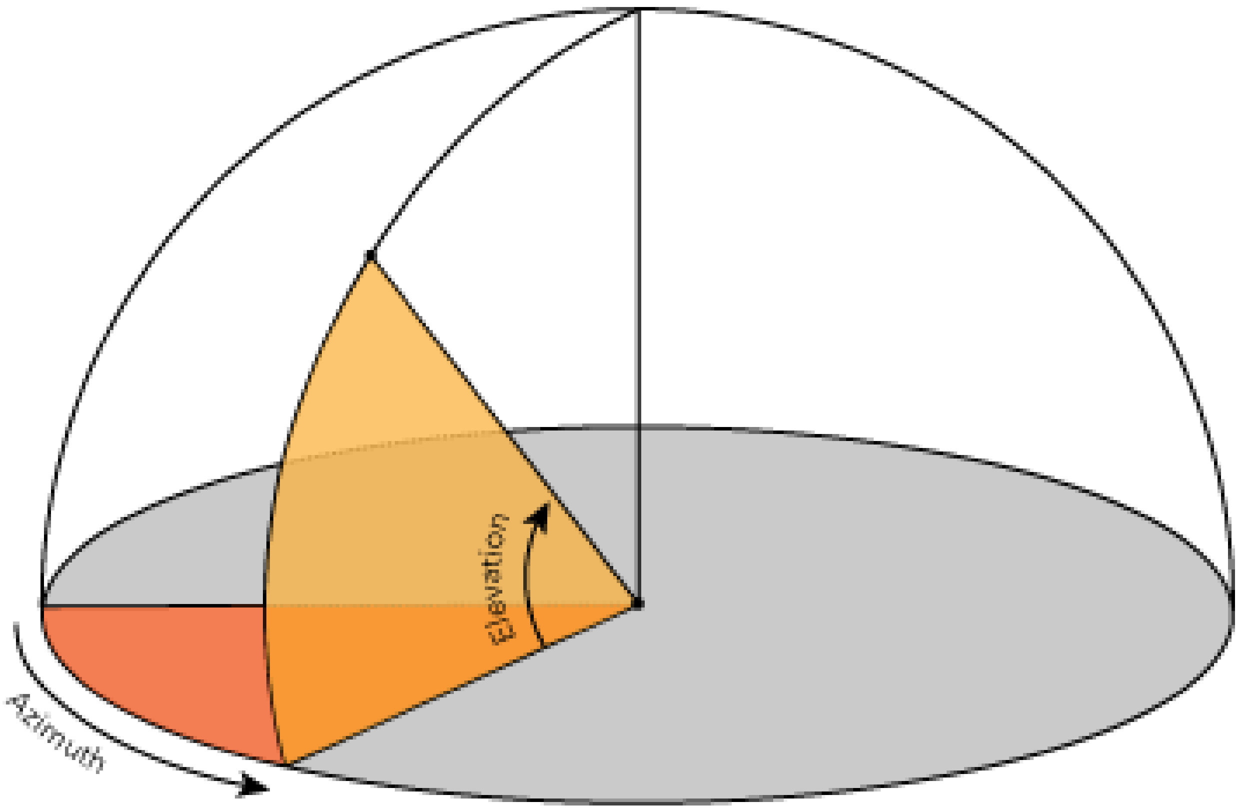
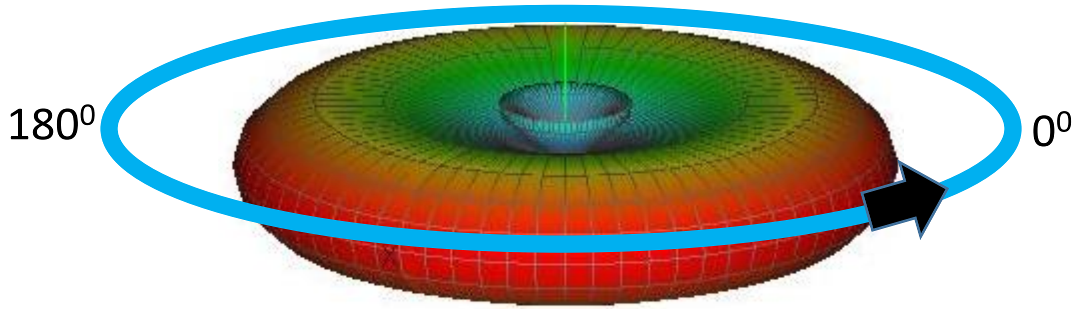
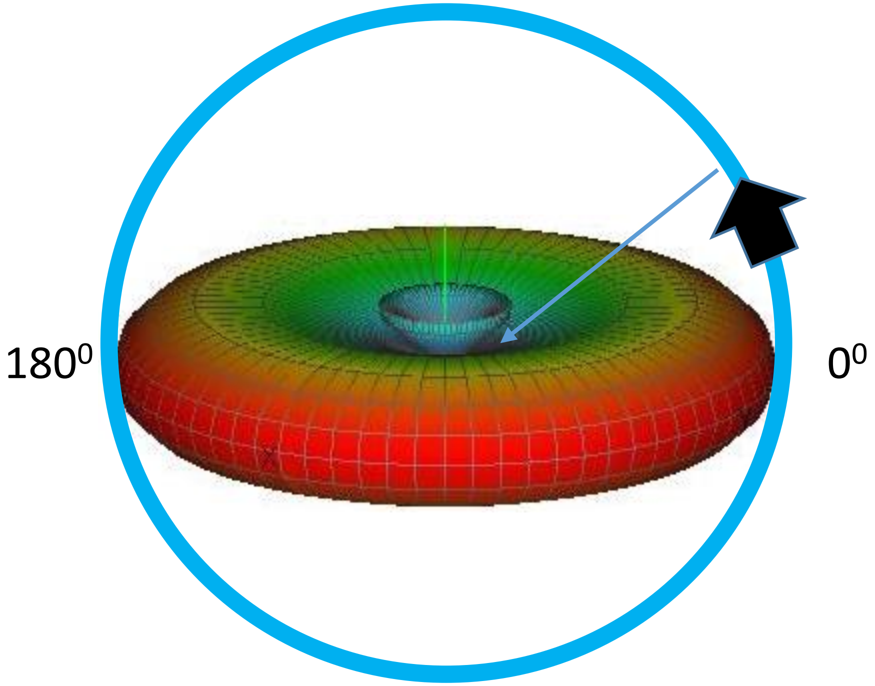
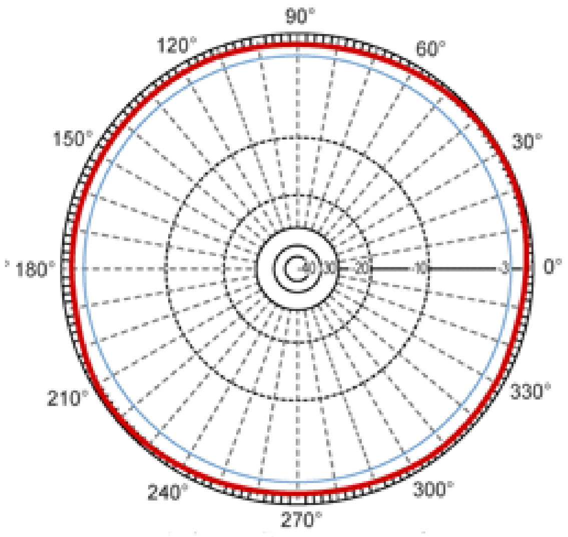
Azimuth and Elevation



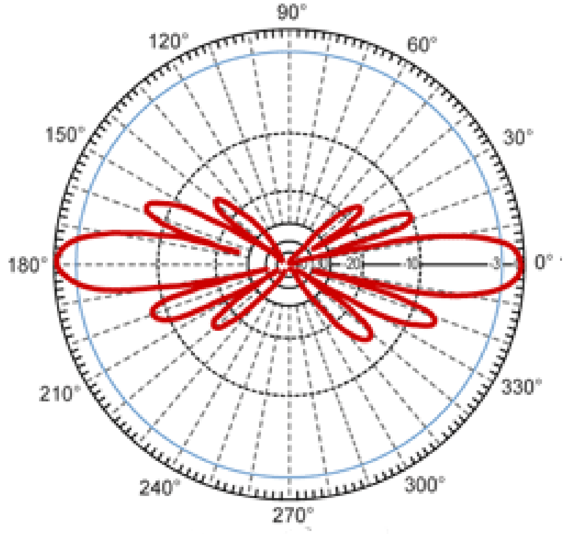




AZIMUTH PATTERN



ELEVATION PATTERN



Five Points about Depiction and Notation

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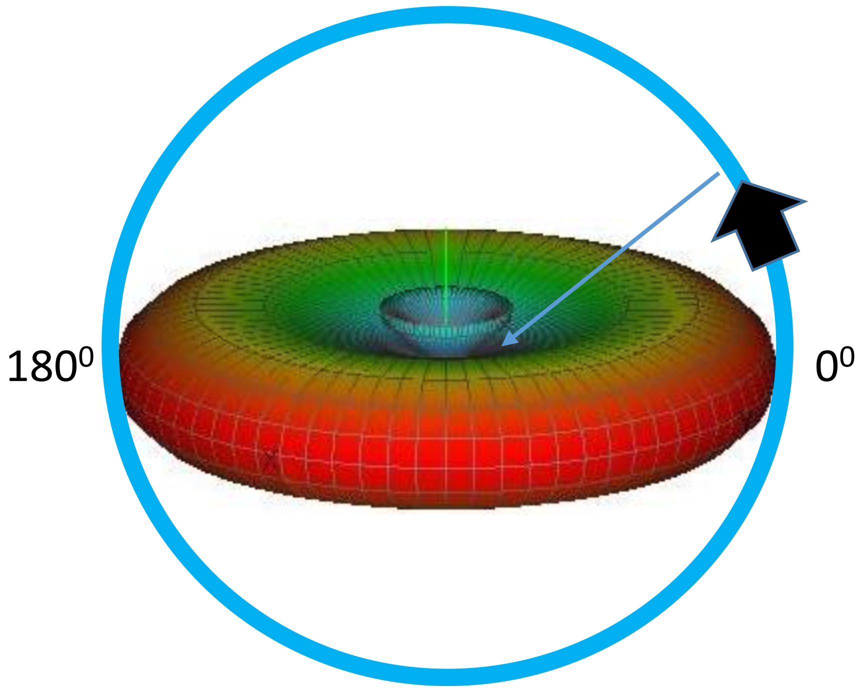
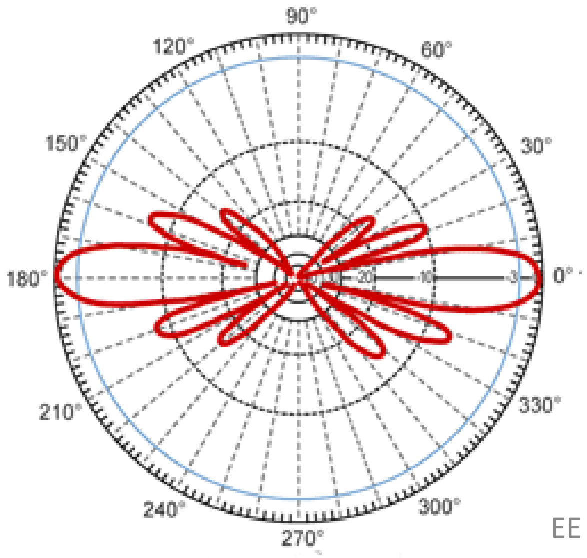
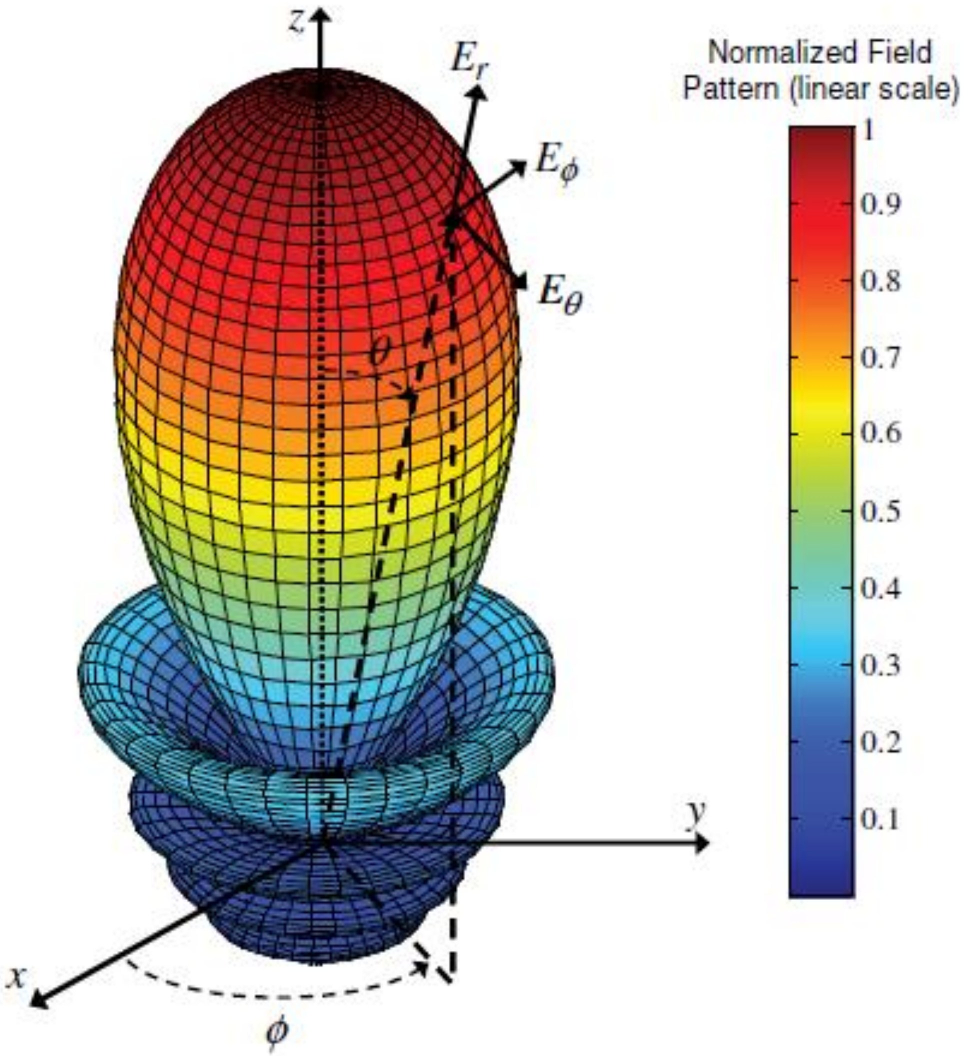


ELEVATION PATTERN





Five Points about Depiction and Notation

1. We will **use vectors** to represents fields in space
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4. We may also show **relative strength** of a field by **color** or by **size**
5. We may use **linear** scale or **log (dB)** scale

Linear and Log (dB) Scales

Linear Value	dB Value
1e-10	-100 dB
1e-9	-90 dB
1e-6	-60 dB
1e-3	-30 dB
1e-1	-10 dB
0.5	-3 dB
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10	10 dB
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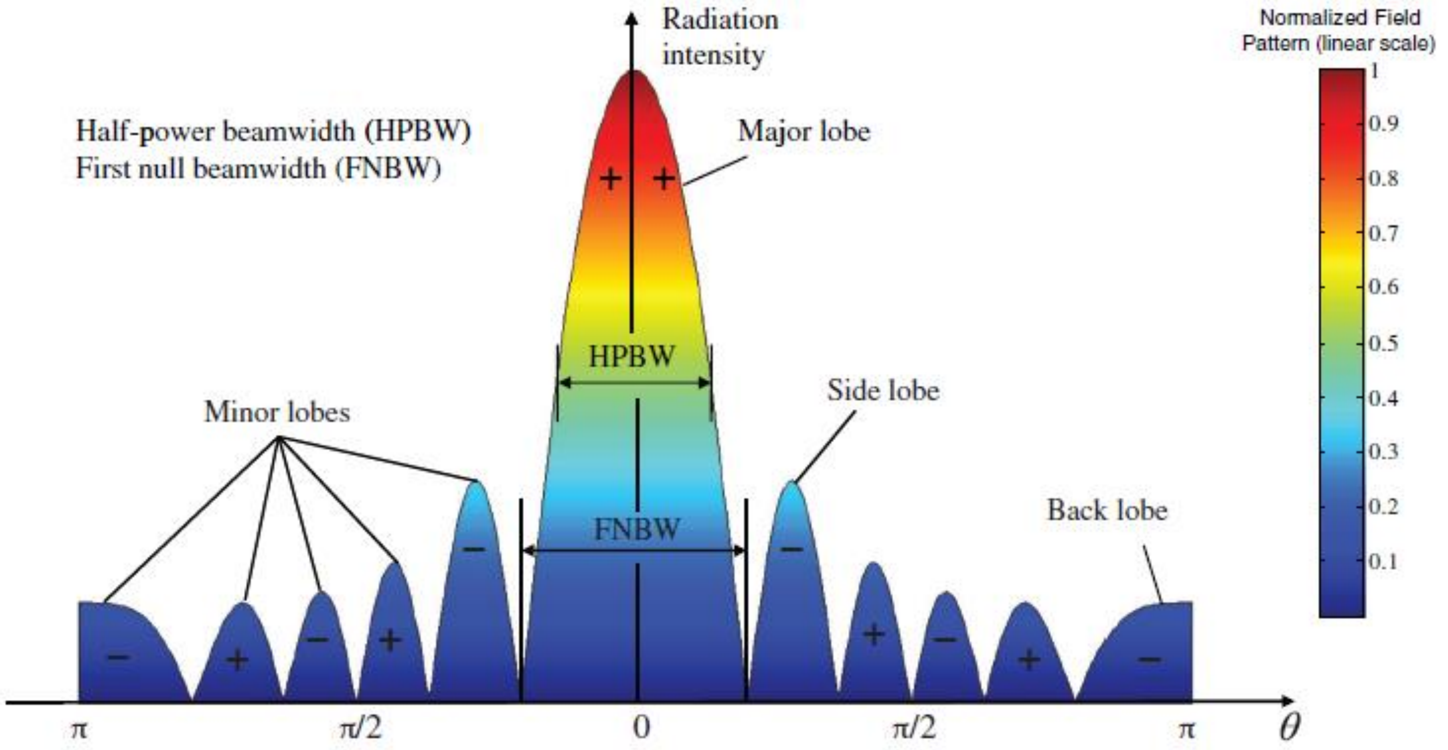
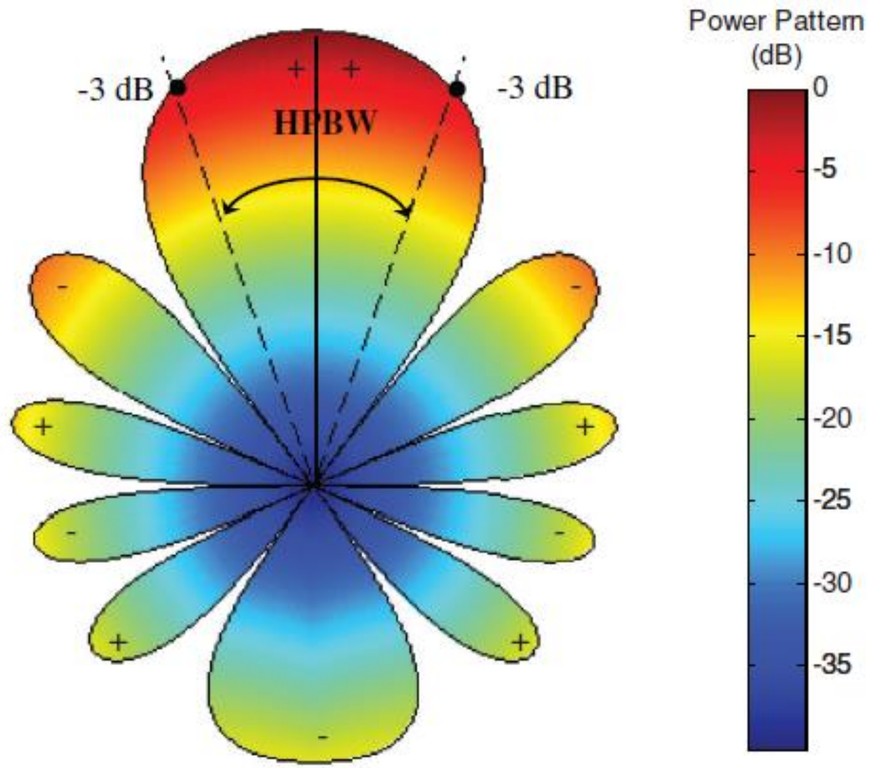
1e6	60 dB

$$\boxed{P_{dB} = 10 \log_{10} P}$$

Power in linear scale

Linear and Log (dB) Scales



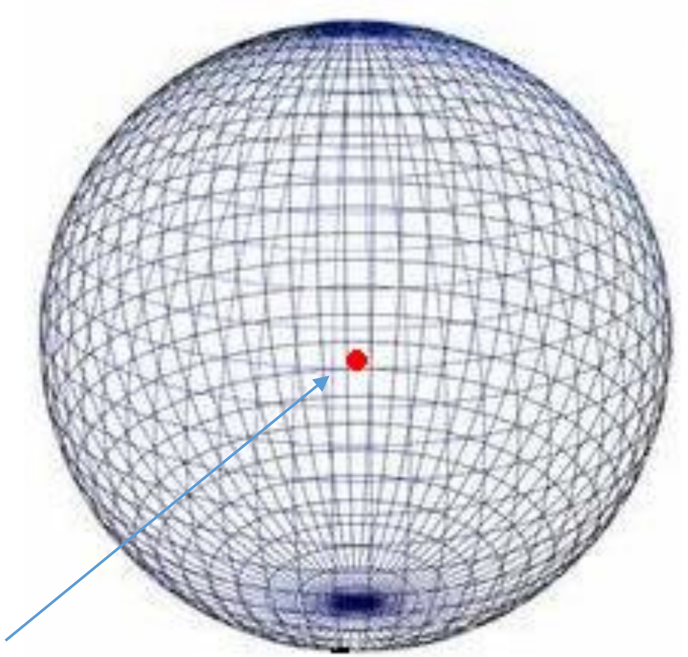


Three important types of radiation pattern

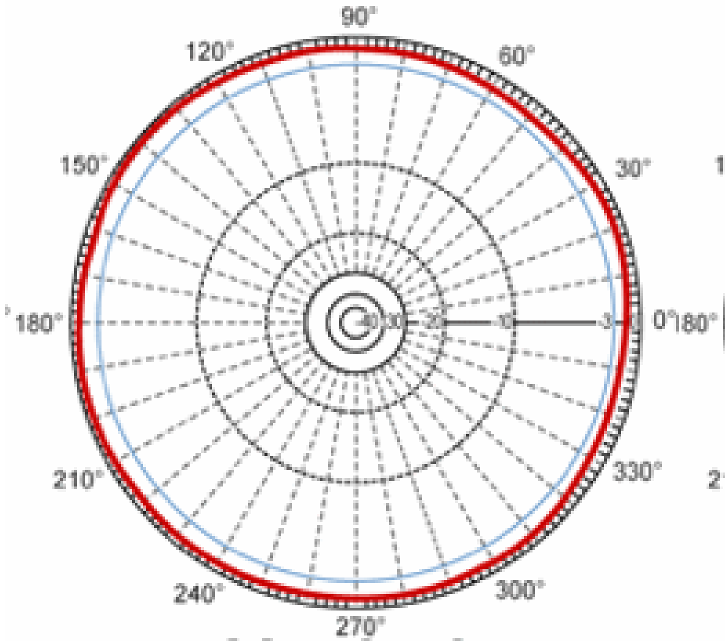
1. Isotropic
2. Omnidirectional
3. Directional

Isotropic = equal radiation in **all** directions

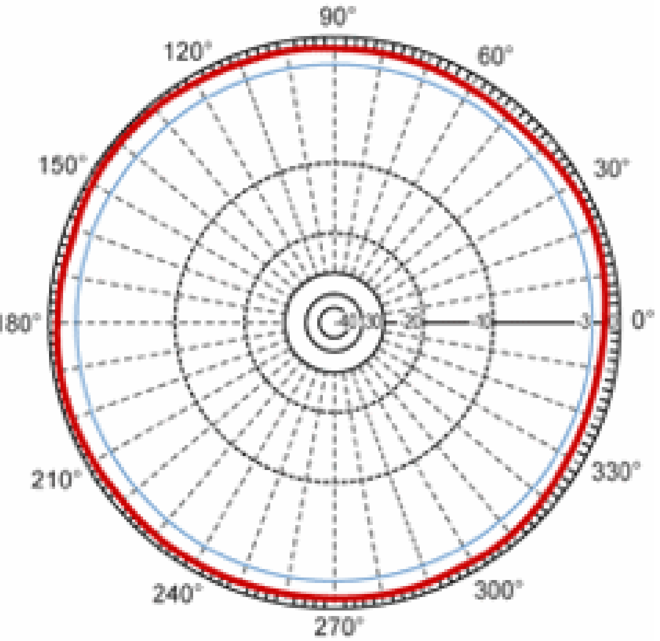
- Not possible in practice
- Used only as a reference pattern



antenna



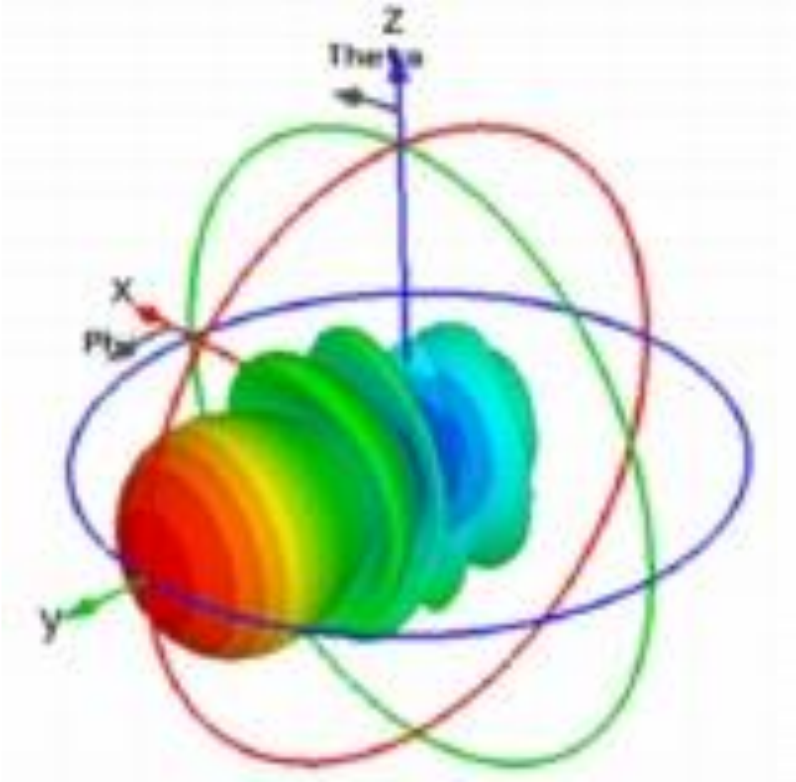
Elevation

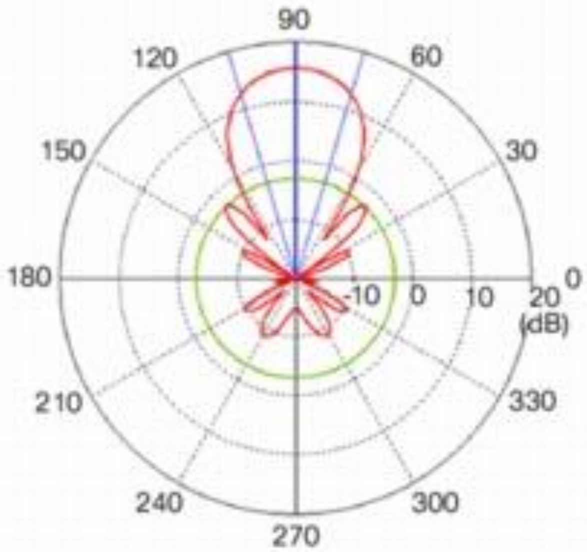


Azimuth

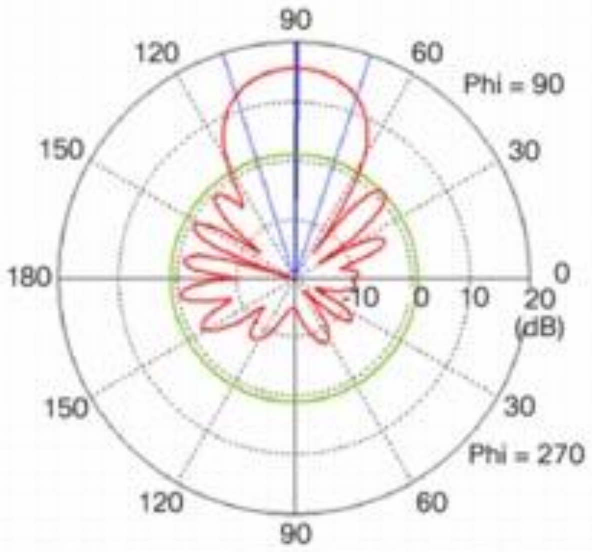
Directional = unequal radiation in various directions

A directional antenna transmits or receives more effectively from some directions compared to others





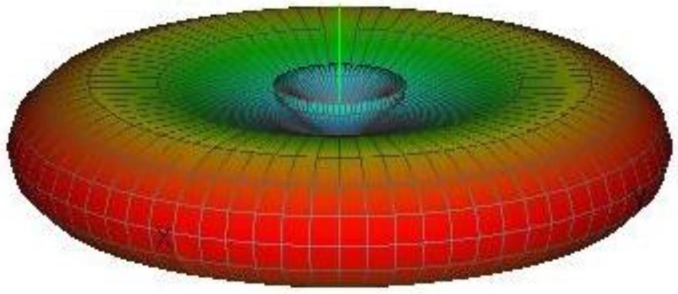
(c) Yagi Antenna Azimuth Plane Pattern



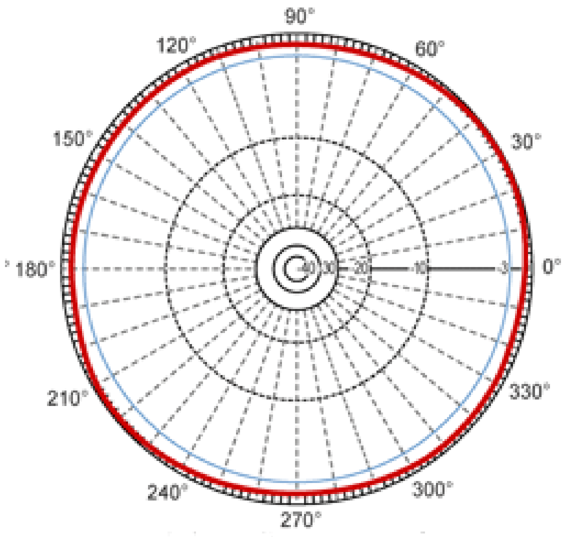
(d) Yagi Antenna Elevation Plane Pattern

Omnidirectional = special case of directional pattern

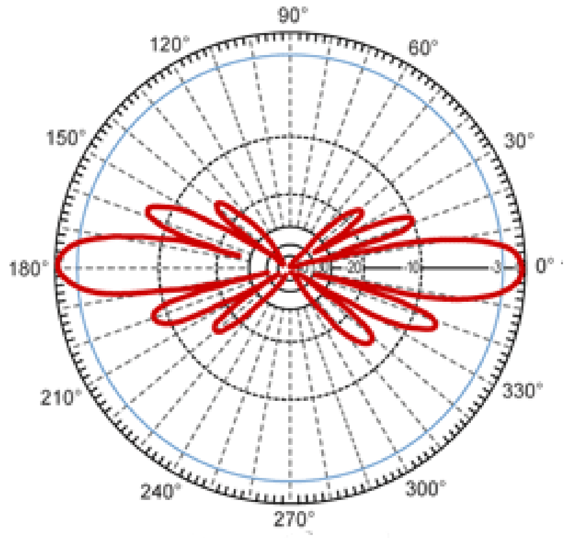
- Equal radiation in one plane (e.g., azimuth)
- Unequal radiation in orthogonal plane (e.g., elevation)



AZIMUTH PATTERN



ELEVATION PATTERN



Questions?? Thoughts??

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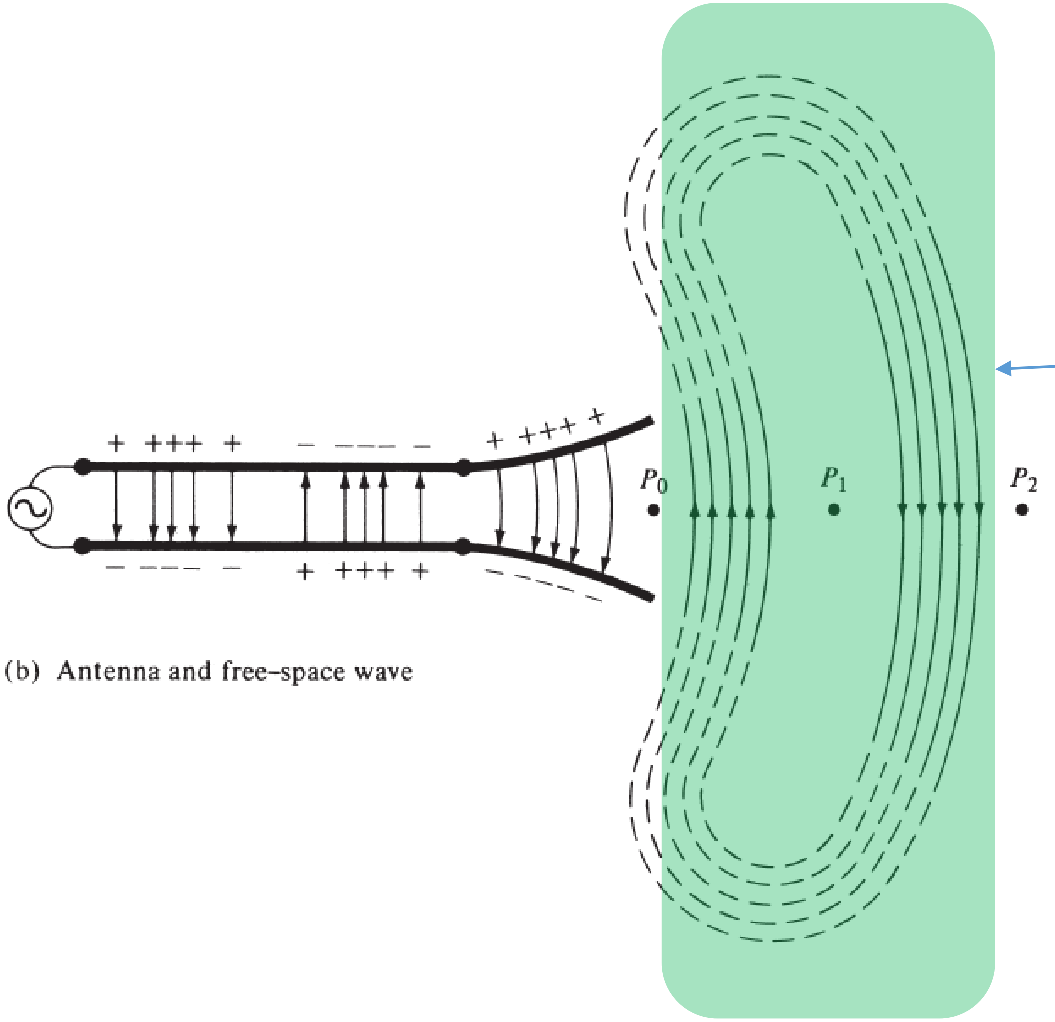
In the last lecture we said that

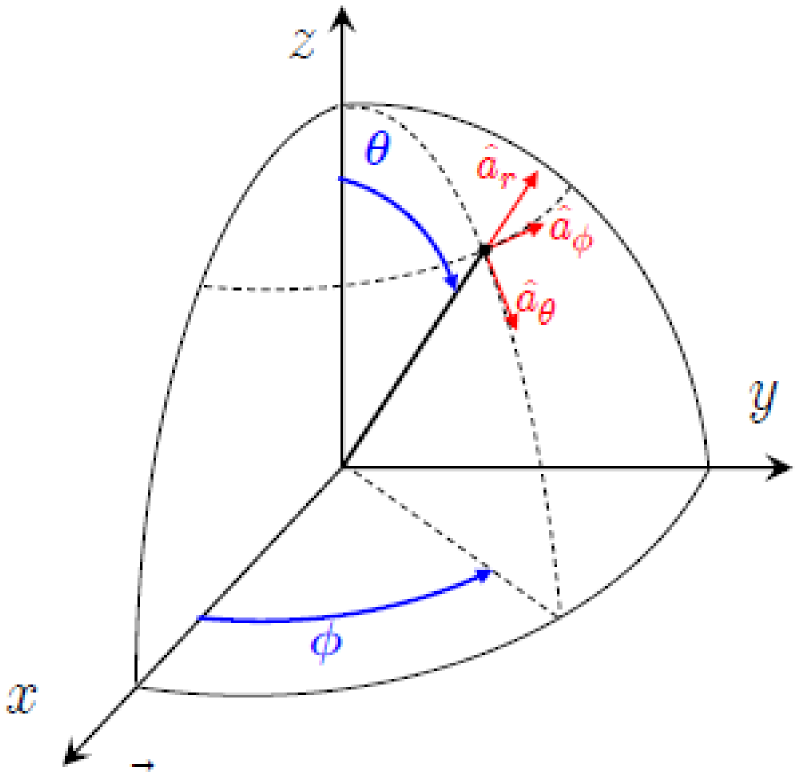


(b) Antenna and free–space wave

We will now focus on the radiated fields (strength, density, direction etc.)

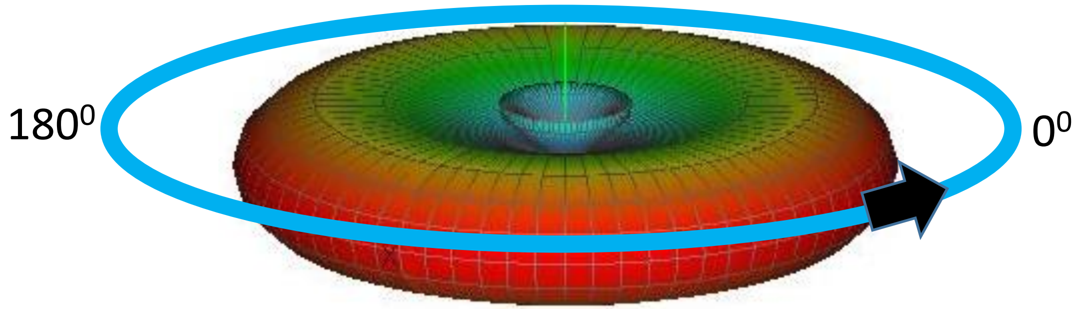
*And we discussed **Five Points** about depiction and notation of radiation patterns*

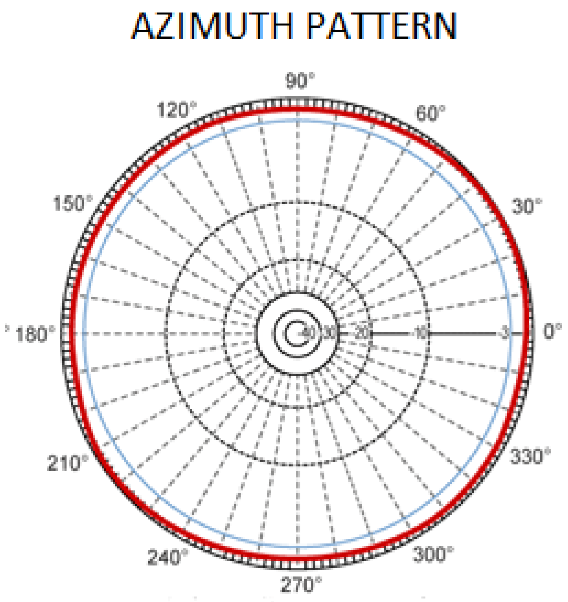
1. We will use vectors to represents fields in space
2. We may use Cartesian coordinates or Spherical Coordinates to represent the vectors in 3D

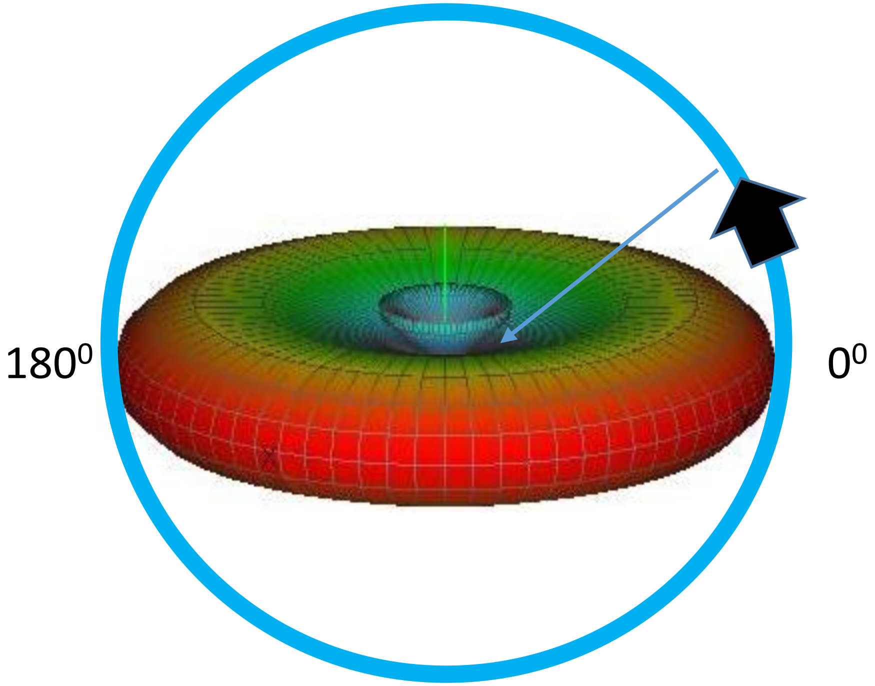


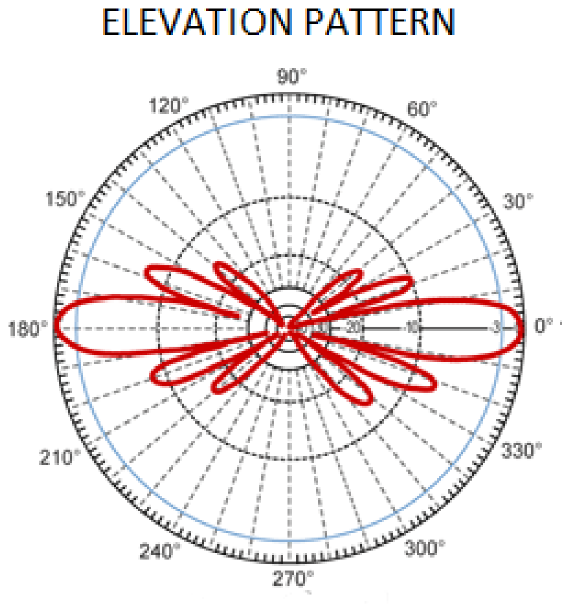
$$\vec{E} = E_r\hat{a}_r + E_\theta\hat{a}_\theta + E_\phi\hat{a}_\phi$$

3. We define two planes of interest: Azimuth and Elevation

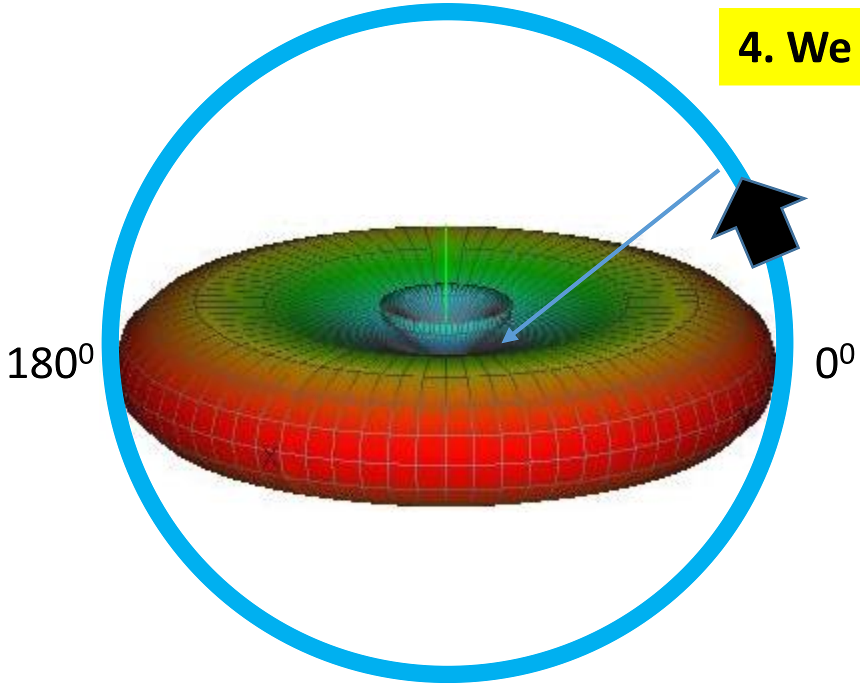




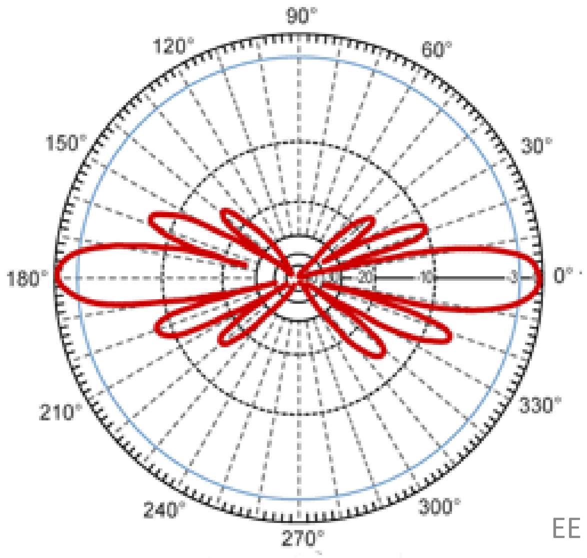


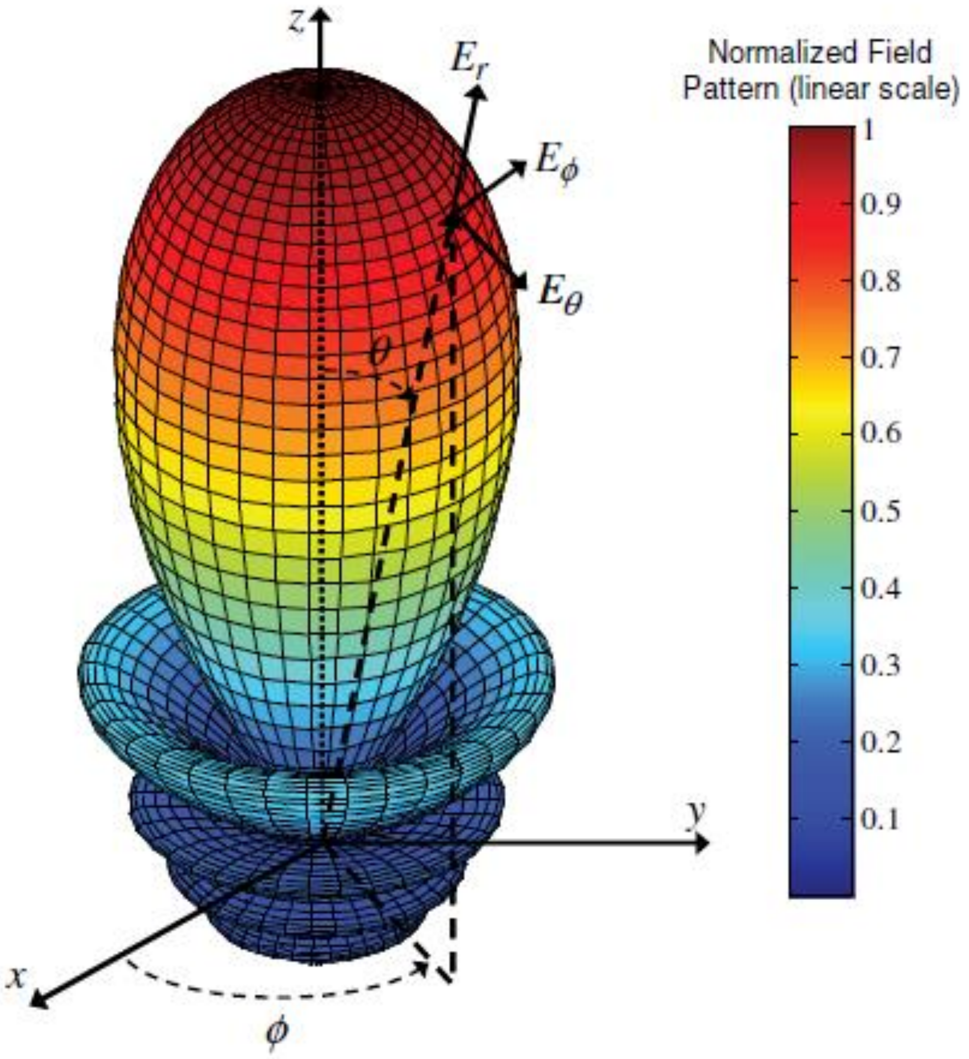


4. We may also show relative strength of a field by color or by size



ELEVATION PATTERN





5. We may use linear scale or log (dB) scale

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$$\boxed{P_{dB} = 10\log_{10} P}$$

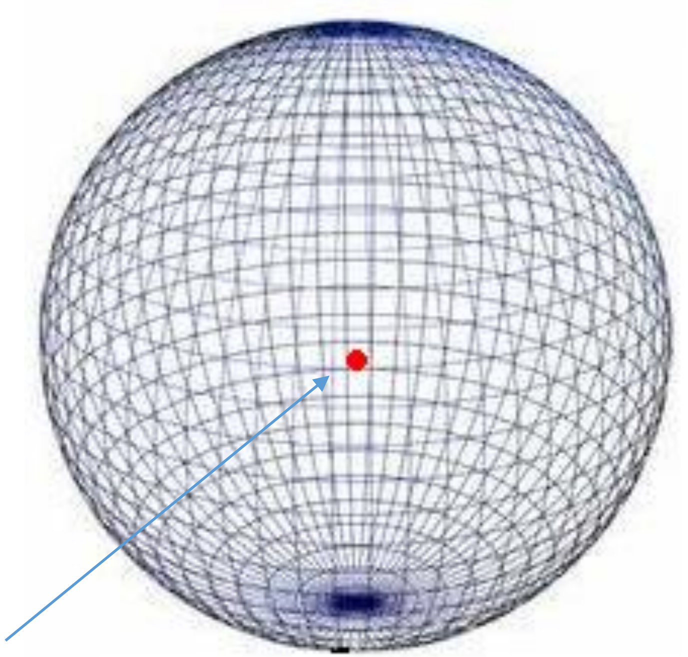
Power in linear scale

And we saw three types of antenna patterns…

1. Isotropic
2. Omnidirectional
3. Directional

Isotropic = equal radiation in **all** directions

- **Not possible in practice**
- **Used only as a reference pattern**



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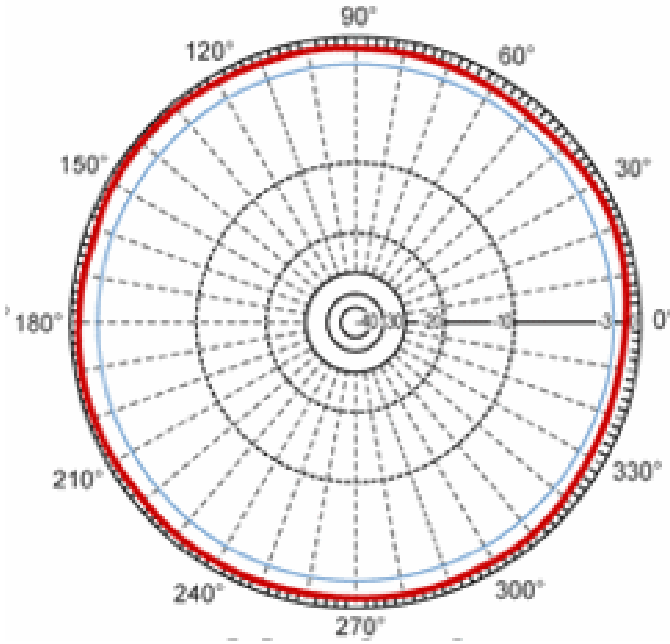
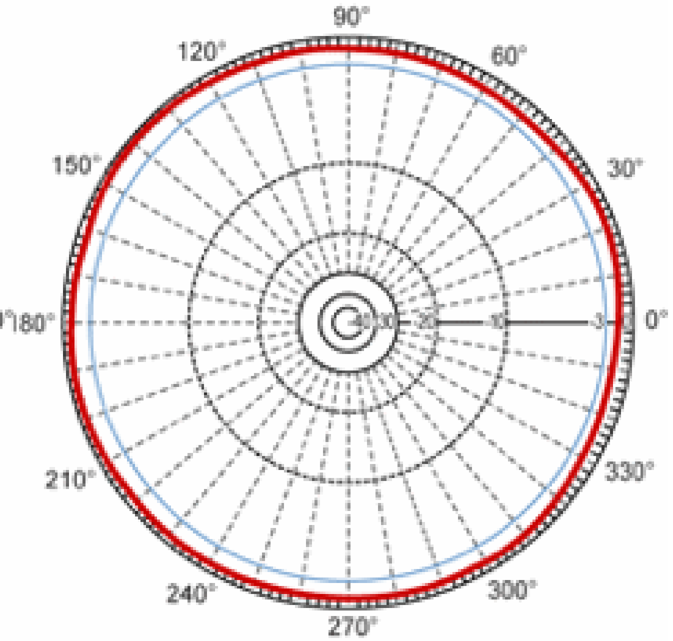


Elevation



Azimuth

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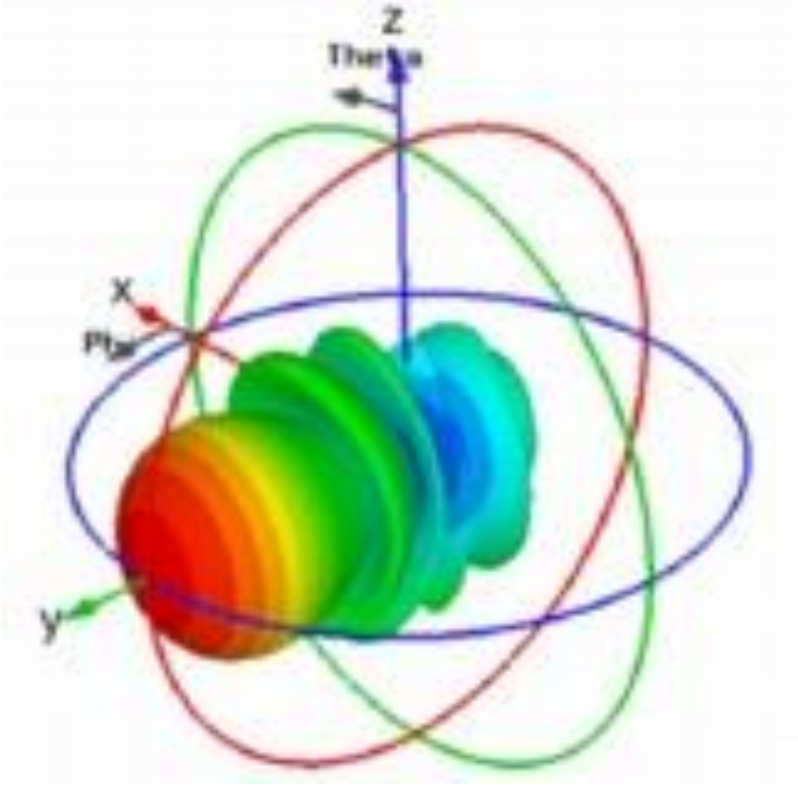
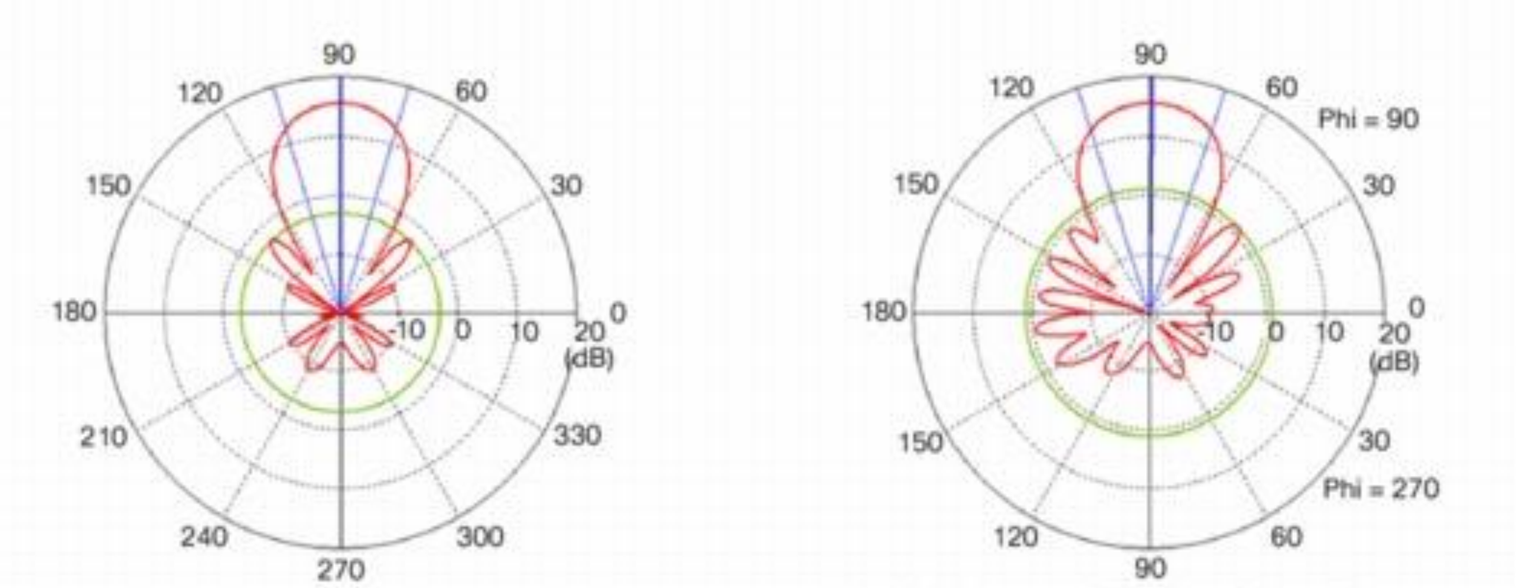


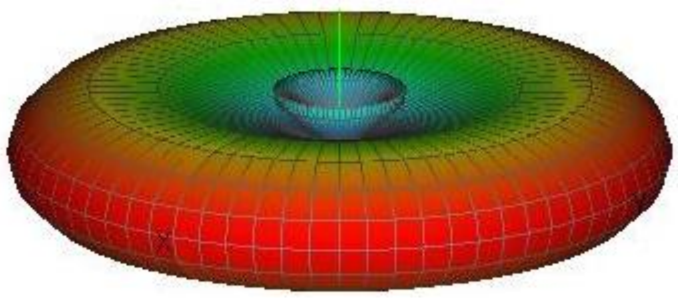


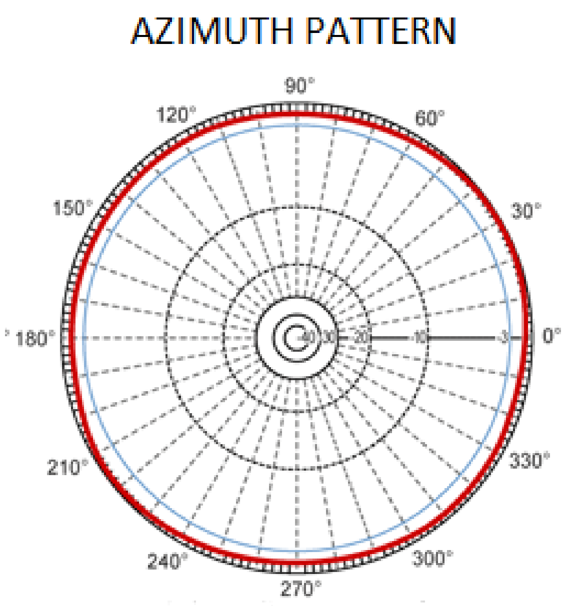
(c) Yagi Antenna Azimuth Plane Pattern

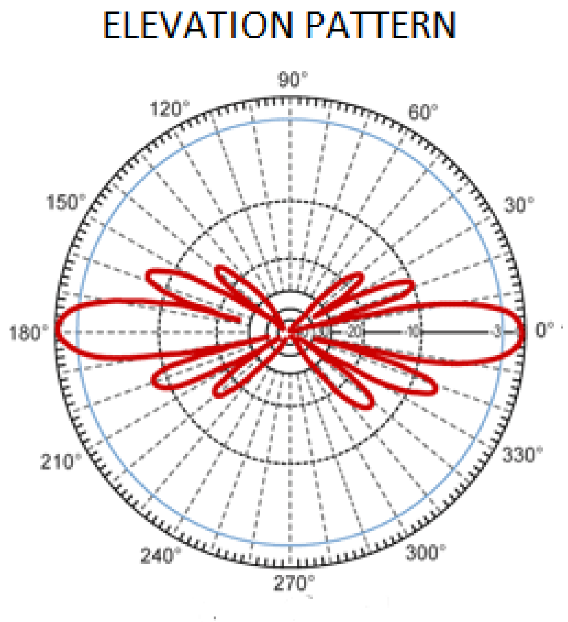
(d) Yagi Antenna Elevation Plane Pattern

Omnidirectional = special case of directional pattern

- **Equal radiation in one plane (e.g., azimuth)**
- **Unequal radiation in orthogonal plane (e.g., elevation)**





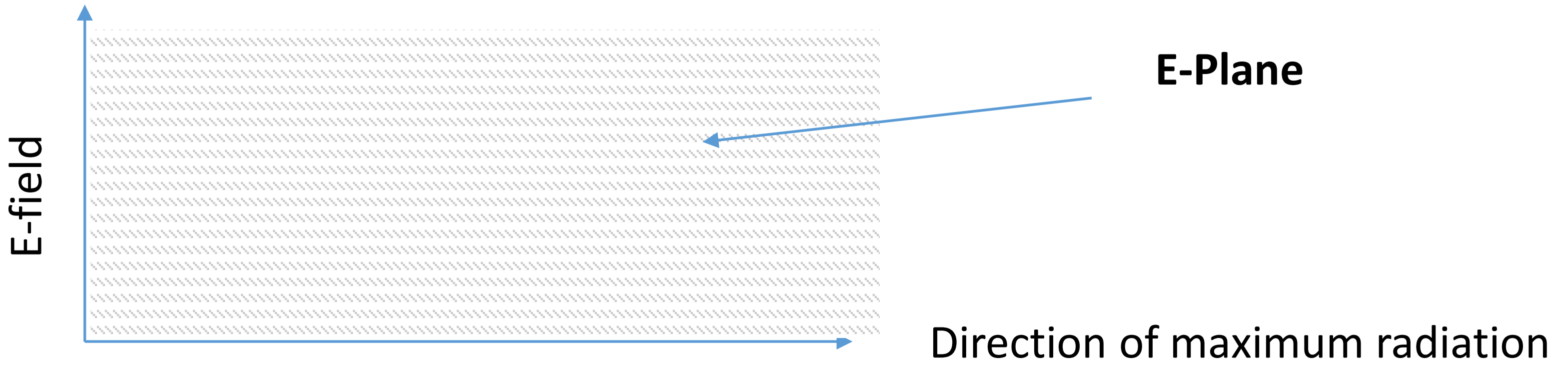


Today …

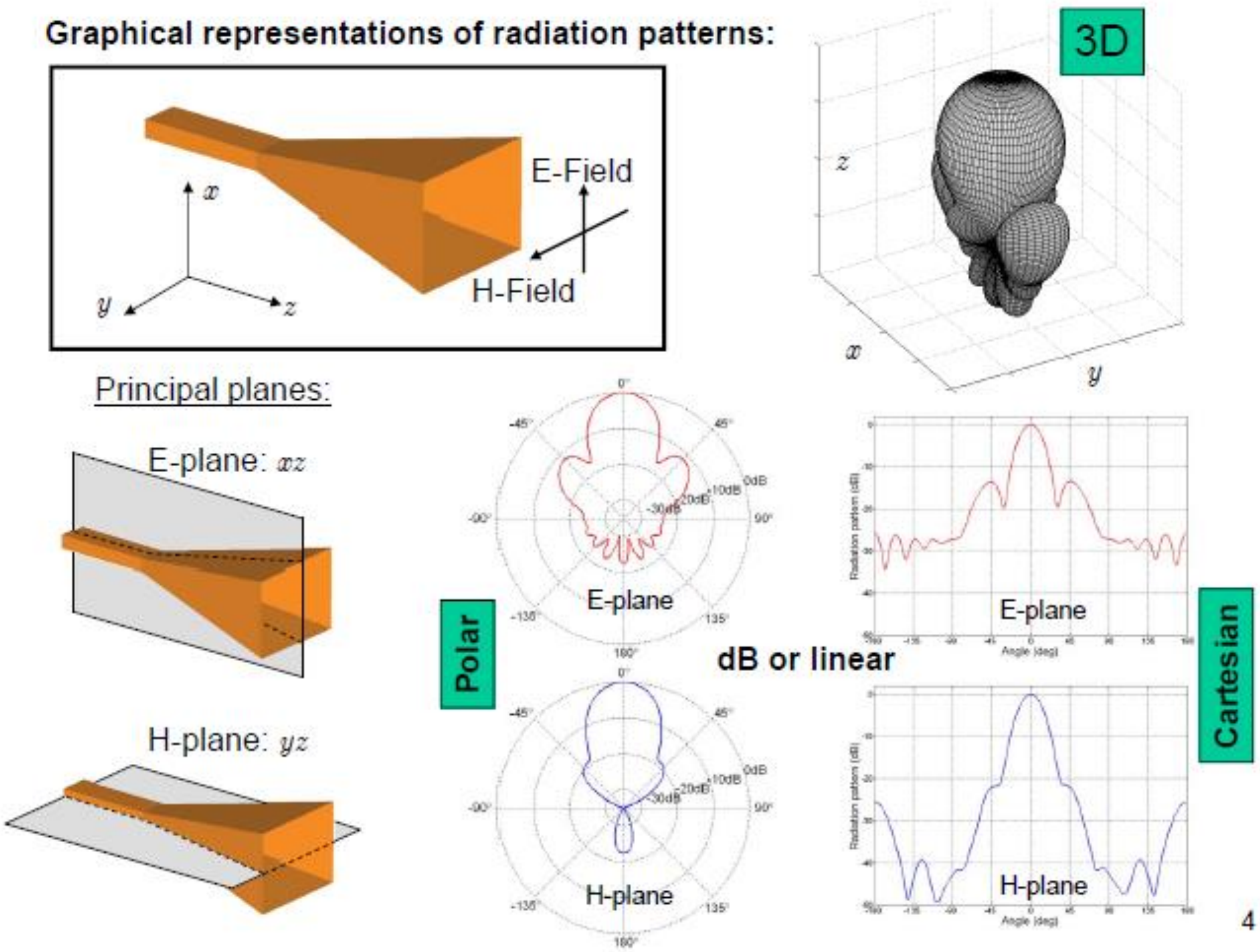
- We will discuss
  - Planes
  - Lobes
  - Beamwidths

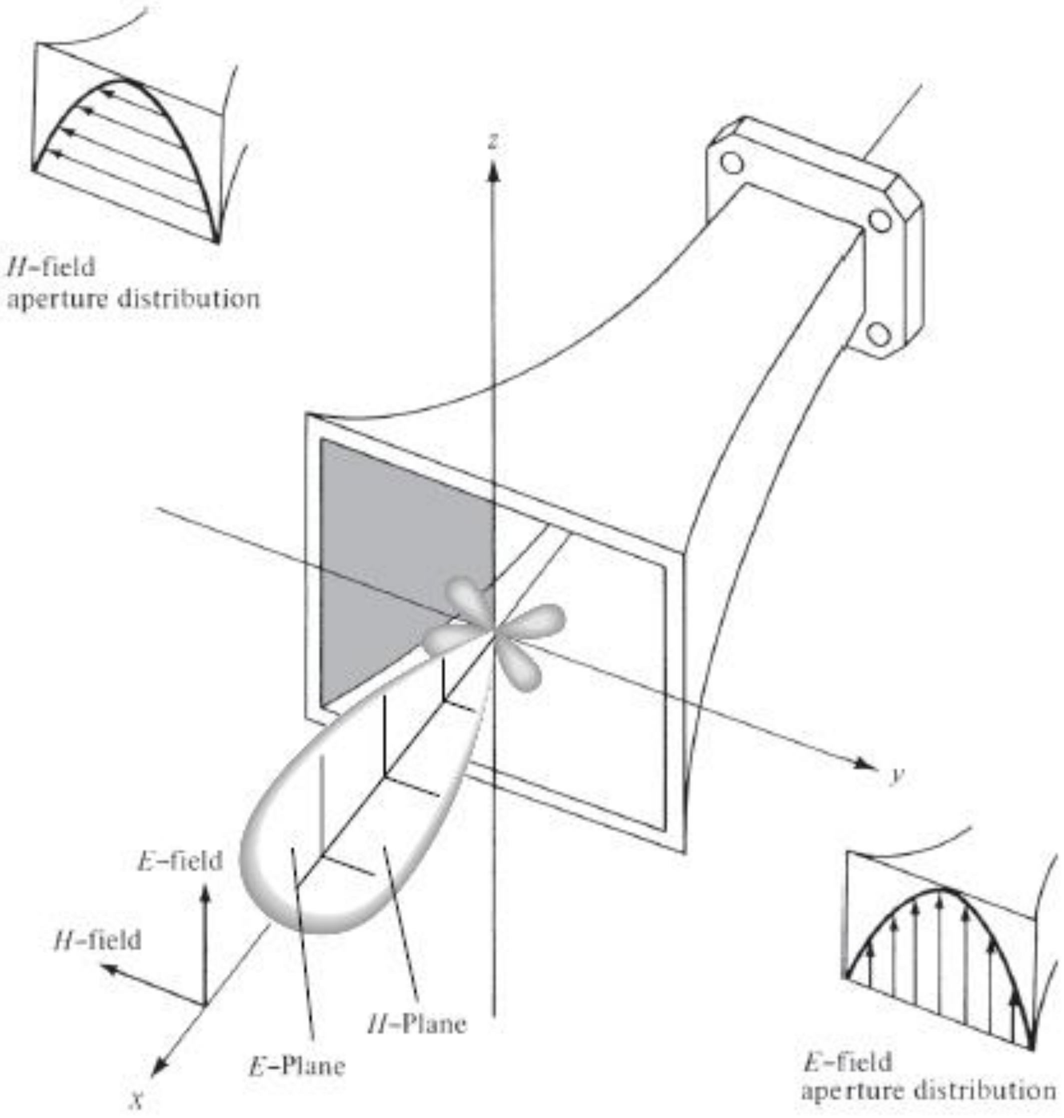
Principal Planes: **E-Plane** & **H-Plane**

- E-Plane
  - *The plane containing the electric-field vector and the direction of maximum radiation*
- H-Plane
  - The plane containing the magnetic-field vector and the direction of maximum radiation



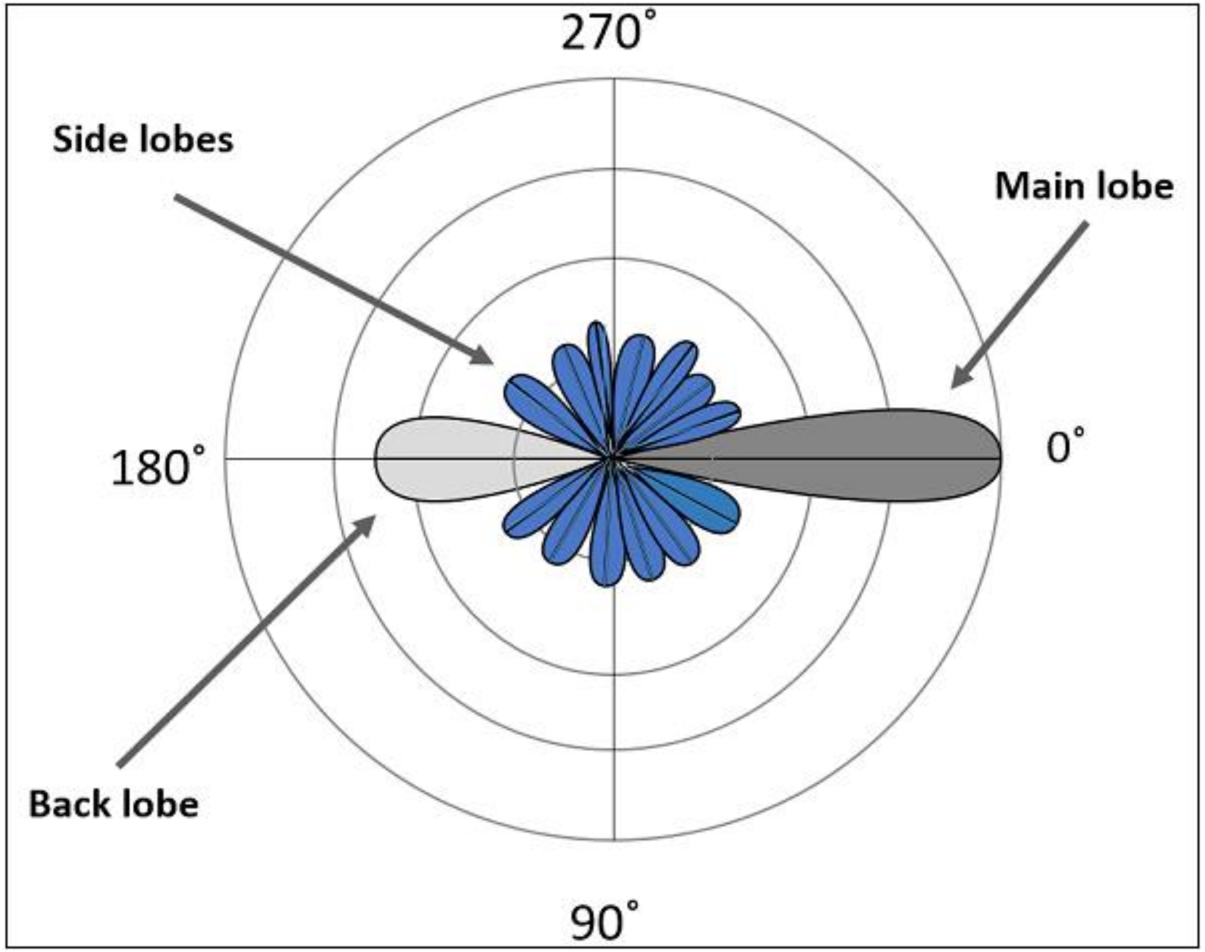
Graphical representations of radiation patterns:

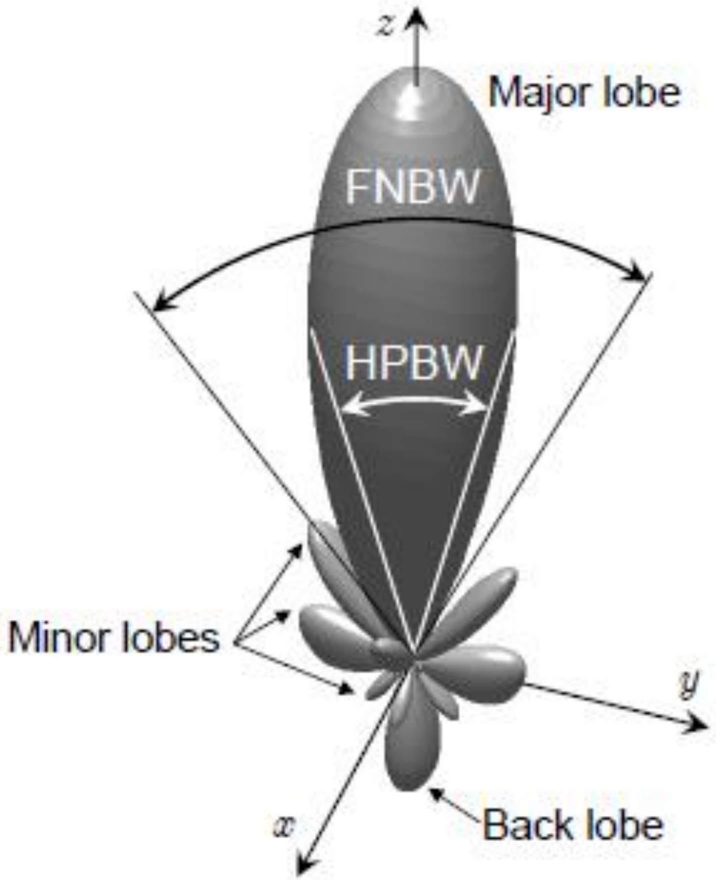




H-field
aperture distribution
z
y
E-field
H-field
H-Plane
E-Plane
x
E-field
aperture distribution

Lobes

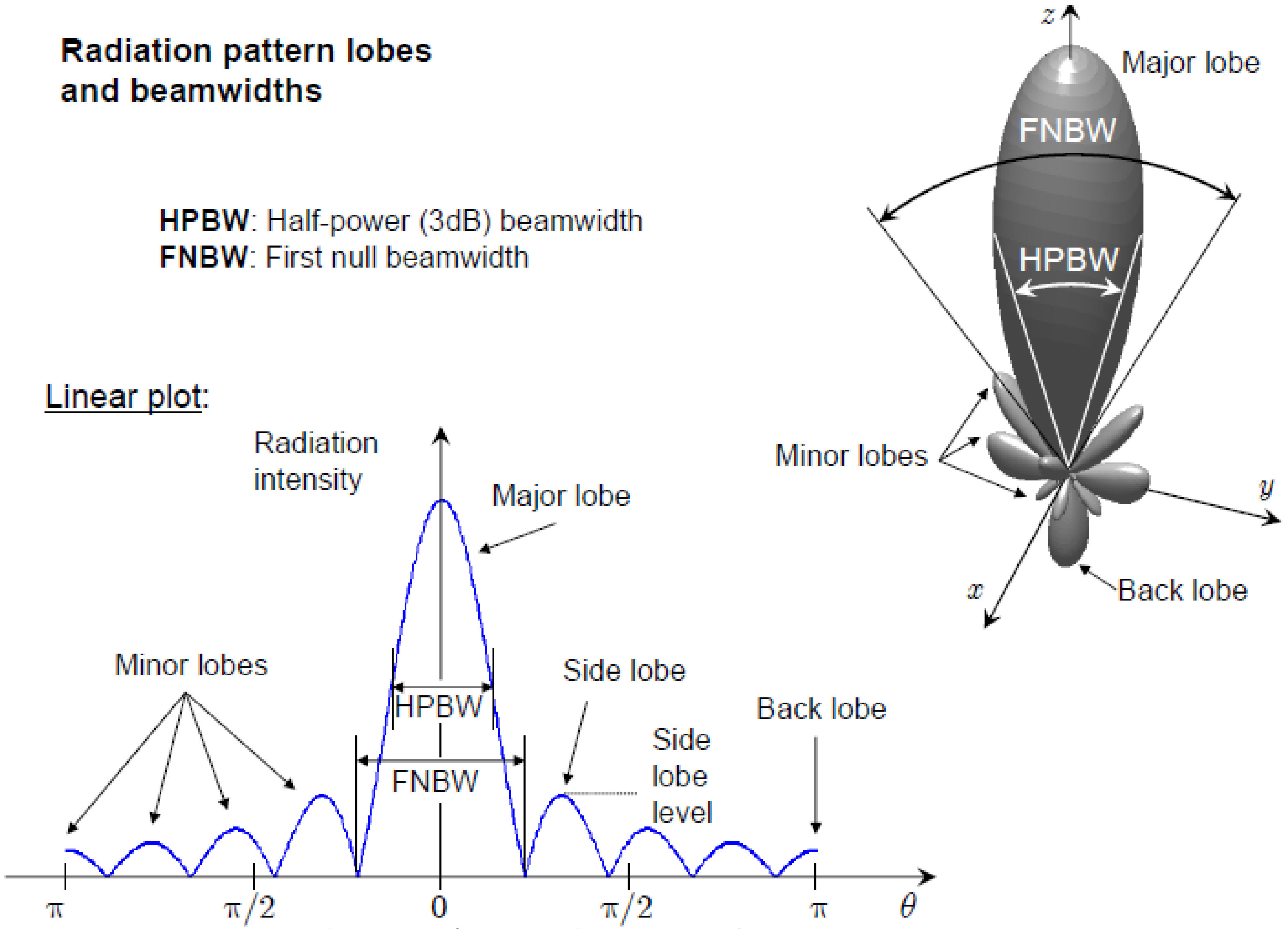

270°
Side lobes
Main lobe
180°
0°
Back lobe
90°

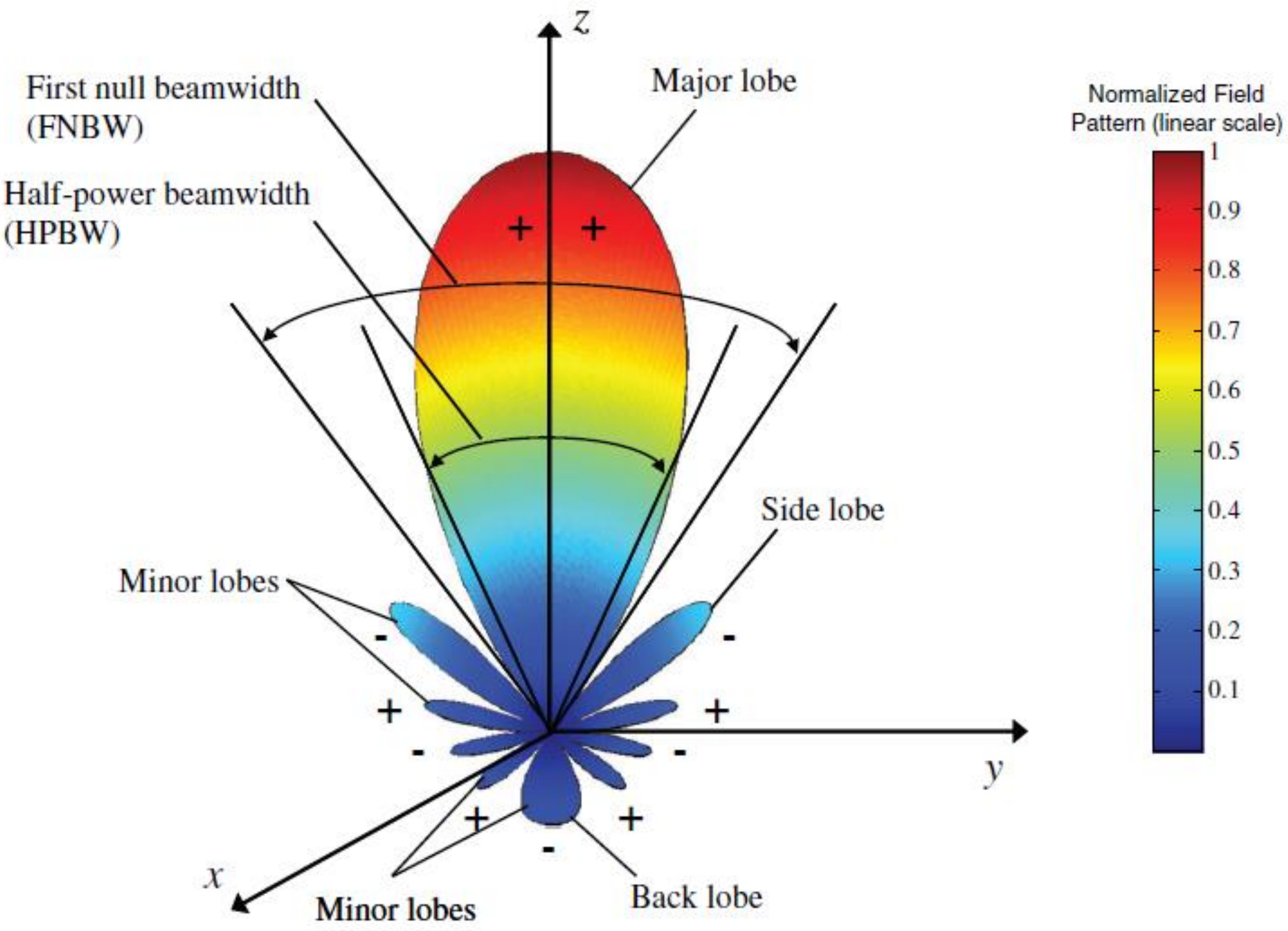

z
Major lobe
FNBW
HPBW
Minor lobes
y
x
Back lobe

Lobes: Main/Major, Minor (Side and Back)

- **Main** or **Major** Lobe
  - *The lobe containing the* ***direction of maximum radiation***
- **Minor** Lobes
  - *A minor lobe is* ***any lobe except a major lobe***
  - *Minor lobes are typically further* ***divided into two categories***
    - **Side** Lobes
      - A lobe in any direction other than the intended lobe
    - **Back** Lobe
      - A lobe whose axis makes an angle of approximately 180° with respect to the beam of an antenna

Width of the Major Lobe





Half-power beamwidth (HPBW) = angle for which power is half of the maximum (recall 0.5 = -3dB)

First null beamwidth (FNBW) = angle for which power hits zero for the first time.

Some Practice …

- Try to identify various lobes and beamwidths of some of the patterns we saw before
  - Isotropic
  - Directional
  - Omnidirectional

Questions?? Thoughts??

EE 328
Wave Propagation and Antennas

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Some Important Measures of an Antenna's Radiation Pattern

- In the previous lectures we covered **basic introduction** to antenna radiation patterns
- Now we will discuss some **formal (mathematical) definitions** for characterizing an antenna's radiation pattern
  - Radiation Power Density
  - Radiation Intensity
  - Average Radiated Power
  - Directivity
  - Efficiency
  - Gain

Let’s talk about a bulb!

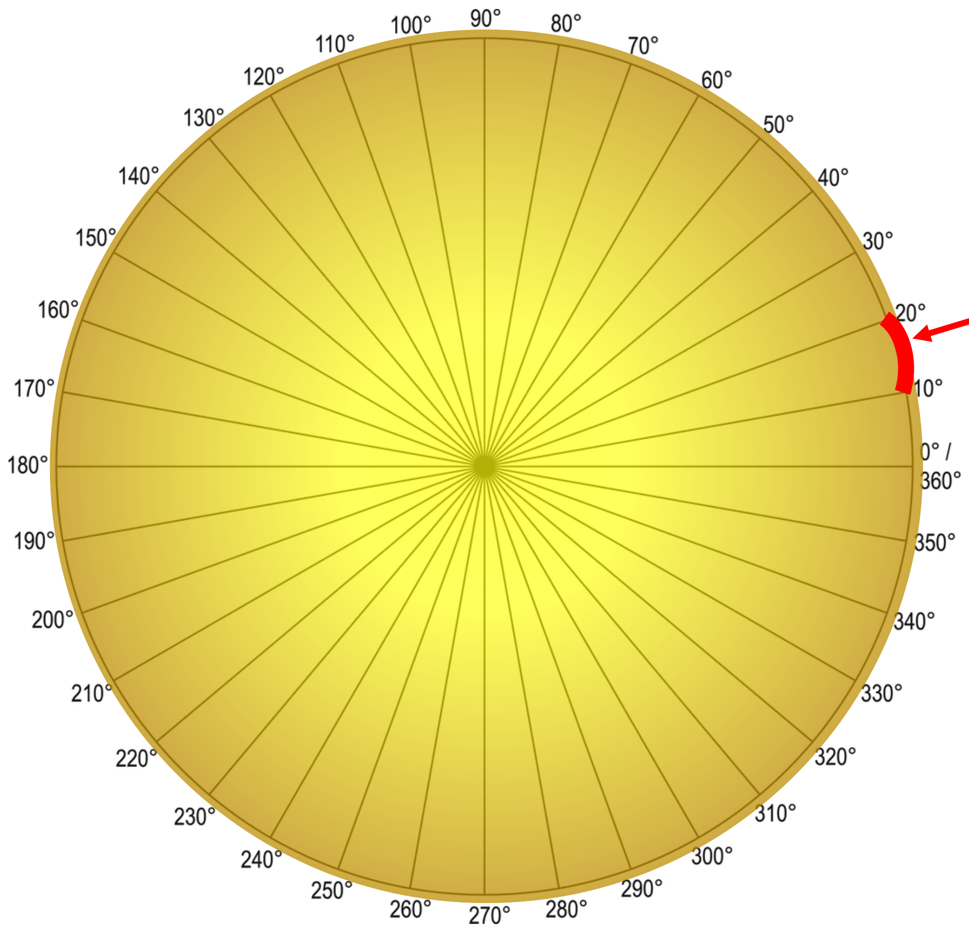
We may be interested in asking how is the light energy distributed?

Or

How much of it reaches a certain point/area?

How much light reaches each of these distances?

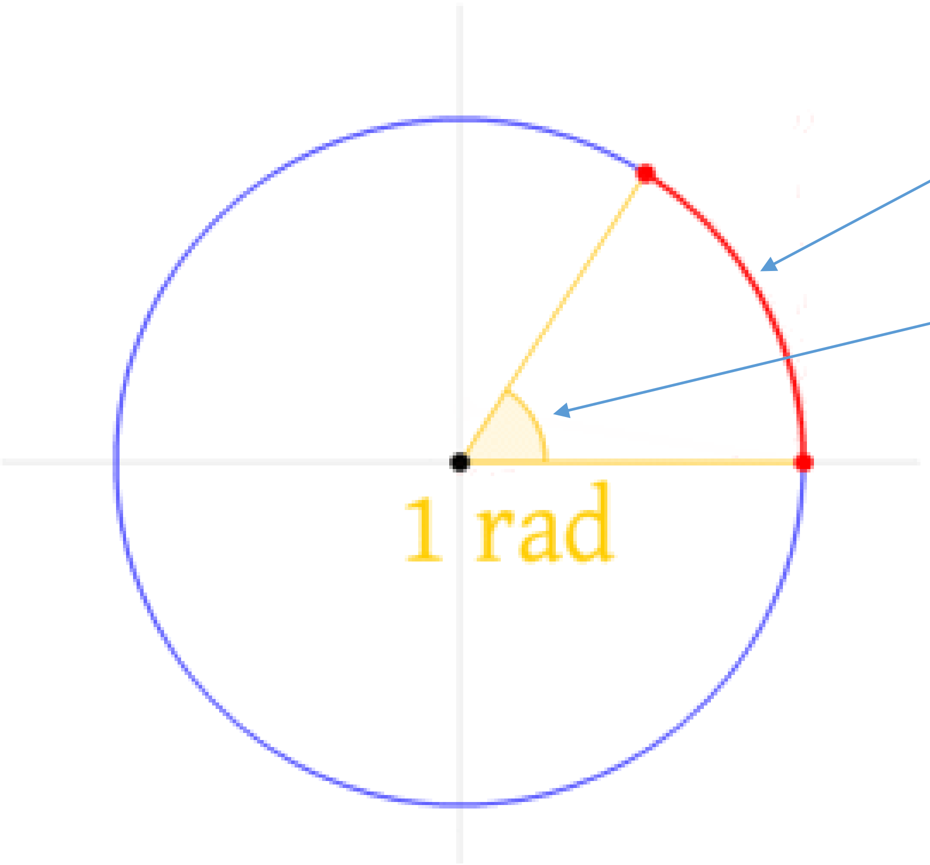
Let's talk about a bulb!



Or we may split one circle into small segments (arcs) and ask:

How much light reaches each segment?

How do we define a segment?

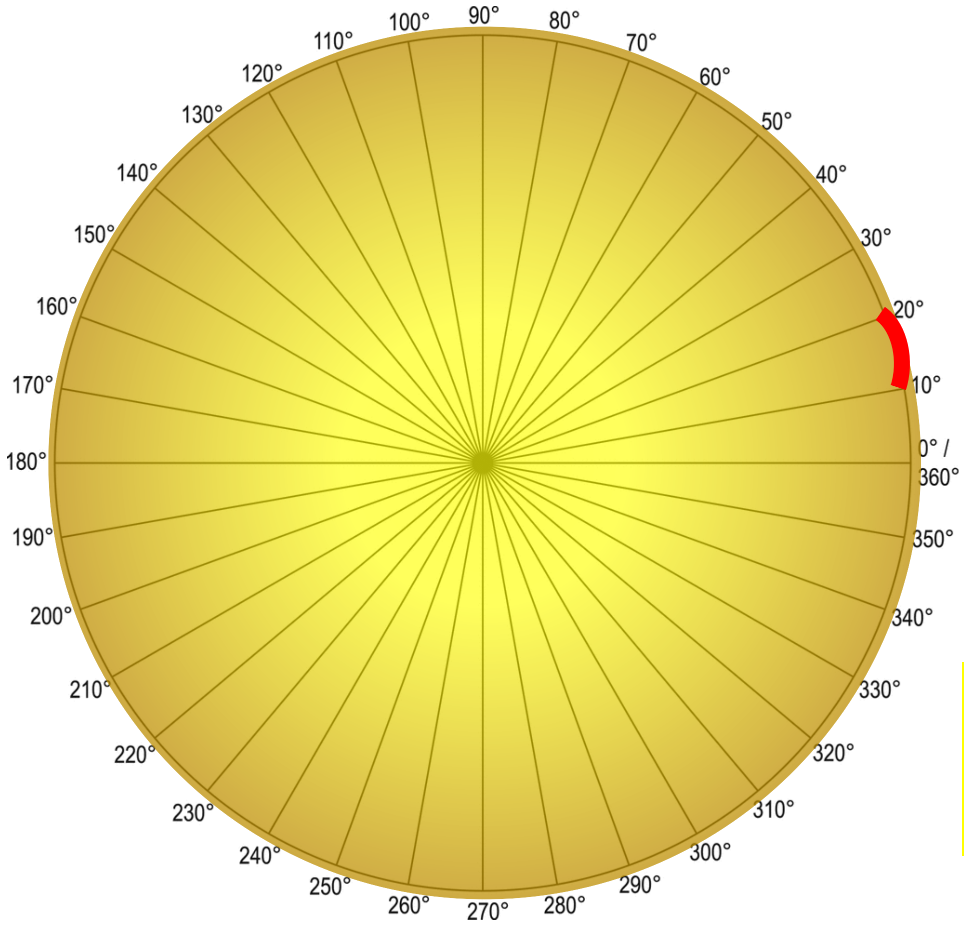


- We can define a segment by its
  - **Length**
  - Or, by the **angle** it makes at the center

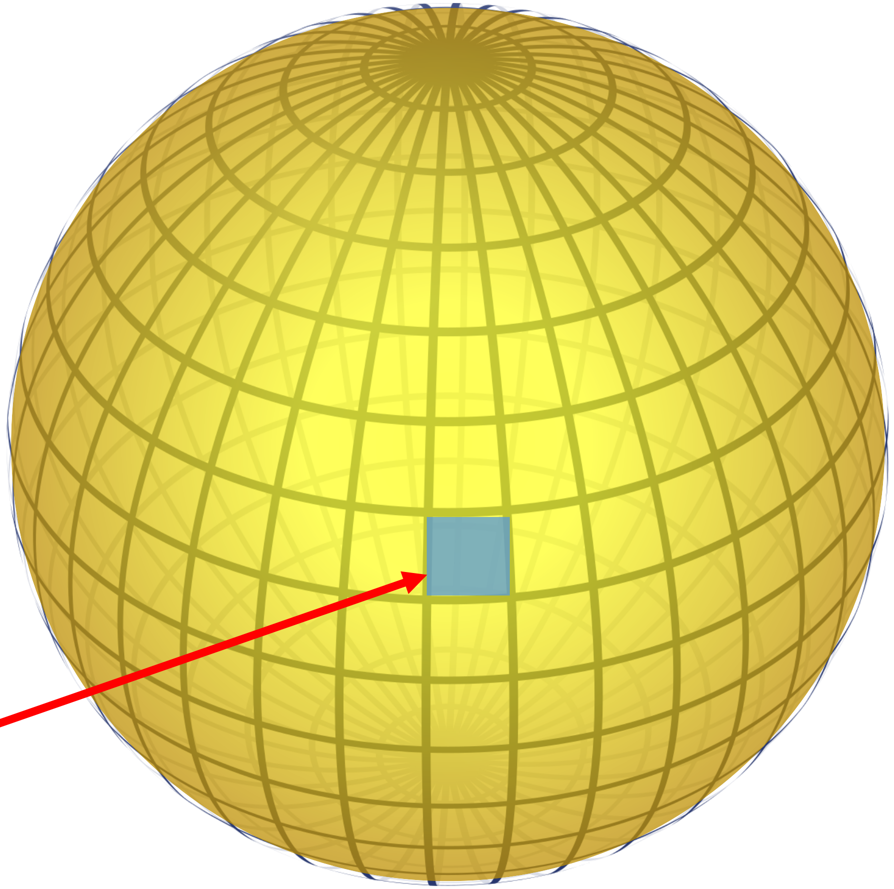
If length of the arc is equal to the circle radius then we say the angle it makes is **one radian**

- 1 radian = 57.3 degrees roughly
- A circle has total of 2π radians (= 360 degrees)

What about 3D Space?

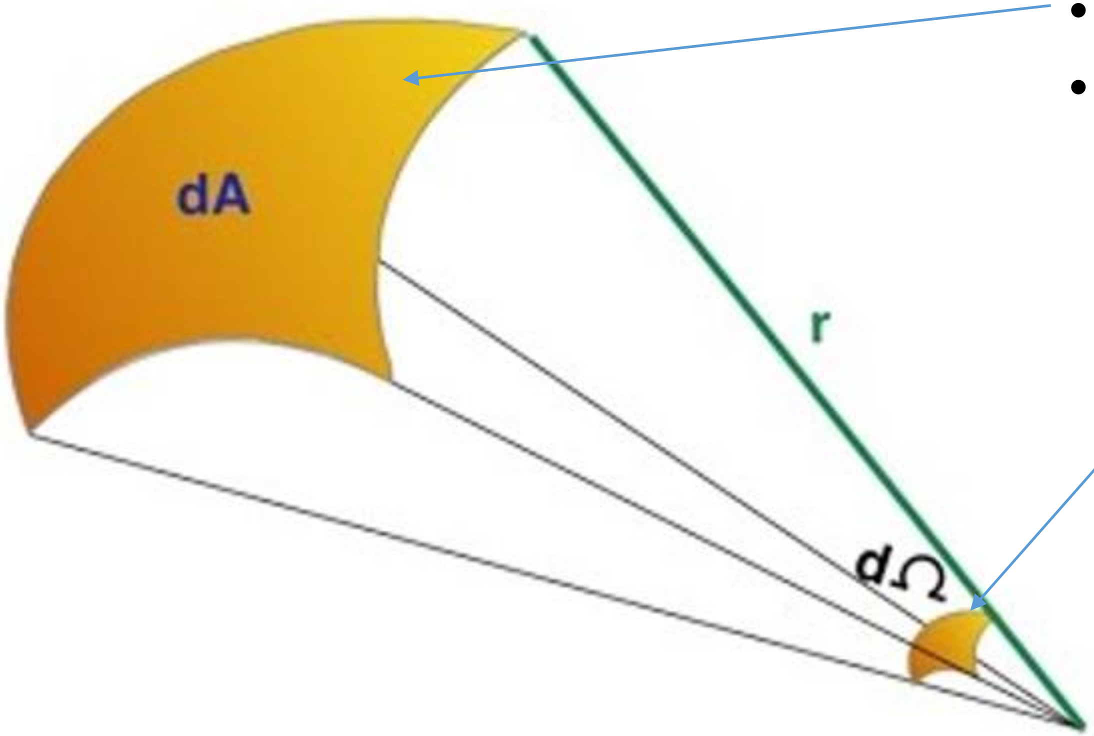


For 3D, we can talk about a **sphere** instead of a **circle**

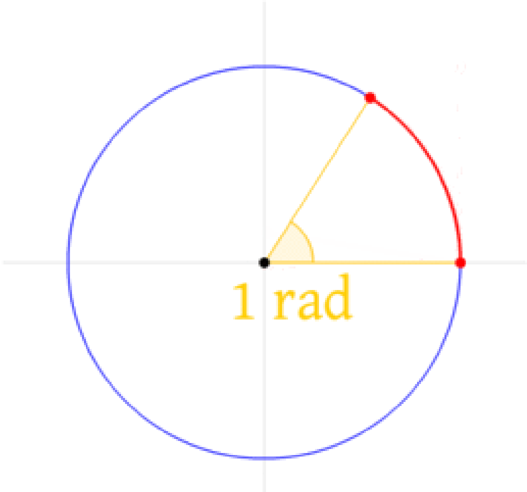


And we can talk about surface **patches** instead of **arcs**

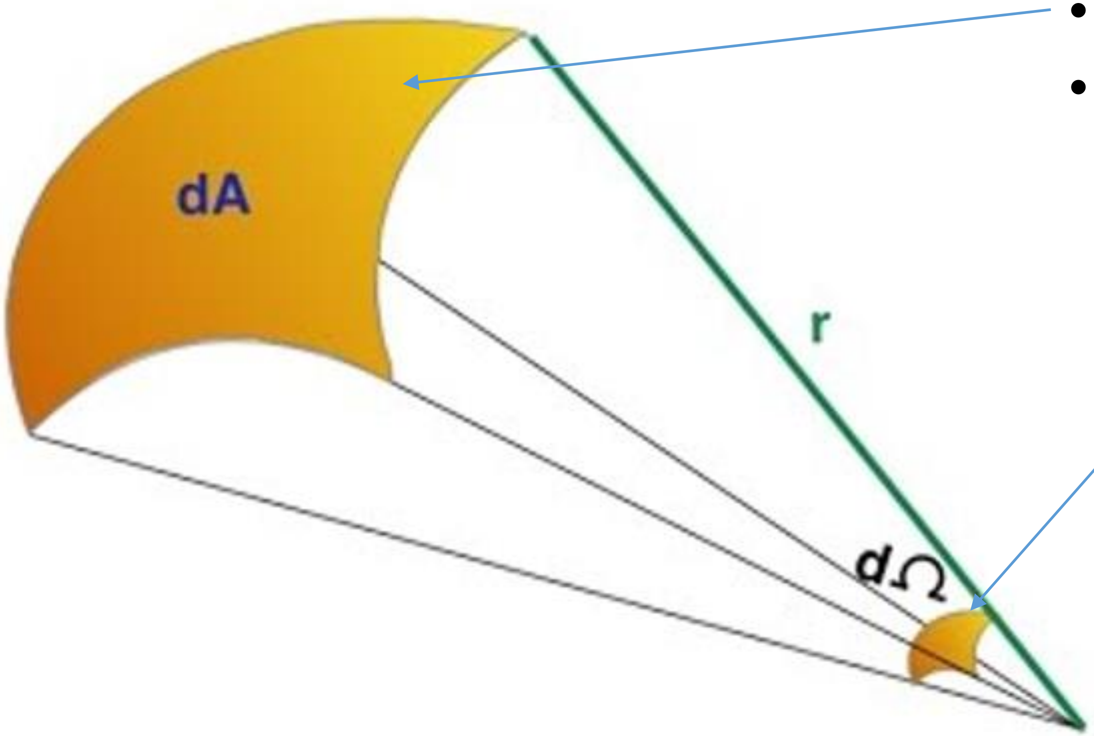
How do we define a surface patch?



- We can define a surface patch by its
  - **Area**
  - Or, by the **Solid Angle** it makes at the center



Solid Angle and Steradian

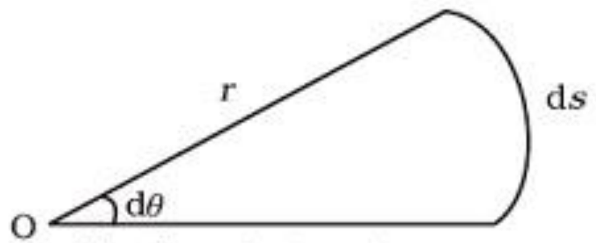


- We can define a surface patch by its
  - **Area**
  - Or, by the **Solid Angle** it makes at the center

If the area of the patch is equal to the square of radius then we say the solid angle it makes is **one steradian**

- A sphere has total of 4π steradians

Some important relations



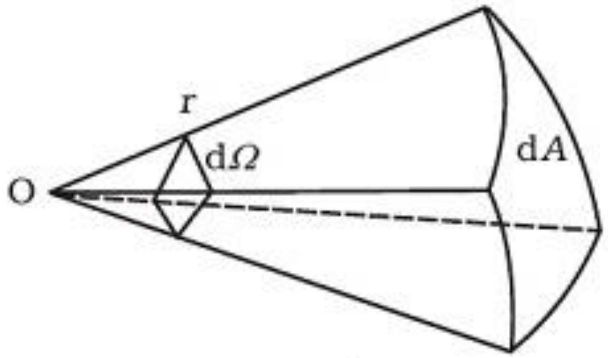
(a) $d\theta = ds/r$ radian

Length of an arc segment is

$$ds = r d\theta$$

If angle is one radian then

$$ds = r$$



(b) $d\Omega = dA/r^2$ steradian

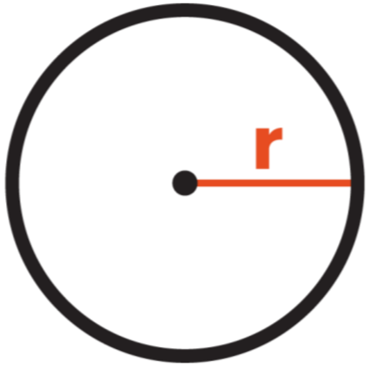
Area of a surface patch is

$$dA = r^2 d\Omega$$

If solid angle is one steradian then

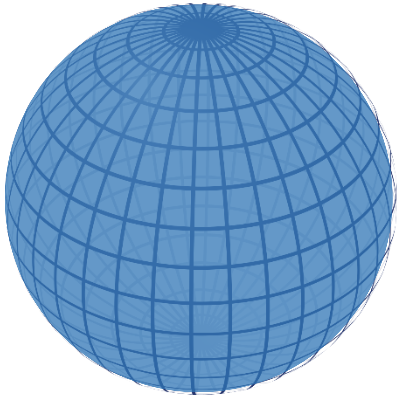
$$dA = r^2$$

Some important relations



Total arc length (circumference) of a circle is
$2\pi r$

A circle has 2π radians

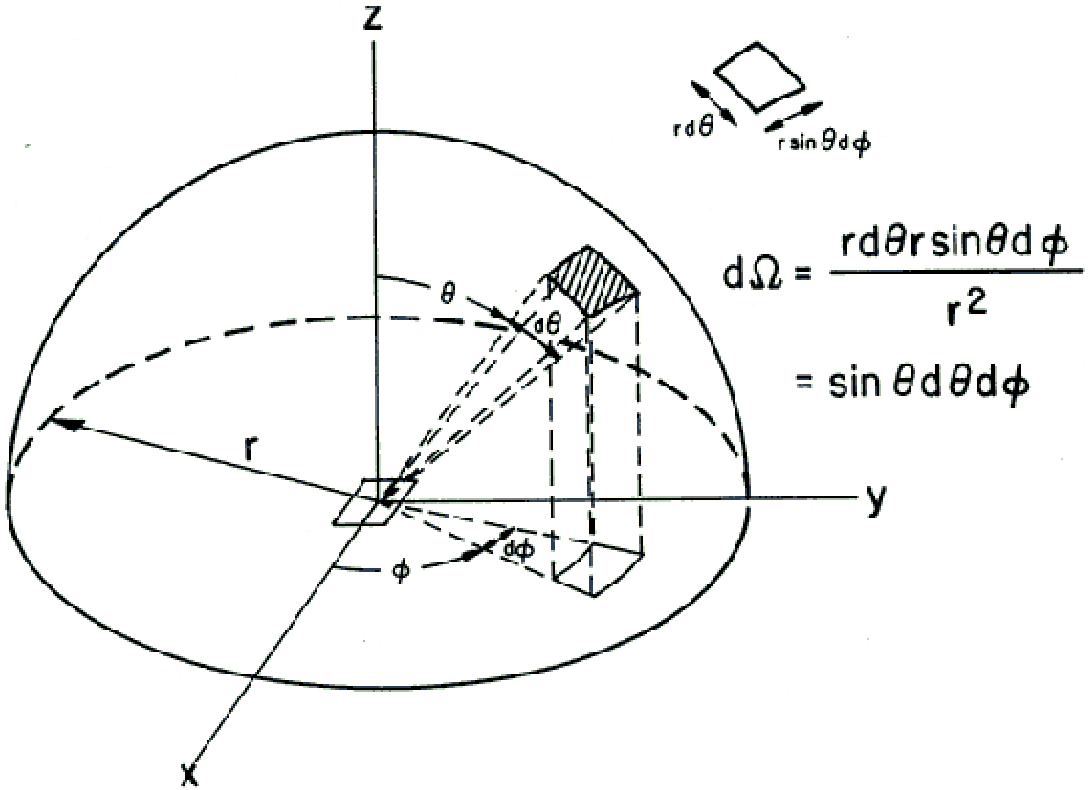


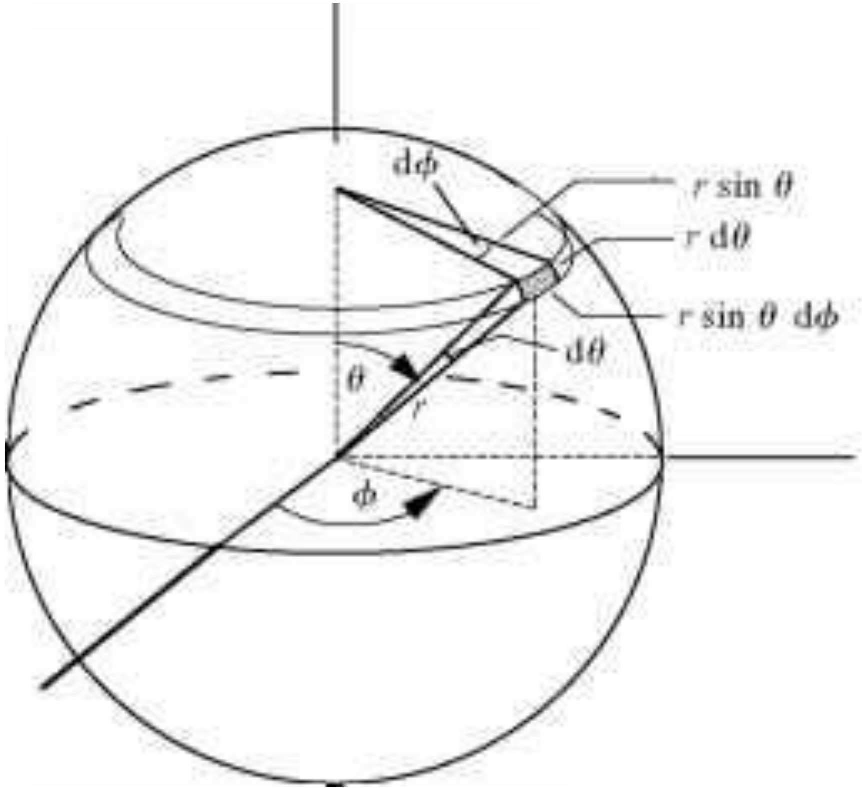
Total surface area of a sphere is
$4\pi r^2$

A sphere has 4π steradians

Relation between Solid Angle (Ω) and θ, ϕ

$$d\Omega = \sin\theta \, d\theta d\phi$$





Area of surface element:

$(r\,d\theta)(r\sin\theta\,d\phi) =$
$r^2\sin\theta\,d\theta\,d\phi$

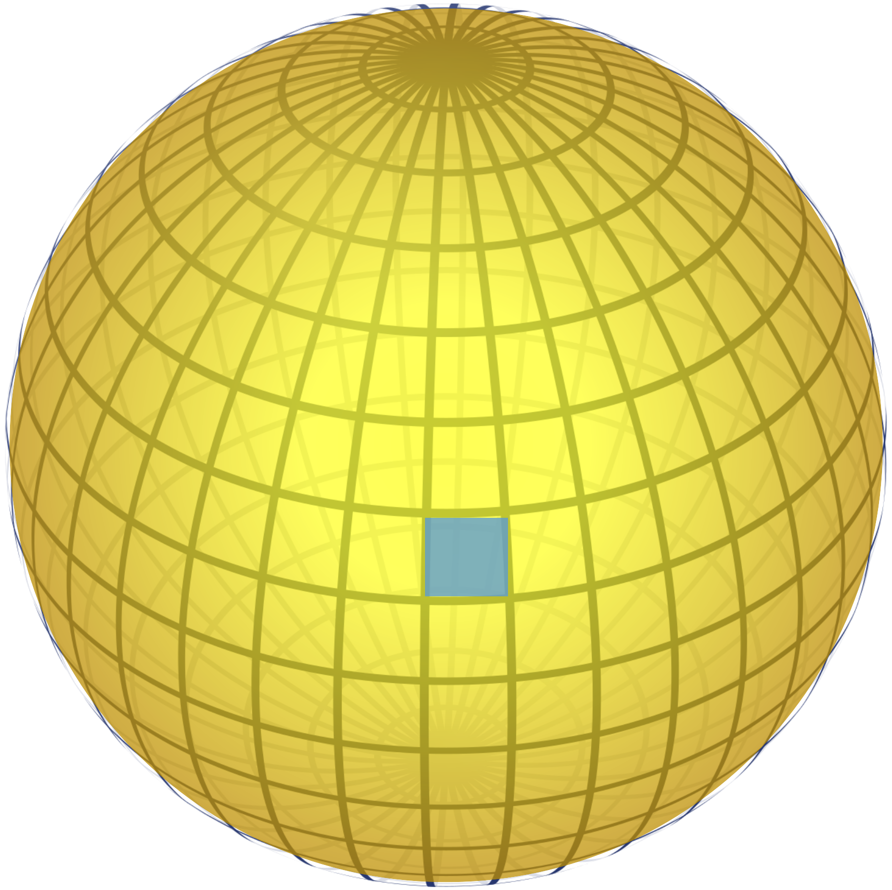
or, for $r = 1$:

$\boxed{d\Omega = \sin\theta\,d\theta\,d\phi}$

Element of solid angle
(steradians)

Radiation Power Density = Power Per Unit Area

$$\frac{Power}{Area}$$



Radiation Power Density = Power Per Unit Area

$$\vec{W} = \vec{S} = \vec{E} \times \vec{H} \quad [\mathrm{W/m^2}]$$

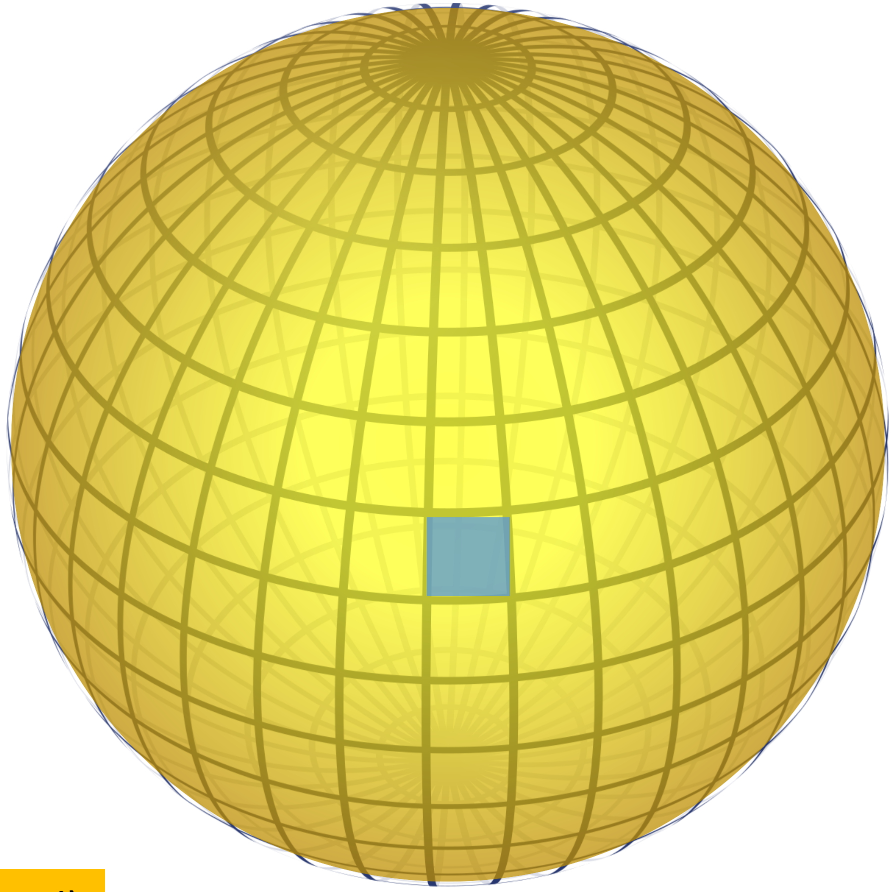
Instantaneous power per unit area

$$\underline{\underline{\vec{W}}}_{av} = \frac{1}{2}\left(\underline{\vec{E}} \times \underline{\vec{H}}^*\right) \quad [\mathrm{W/m^2}]$$

Time-averaged power per unit area

$$\vec{W}_{rad} = \frac{1}{2}\mathrm{Re}\left[\underline{\vec{E}} \times \underline{\vec{H}}^*\right] \quad [\mathrm{W/m^2}]$$

Far-field average power per unit area (mostly real valued)



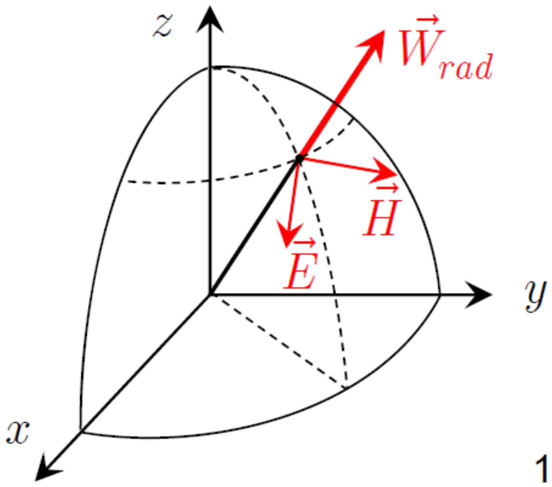
In the far-field the radiation power density is radial

$$\vec{W}_{rad} = \frac{1}{2}\mathrm{Re}\left[\underline{\vec{E}} \times \underline{\vec{H}}^*\right] \quad \left[\mathrm{W/m^2}\right]$$

Far-field average power per unit area (mostly real valued)

$$\vec{W}_{rad} = W_{rad}\hat{a}_r$$

Far-field average power per unit area is radial, i.e., directed along the direction of wave travel



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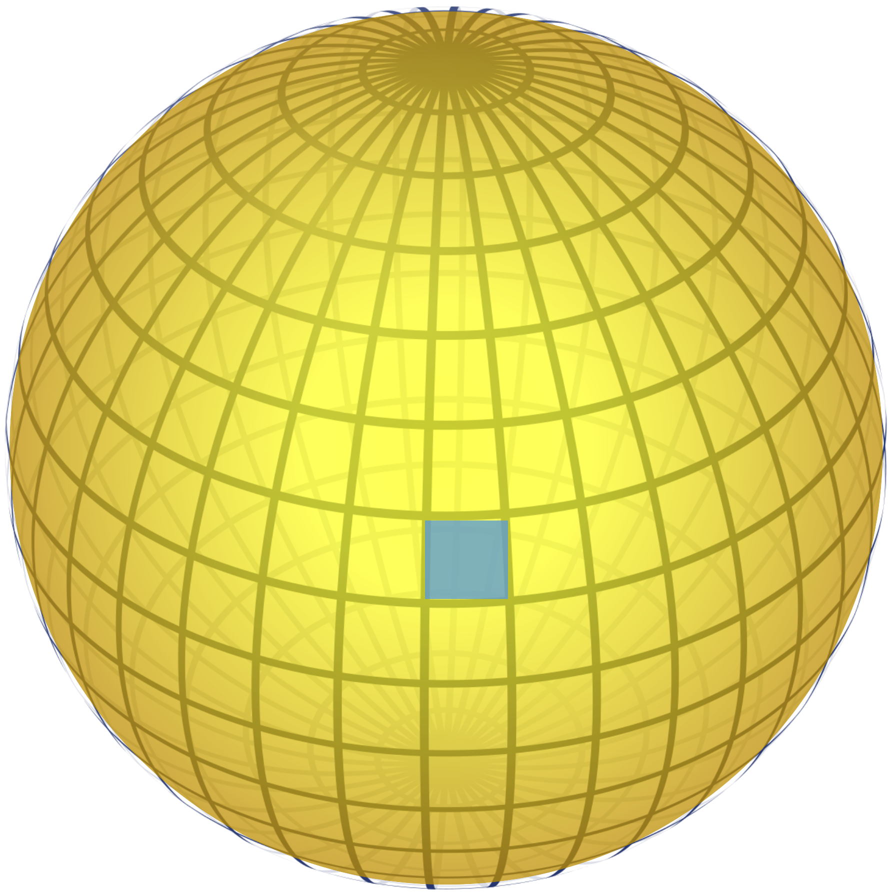
Radiation Power Density = Power Per Unit Area

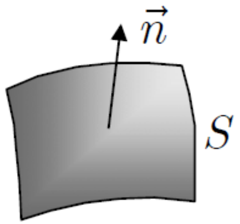
e.g., if W_{rad} = 10 w/m^2 and shown area patch is 1 m^2 then 10 watts of power from the antenna falls on the patch.



W_{rad} can be used to find average power passing through any closed surface S

$$P_{rad} = \oiint_S \vec{W}_{rad} \cdot \vec{n} da = \frac{1}{2} \oiint_S \mathrm{Re}\left[\vec{E} \times \vec{H}^*\right] d\vec{s} \quad [\mathrm{W}]$$

Far-field average power through surface ***S***



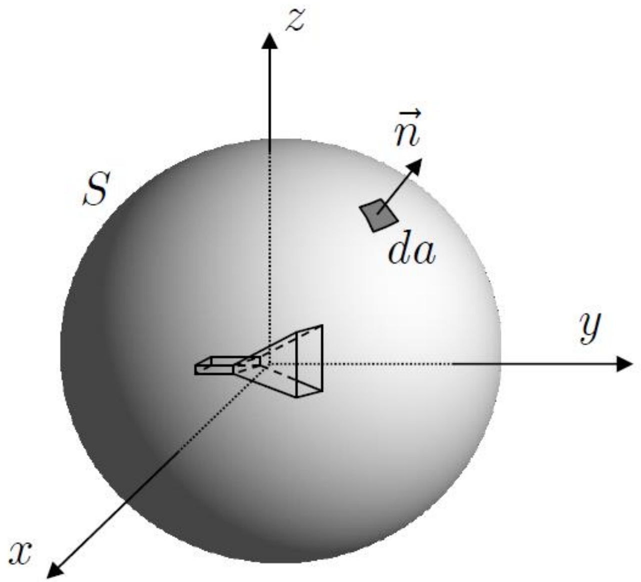
W_{rad} can **also** be used to find average power radiated by an antenna

$$P_{rad} = \oiint_S \vec{W}_{rad} \cdot \vec{n}\, da$$

$$= \int_0^{2\pi} \int_0^{\pi} W_{rad}\, r^2 \sin\theta \, d\theta \, d\phi \quad [\mathrm{W}]$$

$$\boxed{\hat{a}_r \parallel \vec{n}}$$

Average power radiated by the antenna is the total power through a sphere



W_{rad} can **also** be used to find average power radiated by an antenna

$$P_{rad} = \oiint_S \vec{W}_{rad} \cdot \vec{n}\, da$$

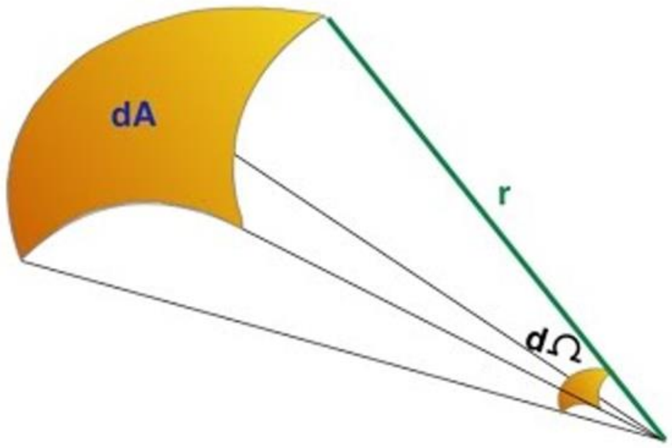
$$= \int_0^{2\pi} \int_0^{\pi} W_{rad}\, r^2 \sin\theta \, d\theta \, d\phi \quad [\mathrm{W}]$$

$$\boxed{\hat{a}_r \parallel \vec{n}}$$

- Example: Isotropic source with W_0 at a distance r_0

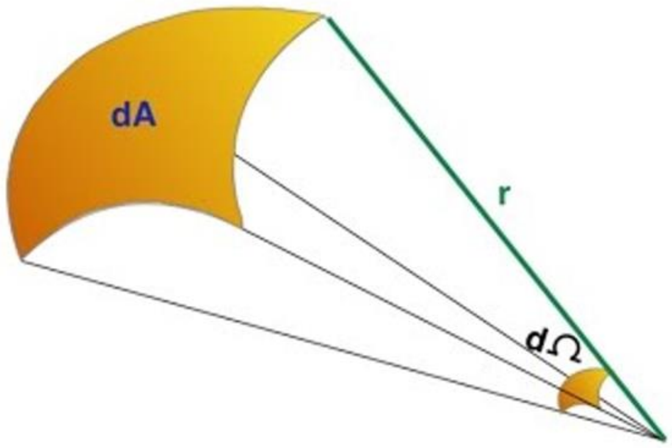
$$P_{rad} = 4\pi r_0^2 \cdot W_0 \quad \Rightarrow \quad W_0 = \frac{P_{rad}}{4\pi r_0^2}$$

Radiation Intensity = Power Per Unit Solid Angle



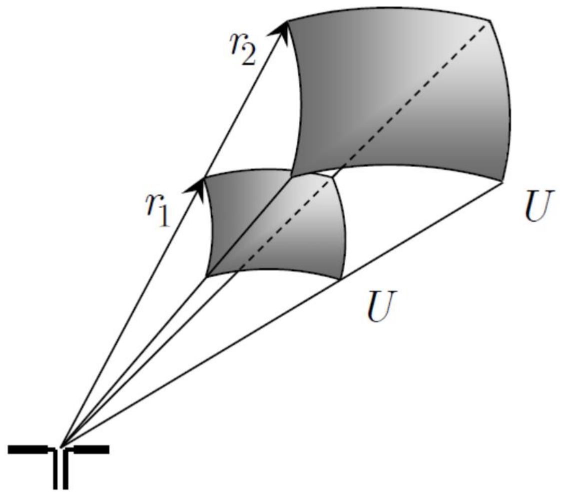
$$\frac{Power}{Solid\ Angle}$$

Radiation Intensity = Power Per Unit Solid Angle



$$\boxed{U = r^2 W_{rad}}\quad [\text{W/unit solid angle}]$$

Radiation Intensity



$$W_{rad}(r_1) \neq W_{rad}(r_2)$$

Radiation Intensity can also be used to find total power radiated by an antenna

$$P_{rad} = \oiint_{\Omega} U(\theta,\phi)\, d\Omega \;\; = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi) \sin\theta d\theta d\phi$$

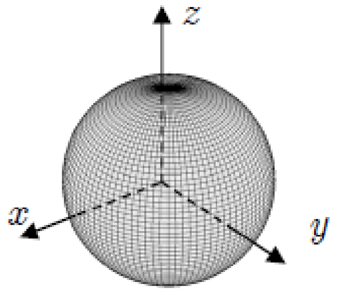
Integrating over the entire sold angle of a sphere

Radiation Intensity can also be used to find total power radiated by an antenna

$$P_{rad} = \oiint_{\Omega} U(\theta,\phi)\, d\Omega \;= \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi) \sin\theta d\theta d\phi$$

- <u>Example</u>: For an isotropic source, $U = U_0$ is independent of angle

$$P_{rad} = U_o \oiint_{\Omega} d\Omega = U_0 \underbrace{\int_0^{2\pi}\int_0^{\pi} \sin\theta d\theta d\phi}_{4\pi} = 4\pi U_0$$

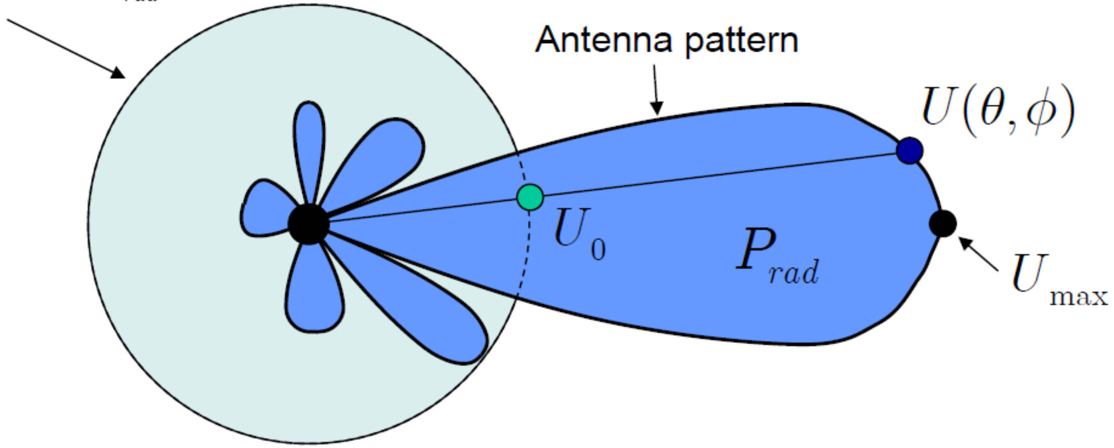


It follows that the radiation intensity of an isotropic source is: $U_0 = \dfrac{P_{rad}}{4\pi}$

Directivity = how directed is the pattern? (and in which direction)

$$\boxed{D(\theta,\phi) = \frac{U(\theta,\phi)}{U_0} = \frac{4\pi U(\theta,\phi)}{P_{rad}}}$$

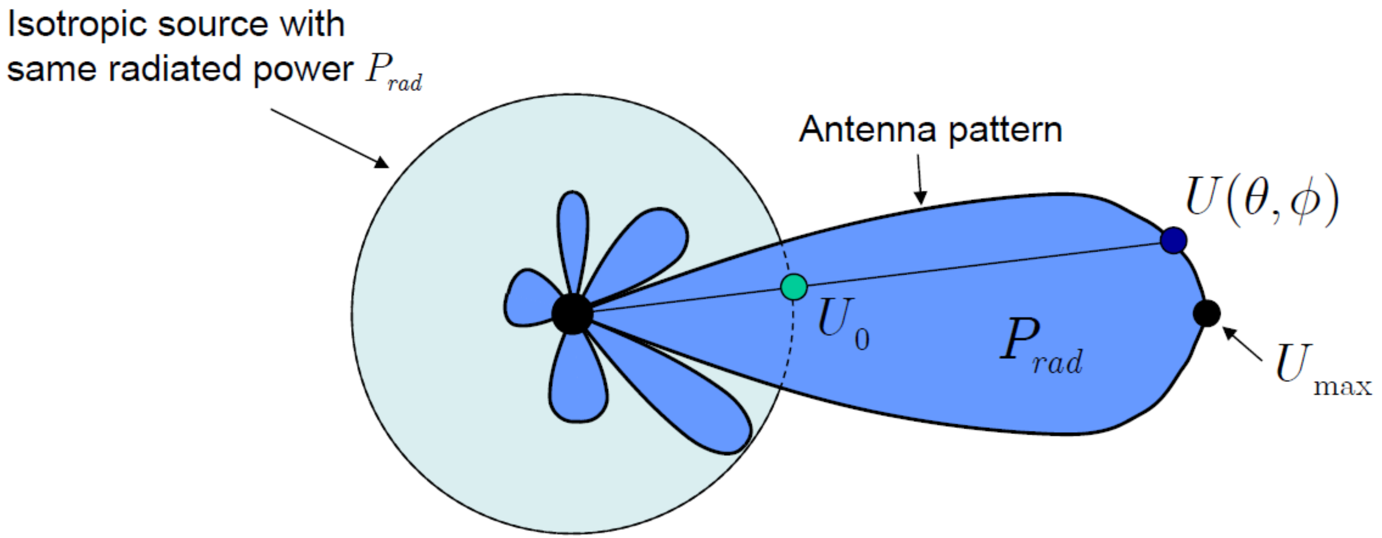
Isotropic source with same radiated power P_{rad}



Directivity = the ratio of the radiation intensity in a given direction to the radiation intensity averaged over all directions (i.e., isotropic)

Maximal Directivity = direction of maximum intensity

$$\boxed{D = D_0 = D_{\max} = \frac{U_{\max}}{U_0} = \frac{4\pi U_{\max}}{P_{rad}}}$$

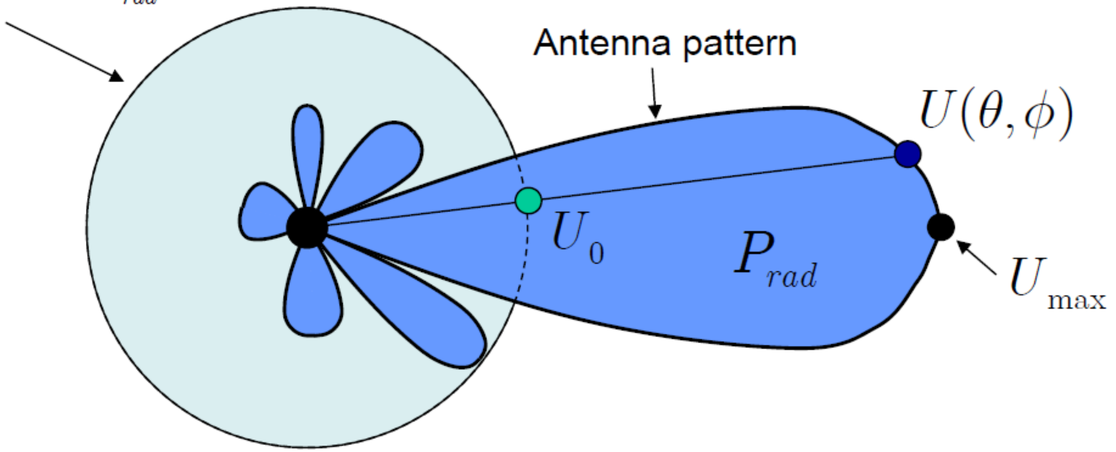


Maximal Directivity = direction of maximum intensity

$$\boxed{D = D_0 = D_{\max} = \frac{U_{\max}}{U_0} = \frac{4\pi U_{\max}}{P_{rad}}}$$

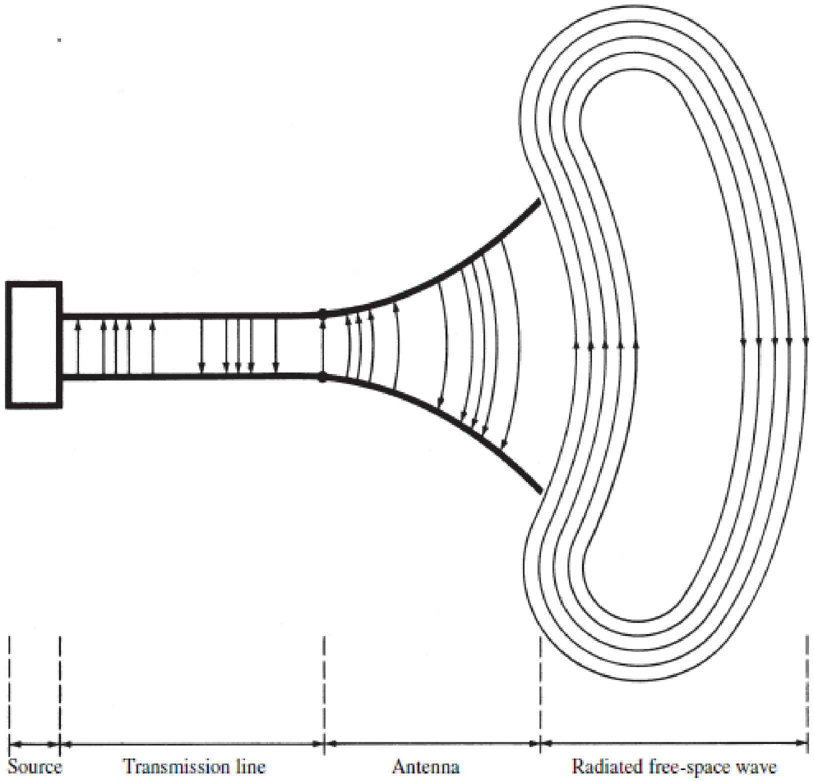
D_0* = 1 *for isotropic antenna

For an antenna (not isotropic)	$0 \le D(\theta,\phi) \le D_0$	
	$D_0 > 1$	(>0 dB)



A “good” antenna system

- ***A good antenna system converts most of the energy provided by the source into radiated energy***.



A “good” antenna system

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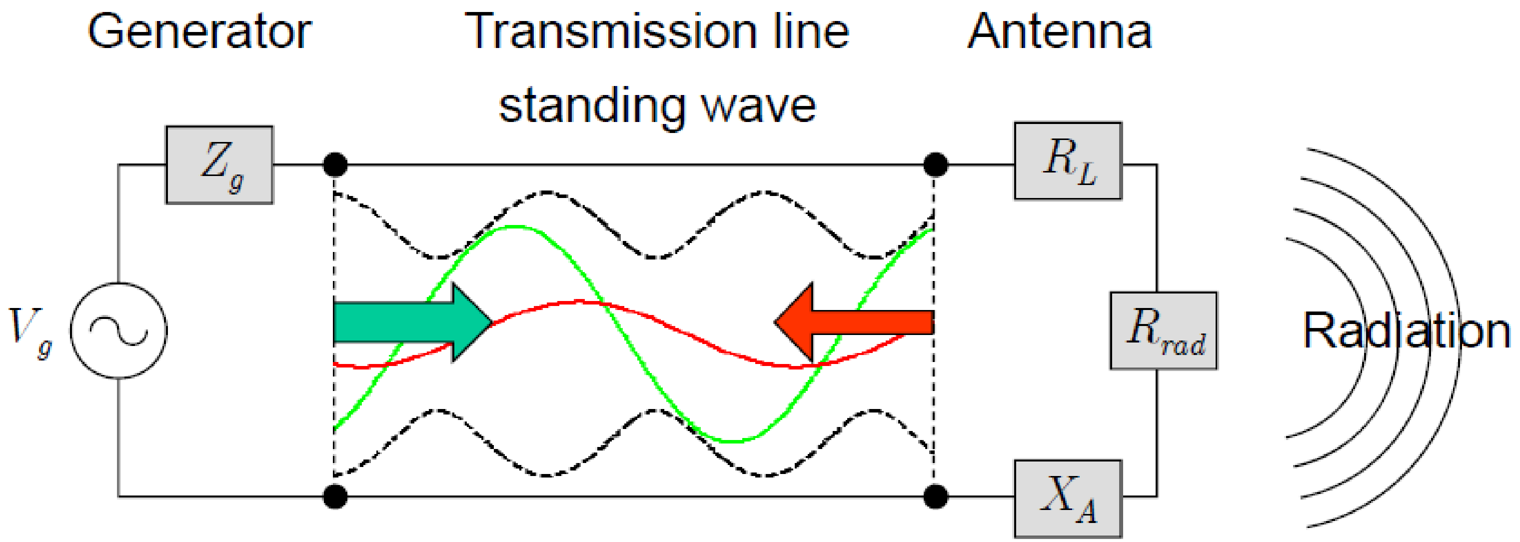


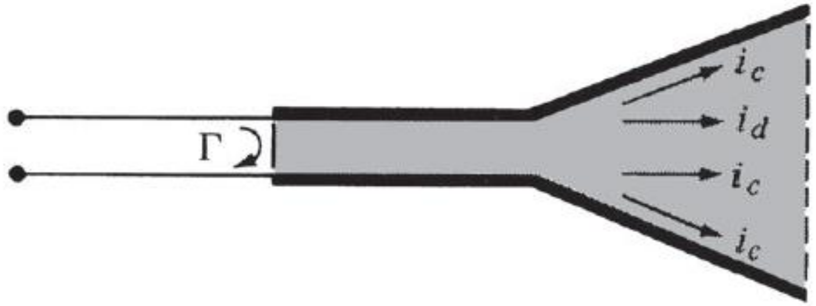
A “good” antenna system

- Some of the problems (leading to energy loss) an antenna system can face
  - Losses in transmission line
  - Losses in antenna
  - Standing waves due to impedance mismatch between antenna and line
- A good antenna system avoids these by
  - Selecting low-loss lines
  - Reducing conduction and dielectric losses in antenna
  - Matching impedance of the antenna to the characteristic impedance of the line

Efficiency = how good is the antenna at avoiding losses?

Major Types of Losses

1. reflections because of the mismatch between the transmission line and the antenna
2. I^2R losses (conduction and dielectric)



Efficiency = how good is the antenna at avoiding losses?

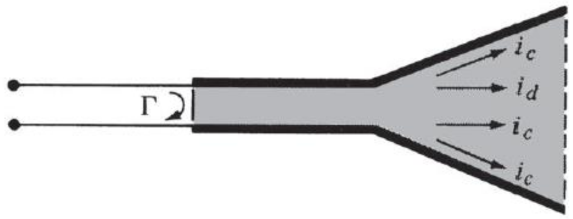
e_r = reflection (mismatch) efficiency $= (1 - |\Gamma|^2)$

$$\boxed{e_0 = e_r e_c e_d}$$

e_c = conduction efficiency

e_d = dielectric efficiency

e_0 = total efficiency (dimensionless)



Efficiency = how good is the antenna at avoiding losses?

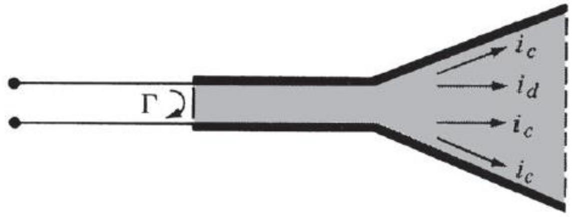
e_r = reflection (mismatch) efficiency $= (1 - |\Gamma|^2)$

$$\boxed{e_0 = e_r e_{cd} = e_{cd}(1 - |\Gamma|^2)}$$

$e_{cd} = e_c e_d$ = antenna radiation efficiency

Γ = voltage reflection coefficient at the input terminals of the antenna

Efficiency = how good is the antenna at avoiding losses?



Γ = voltage reflection coefficient at the input terminals of the antenna

$$\Gamma = (Z_{in} - Z_0)/(Z_{in} + Z_0)$$

$Z_{in} =$ Antenna input impedance

$Z_0 =$ Characterisic impedance of the transmission line

$$\text{VSWR} = \text{voltage standing wave ratio} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Gain = how good is the antenna at avoiding losses and how directed is it?

$$\boxed{\text{Gain} = 4\pi \frac{\text{radiation intensity}}{\text{total input (accepted) power}} = 4\pi \frac{U(\theta,\phi)}{P_{in}}}$$

$$P_{in} = P_{rad} \,/\, e_{cd}$$

$$P_{rad} = e_{cd} \cdot P_{in}$$

Gain depends on both, **antenna efficiency** and **directivity**.

$$\boxed{G(\theta,\phi) = e_{cd}\frac{4\pi U(\theta,\phi)}{P_{rad}} = e_{cd} D(\theta,\phi)}$$

Gain = how good is the antenna at avoiding losses and how directed is it?

$$\boxed{G_0 = G(\theta, \phi)|_{\max} = e_{cd} D(\theta, \phi)|_{\max} = e_{cd} D_0}$$

Gain along direction of maximum radiation

Questions?? Thoughts??

EE 328
Wave Propagation and Antennas

with

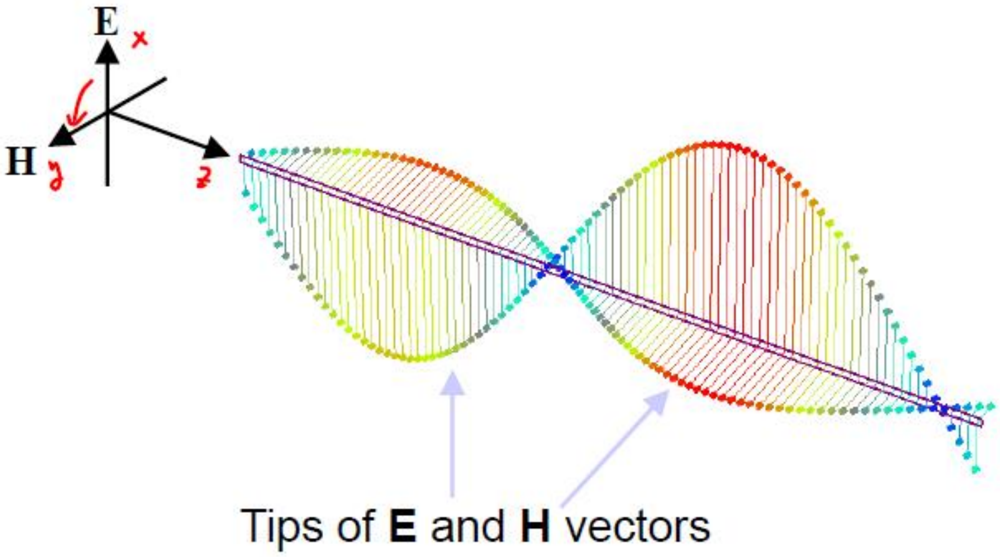
Dr. Naveed R. Butt

@

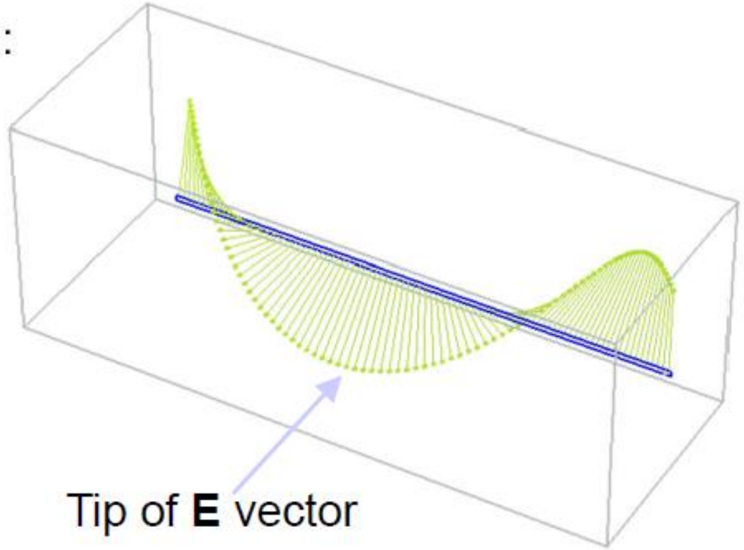
Jouf University

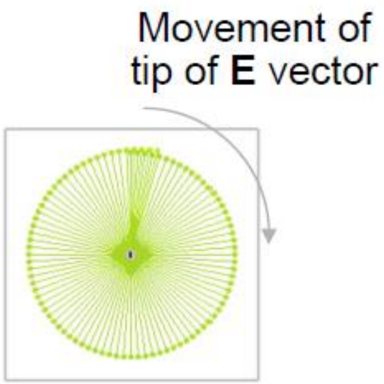
Polarization – *How the Field Vectors are Rotating*

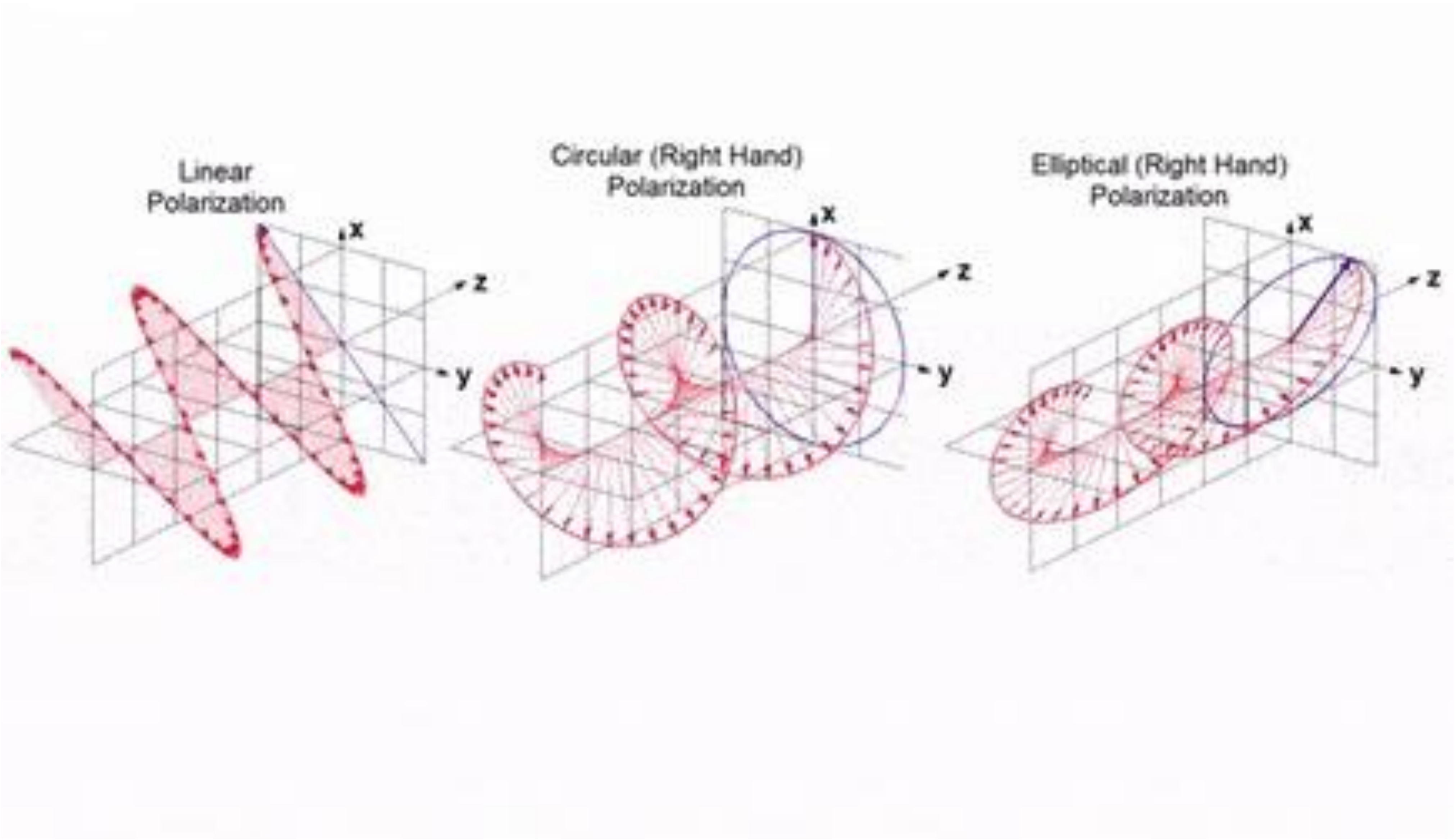
Linear polarization:

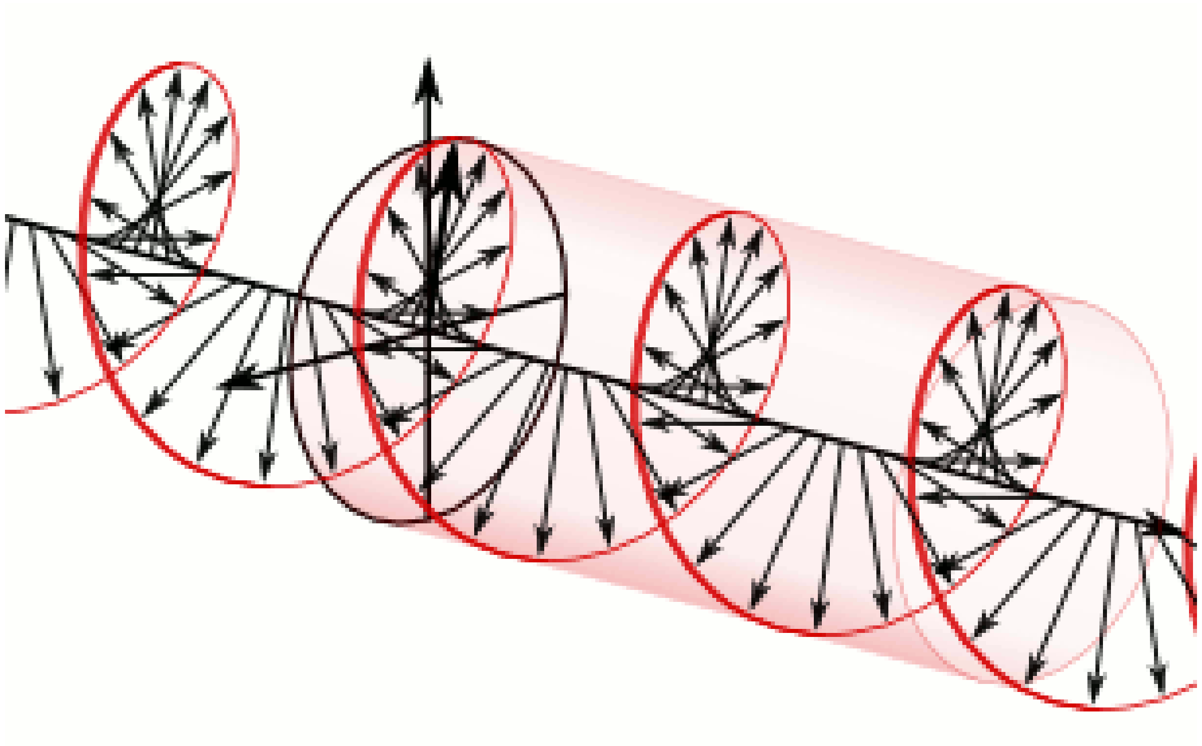


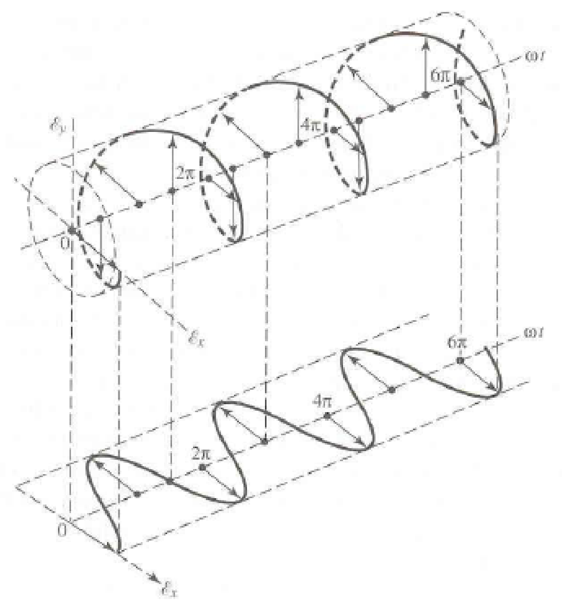
Circular polarization:






Linear
Polarization
Circular (Right Hand)
Polarization
Elliptical (Right Hand)
Polarization
x
z
y
x
z
y
x
z
y





Polarization is defined by the time-varying direction and magnitude of the electric field vector.

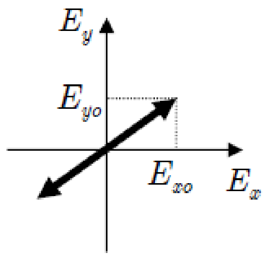
Let us consider a wave traveling along the $-z$ direction:

$$\begin{cases} E_x = E_{xo}\cos(\omega t + kz) \\ E_y = E_{yo}\cos(\omega t + kz + \Delta\phi) \end{cases}$$

We observe the behavior of the E-field vector at a fixed point as a function of time

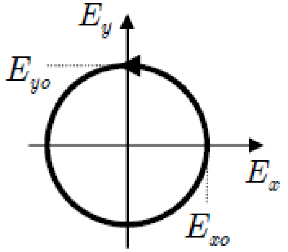
<u>Linear polarization:</u> $\Delta\phi = n\pi$

$n = 0, 1, 2, \ldots$



<u>Circular polarization:</u> $E_{xo} = E_{yo}$

$$\Delta\phi = \begin{cases} +(2n + \frac{1}{2})\pi & \text{clockwise} \\ -(2n + \frac{1}{2})\pi & \text{counter clockwise} \end{cases}$$

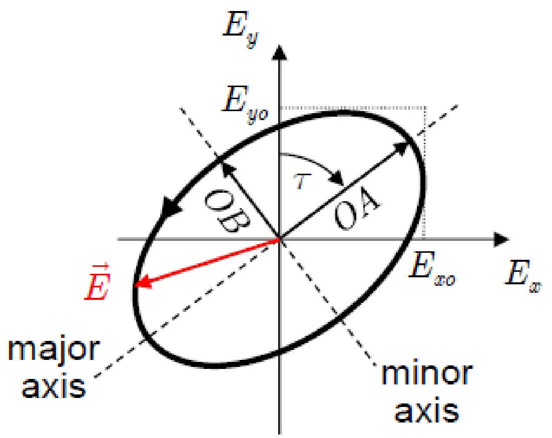


$\otimes \vec{k}$

Elliptical polarization:

either $\Delta\phi = \pm(2n + \frac{1}{2})\pi$ and $E_{xo} \neq E_{yo}$

or $\Delta\phi \neq \pm n\frac{\pi}{2}$



Axial Ratio: $AR = \frac{\text{major axis}}{\text{minor axis}} = \frac{OA}{OB}$

$$1 \leq AR < \infty$$

$$0\,\text{dB} \leq AR\ [\text{dB}] < \infty\,\text{dB}$$

$$OA = \sqrt{\frac{1}{2}\left[E_{x0}^2 + E_{y0}^2 + \left[E_{x0}^4 + E_{y0}^4 + 2E_{x0}^2E_{y0}^2\cos(2\Delta\phi)\right]^{\frac{1}{2}}\right]}$$

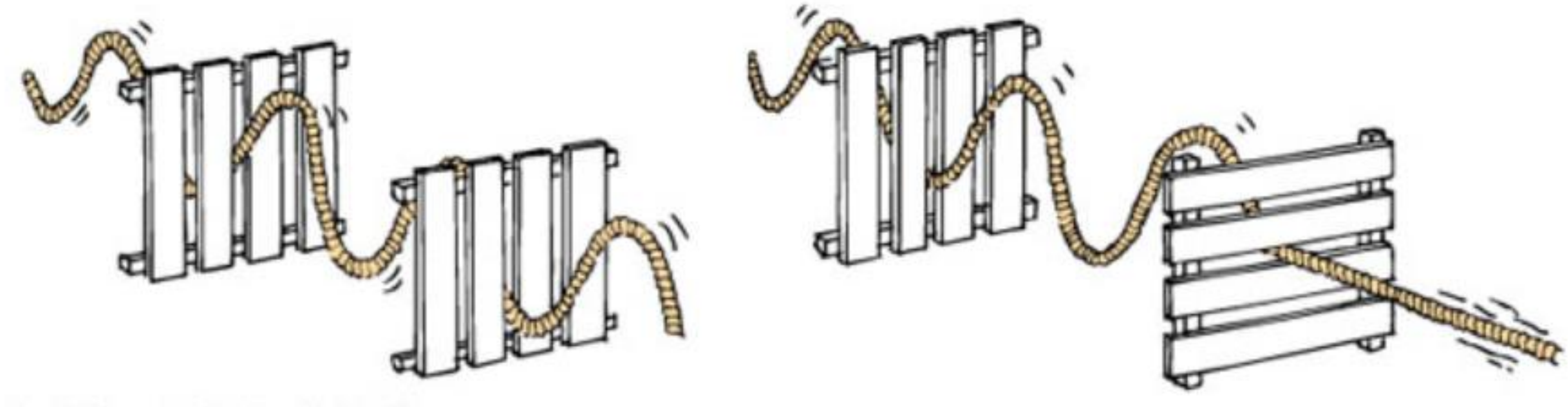
$$OB = \sqrt{\frac{1}{2}\left[E_{x0}^2 + E_{y0}^2 - \left[E_{x0}^4 + E_{y0}^4 + 2E_{x0}^2E_{y0}^2\cos(2\Delta\phi)\right]^{\frac{1}{2}}\right]}$$

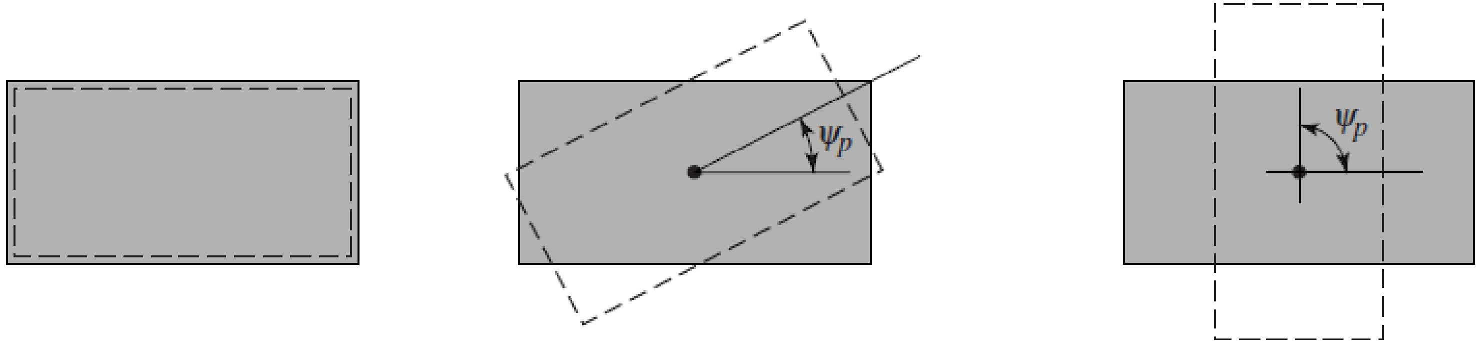
The tilt of the ellipse, relative to the y-axis, is represented by the angle:

$$\tau = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left[\frac{2E_{x0}E_{y0}}{E_{x0}^2 - E_{y0}^2}\cos(\Delta\phi)\right]$$

The **polarization ellipse** is completely determined by the axial ratio *and* the tilt *and* the rotation direction.

Polarization Loss Factor (PLF) – *How much loss does polarization mismatch cause*



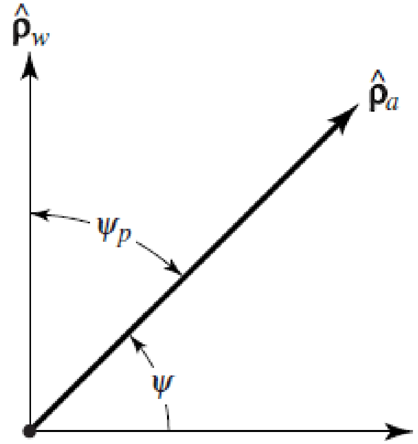


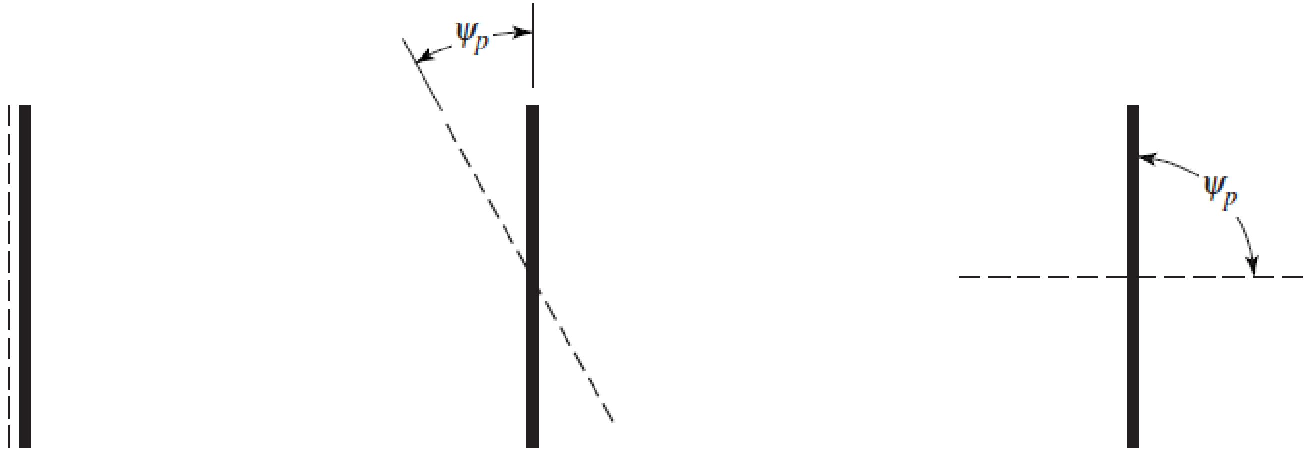
PLF = $|\hat{\rho}_w \cdot \hat{\rho}_a|^2 = 1$
(aligned)

PLF = $|\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \cos^2 \psi_p$
(rotated)

PLF = $|\hat{\rho}_w \cdot \hat{\rho}_a|^2 = 0$
(orthogonal)

(a) PLF for transmitting and receiving aperture antennas





PLF = $|\hat{\rho}_w \cdot \hat{\rho}_a|^2 = 1$
(aligned)

PLF = $|\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \cos^2 \psi_p$
(rotated)

PLF = $|\hat{\rho}_w \cdot \hat{\rho}_a|^2 = 0$
(orthogonal)

(b) PLF for transmitting and receiving linear wire antennas

In general, the polarization of the receiving antenna will not be the same as the polarization of the incoming (incident) wave. This is commonly stated as "polarization mismatch." The amount of power extracted by the antenna from the incoming signal will not be maximum because of the polarization loss. Assuming that the electric field of the incoming wave can be written as

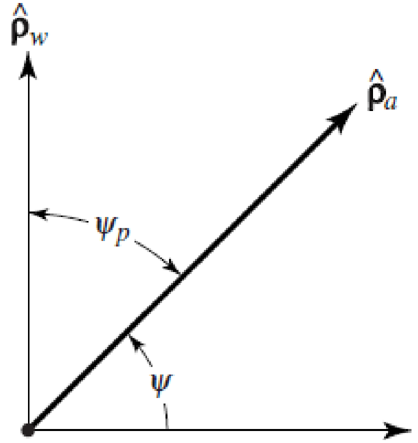
$$\mathbf{E}_i = \hat{\rho}_w E_i$$

where $\hat{\rho}_w$ is the unit vector of the wave, and the polarization of the electric field of the receiving antenna can be expressed as

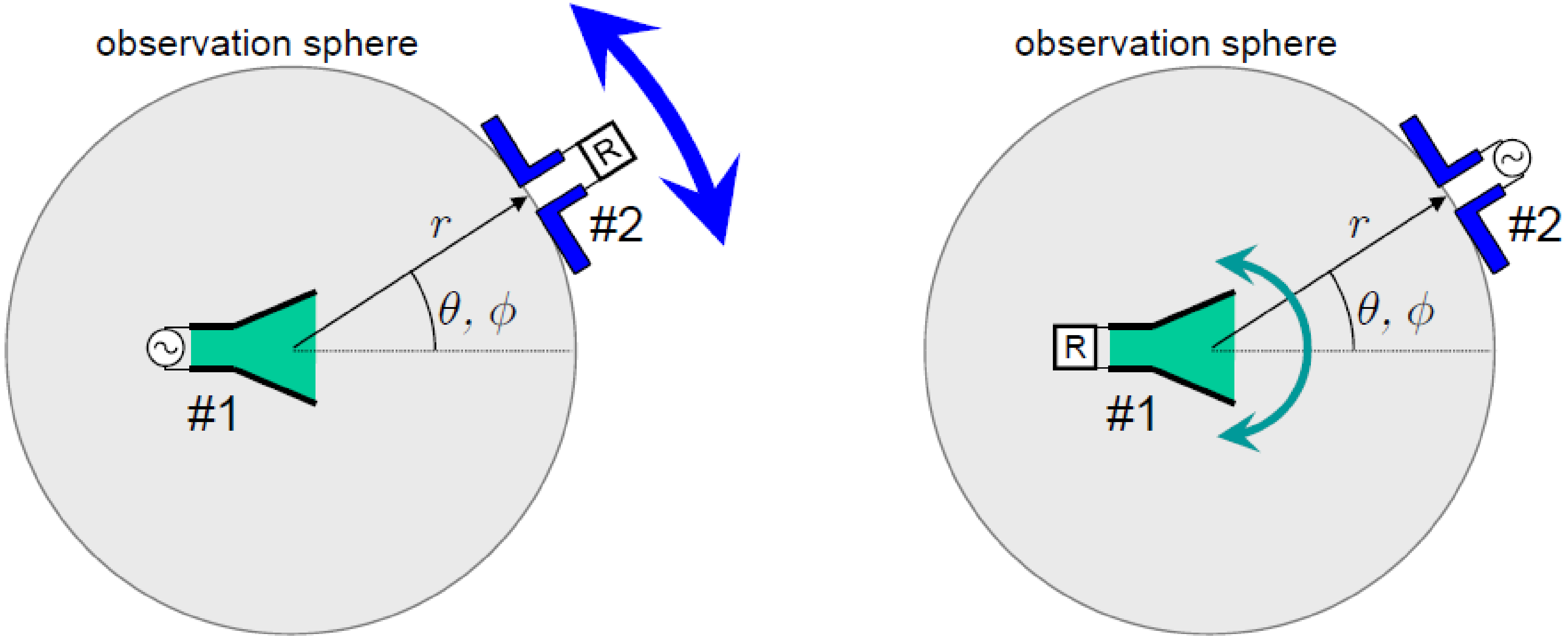
$$\mathbf{E}_a = \hat{\rho}_a E_a$$

where $\hat{\rho}_a$ is its unit vector (polarization vector), the polarization loss can be taken into account by introducing a *polarization loss factor* (PLF). It is defined, based on the polarization of the antenna in its transmitting mode, as

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |\cos\psi_p|^2 \text{ (dimensionless)}$$



Reciprocity – *When antennas have same reception patterns as transmission patterns*



Antenna arrangements for pattern measurements

<u>Reciprocity theorem:</u>

If neither the antenna nor the medium contain non-reciprocal elements, then

- The antenna has the same pattern for the receiving and the transmitting mode.
- Moving either antenna #1 or antenna #2 according to the above figure will result in the same pattern measurement.

Input Impedance – *The Impedance at the Terminals of an Antenna*

Input impedance:

- The impedance presented by an antenna at its terminals

or

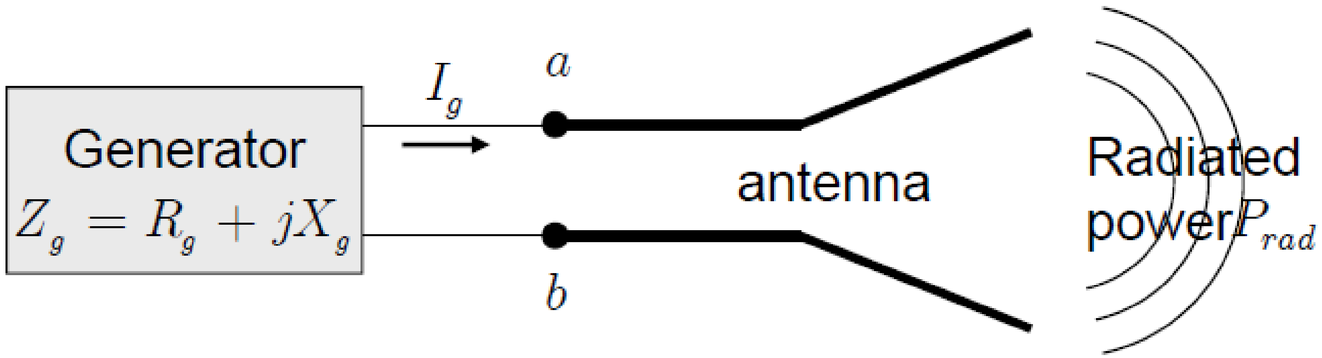
- Ratio of the voltage to current at a pair of terminals

or

- Ratio of the appropriate components of the electric to magnetic fields (integrated)

a) Transmitting mode

Input impedance of the antenna at terminals a-b :



$$\boxed{\begin{aligned} Z_A &= R_A + jX_A \\ &= R_L + R_{rad} + jX_A \end{aligned}}$$

X_A : Antenna reactance

R_L : Loss resistance

R_{rad} : **Radiation resistance**

Power radiated by the antenna $P_{rad} = \dfrac{1}{2}\left|I_g\right|^2 R_{rad}$

Equivalent circuits of the transmitting antenna:

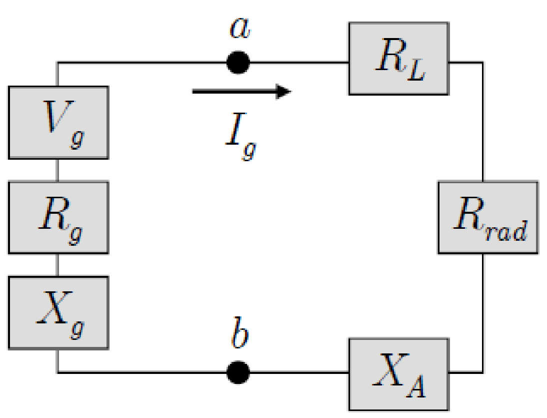
Current in the loop (Thevenin)

$$I_g = \frac{V_g}{Z_{total}} = \frac{V_g}{Z_A + Z_g} = \frac{V_g}{(R_{rad} + R_L + R_g) + j(X_A + X_g)}$$

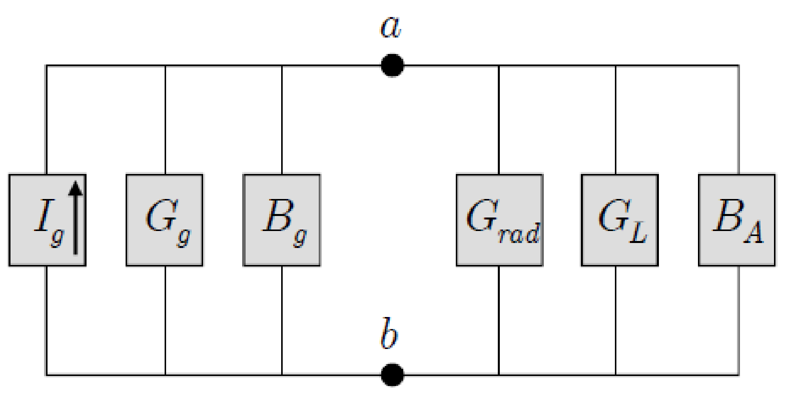
Magnitude of current

$$|I_g| = \frac{|V_g|}{\left[(R_{rad} + R_L + R_g)^2 + (X_A + X_g)^2\right]^{1/2}}$$

where $|V_g|$ is the peak generator voltage



Thevenin equivalent



Norton equivalent

- Power delivered to antenna for radiation:

$$P_{rad} = \frac{1}{2}|I_g|^2 R_{rad} = \frac{|V_g|^2}{2} \frac{R_{rad}}{(R_{rad} + R_L + R_g)^2 + (X_A + X_g)^2}$$

- Dissipated power $$P_L = \frac{1}{2}|I_g|^2 R_L = \frac{|V_g|^2}{2} \frac{R_L}{(R_{rad} + R_L + R_g)^2 + (X_A + X_g)^2}$$

- Remaining power dissipated as heat in internal resistor of generator $$P_g = \frac{|V_g|^2}{2} \frac{R_g}{(R_{rad} + R_L + R_g)^2 + (X_A + X_g)^2}$$

Conjugate matching:

Maximum power delivered to the antenna occurs for conjugate matching

$$\boxed{\begin{aligned} R_{rad} + R_L &= R_g \\ X_A &= -X_g \end{aligned}}$$

For this case

$$\left.\begin{aligned} P_{rad} &= \frac{|V_g|^2}{8}\left[\frac{R_{rad}}{(R_{rad}+R_L)^2}\right] \\ P_L &= \frac{|V_g|^2}{8}\left[\frac{R_L}{(R_{rad}+R_L)^2}\right] \\ P_g &= \frac{|V_g|^2}{8R_g} \end{aligned}\right\} \Longrightarrow \quad P_g = P_{rad} + P_L$$

For conjugate matching, the generator supplies

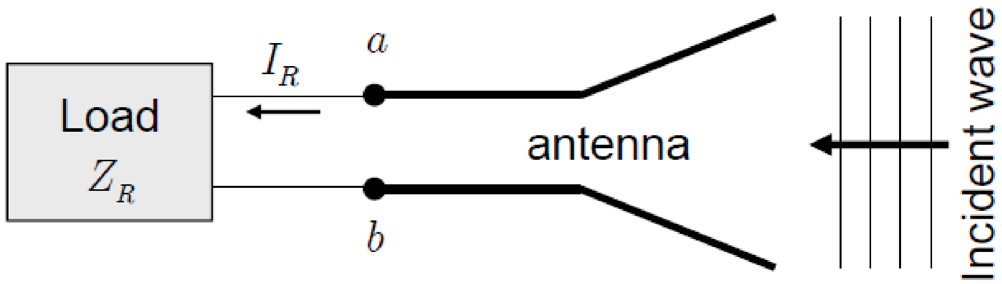
$$P_S = \frac{1}{2}V_g I_g^* = \frac{1}{2}V_g\left[\frac{V_g^*}{2(R_{rad}+R_L)}\right] = \frac{|V_g|^2}{4}\left[\frac{1}{R_{rad}+R_L}\right]$$

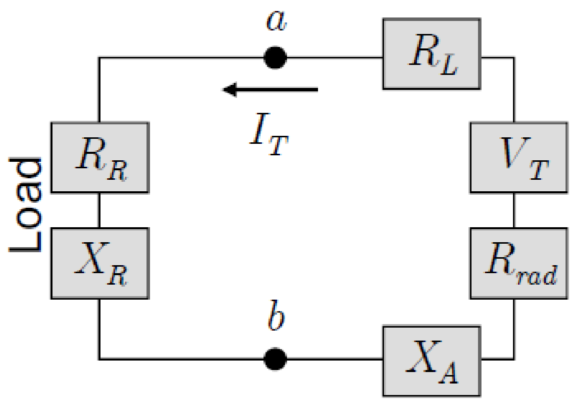
Half of this power is dissipated as heat in the internal resistance of generator, half the power is delivered to the antenna

For a lossless antenna $(e_{cd} = 1)$
(maximal achievable P_{rad})

$$P_{rad} = P_g = \frac{1}{2}P_S$$

b) Antenna in receiving mode





Thevenin equivalent

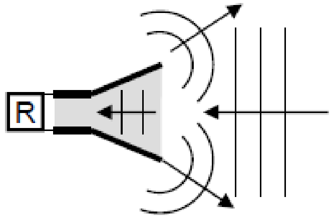
Under conjugate matching conditions:

$$P_R = \frac{|V_R|^2}{2}\left[\frac{R_R}{4(R_{rad}+R_L)^2}\right] = \frac{|V_R|^2}{8}\left[\frac{1}{R_{rad}+R_L}\right] = \frac{|V_R|^2}{8R_R}$$

Power delivered to the load

$$P_{rad} = \frac{|V_R|^2}{2}\left[\frac{R_{rad}}{4(R_{rad}+R_L)^2}\right] = \frac{|V_R|^2}{8}\left[\frac{R_{rad}}{R_{rad}+R_L}\right]$$

Scattered power (reradiated power)



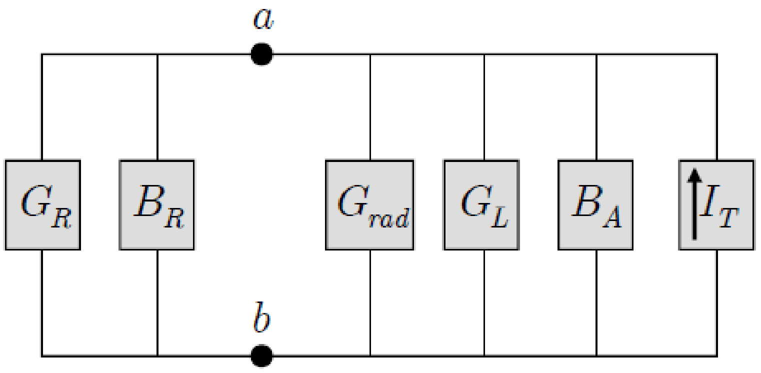
$$P_L = \frac{|V_R|^2}{8}\left[\frac{R_L}{(R_{rad}+R_L)^2}\right]$$

Losses

Captured power

$$P_c = \frac{1}{2}V_R I_R^* = \frac{1}{2}V_R\left[\frac{V_R^*}{2(R_{rad}+R_L)}\right] = \frac{|V_R|^2}{4}\left[\frac{1}{R_{rad}+R_L}\right]$$

Lossless case: $P_R = P_{rad} = \frac{1}{2}P_c$



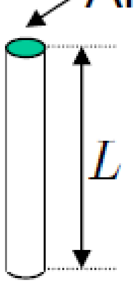
Norton equivalent

Antenna Radiation Efficiency – *How Good in an Antenna in Converting Provided Power to Radiation*

Antenna Radiation Efficiency

$$e_{cd} = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_L + R_{rad}}$$ [dimensionless]

Area A

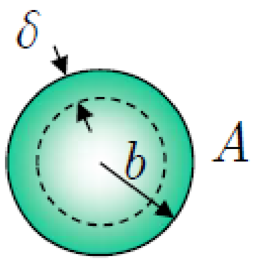


Conductivity σ

<u>Conduction losses:</u>

For a metal rod of length L and radius b

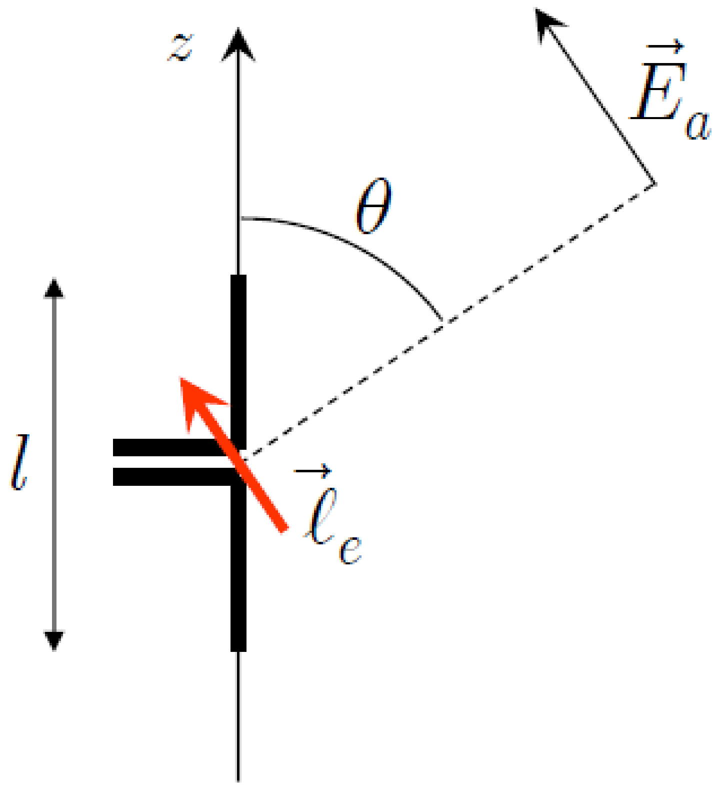
$$R_{DC} = \frac{1}{\sigma}\frac{L}{A}$$



$$R_{RF} = \frac{1}{\sigma}\frac{L}{2\pi b\delta} = \frac{L}{2\pi b}\sqrt{\frac{\omega\mu_0}{2\sigma}}$$

Skin depth $\delta = \sqrt{\frac{2}{\omega\mu_0\sigma}}$

Effective Length – *How relative orientation of wire antenna and Electric field affect the "useful" length of the antenna*



Effective length (effective height)

The (vector) effective length or effective height is a far-field quantity (applied mostly to wire antennas)

$$\vec{\ell}_e = \hat{a}_\theta l_\theta(\theta,\phi) + \hat{a}_\phi l_\phi(\theta,\phi)$$

<u>Transmitting mode</u>

$\vec{\ell}_e$ relates the far-zone field $\vec{E}_a$ with the current I_{in} at its terminals according to

$$\vec{E}_a = \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi \equiv -j\eta \underbrace{\frac{kI_{in}}{4\pi r}}_{1/r \text{ term}} \vec{\ell}_e \underbrace{e^{-jkr}}_{\text{Oscillating term}}$$

$$k = \frac{2\pi}{\lambda}$$

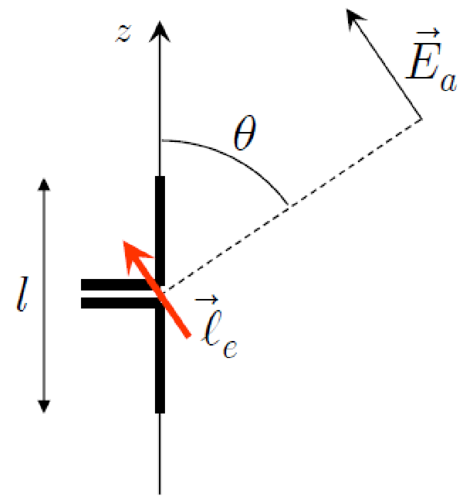
<u>Receiving mode</u>

Useful for relating the open-circuit voltage V_{oc} to the incoming field

$$V_{oc} = \vec{E}_i \cdot \vec{\ell}_e$$

Remarks:

- The vector effective length is only a function of the direction (**far-field** quantity).
- In receiving mode:
  - Maximal open-circuit voltage $V_{oc} = \vec{E}_{ic} \cdot \vec{\ell}_e$ will occur for a wave impinging at normal incidence (θ = 90°).
  - Polarization efficiency is taken into account.



Example:

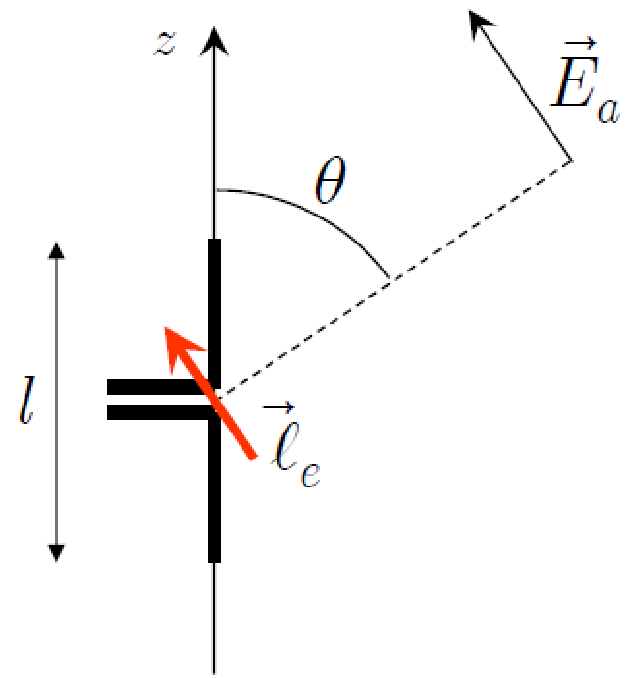
Small dipole ($l < \lambda/10$)

Triangular current distribution

Far-field pattern:

will be derived later

$$\vec{E}_a = \hat{a}_\theta j\eta \frac{kI_{in}le^{-jkr}}{8\pi r}\sin\theta$$



Vector effective length:

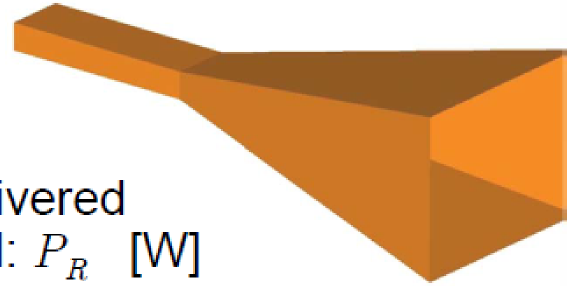
$$\vec{\ell}_e = \hat{a}_\theta \frac{l}{2}\sin\theta$$

$$\begin{cases} \vec{\ell}_e(\theta = 90^\circ) = \hat{a}_\theta \dfrac{l}{2} \\ \vec{\ell}_e(\theta = 0^\circ) = \vec{0} \end{cases}$$

Effective Area – *How good is an aperture antenna at capturing incident radiation*

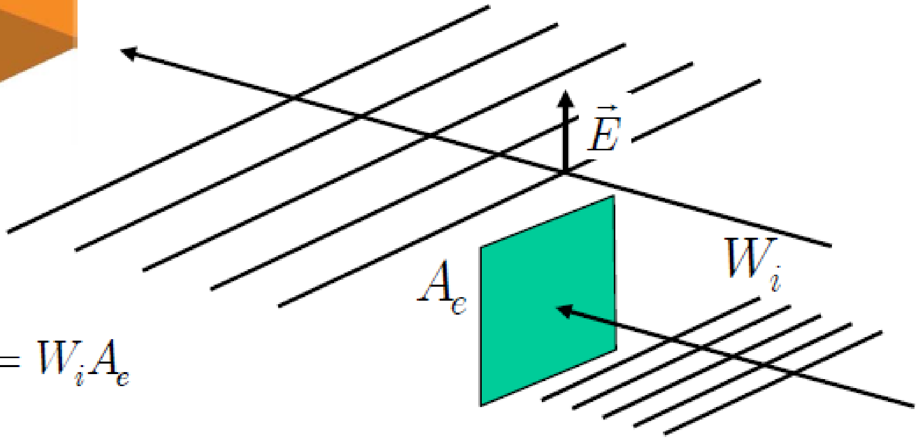
Effective area (aperture)

The effective area describes the power-capturing characteristics of an antenna and is especially applied to aperture antennas.



Power delivered to the load: P_R [W]

Plane wave with power density W_i [W/m²]



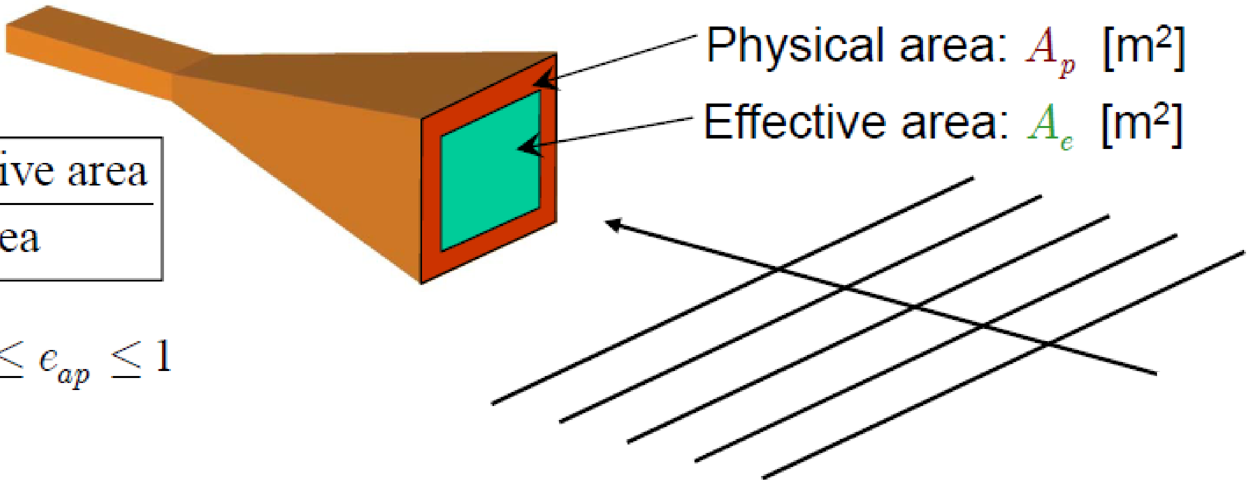
Effective area: A_e [m²]

$$\boxed{A_e = \frac{P_R}{W_i}} \quad \Leftrightarrow \quad P_R = W_i A_e$$

Aperture efficiency

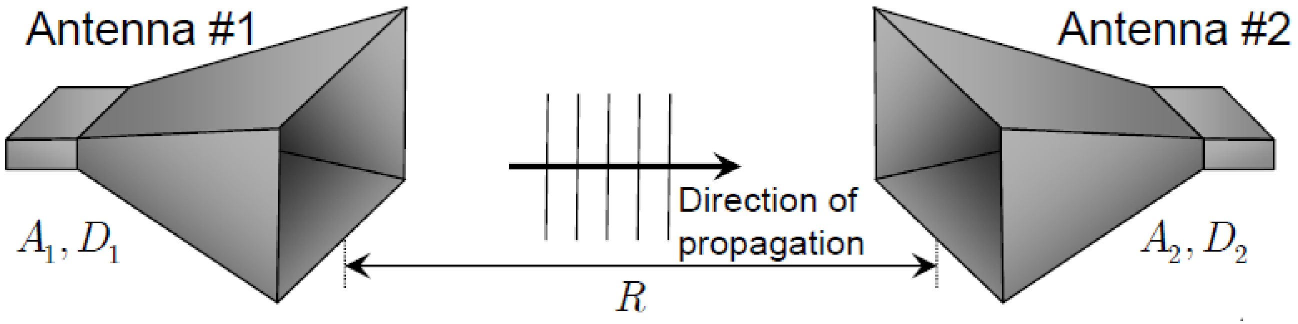
(for aperture antennas)

$$\boxed{e_{ap} = \frac{A_{em}}{A_p} = \frac{\text{maximum effective area}}{\text{physical area}}}$$



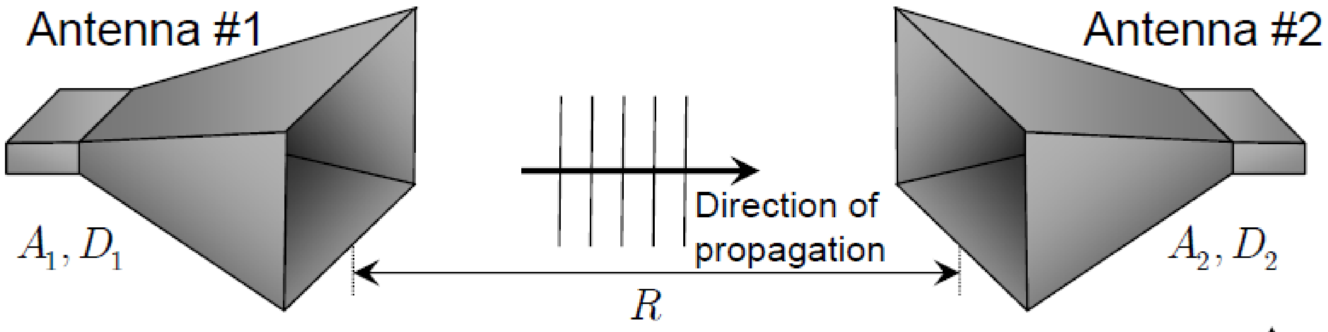
For aperture antennas: $0 \le e_{ap} \le 1$

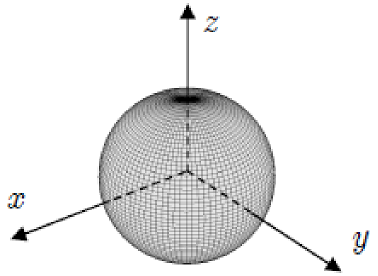
*How **directivity of transmitting antenna** and **effective area of receiving antenna** affect the transmission and reception of radiation*



Maximum directivity and maximum effective Area

$$\boxed{D_0 \Longleftrightarrow A_{em}}$$





<u>Transmission:</u>

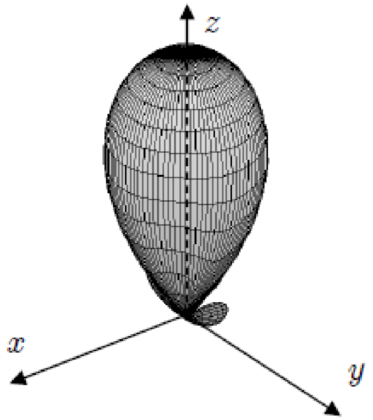
The radiated power density of an isotropic radiator at distance R is

$$W_0 = \frac{P_t}{4\pi R^2}$$

Indices: t: transmitted
r: received

If we include the directivity of the transmitting antenna (#1)

$$W_1 = W_0 D_1 = \frac{P_t D_1}{4\pi R^2}$$

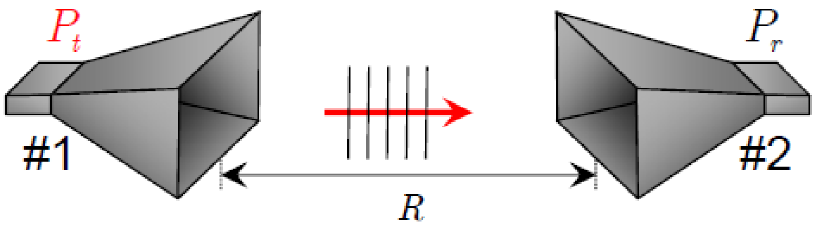


Reception:

The power collected at the receiver and delivered to the load is

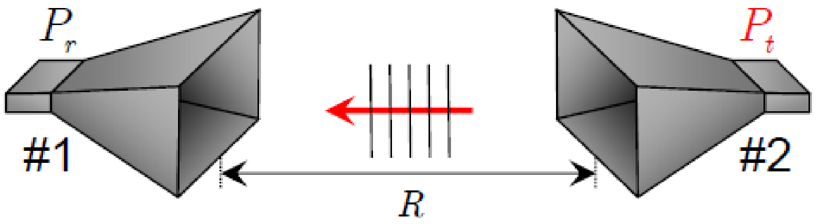
$$P_r = W_1 A_2 = \frac{P_t D_1 A_2}{4\pi R^2}, \qquad \frac{P_r}{P_t} = \frac{D_1 A_2}{4\pi R^2}$$

A_2 = effective area of receiving antenna



Interchanging transmitting and receiving antennas

$$\frac{P_r}{P_t} = \frac{D_2 A_1}{4\pi R^2}$$



Reciprocity:

$$\Rightarrow \; D_1 A_2 = D_2 A_1$$

Maximum directivity

$$\boxed{\frac{D_{0t}}{A_{tm}} = \frac{D_{0r}}{A_{rm}}}$$

Isotropic transmitter:

$$A_{tm} = \frac{A_{rm}}{D_{0r}}\bigg|_{D_{0t}=1} = \ldots = \frac{\lambda^2}{4\pi}$$

Maximum effective aperture vs. directivity:

(Antenna losses included)

$$\boxed{A_{em} = \frac{\lambda^2}{4\pi} G_0 = e_{cd} \frac{\lambda^2}{4\pi} D_0}$$

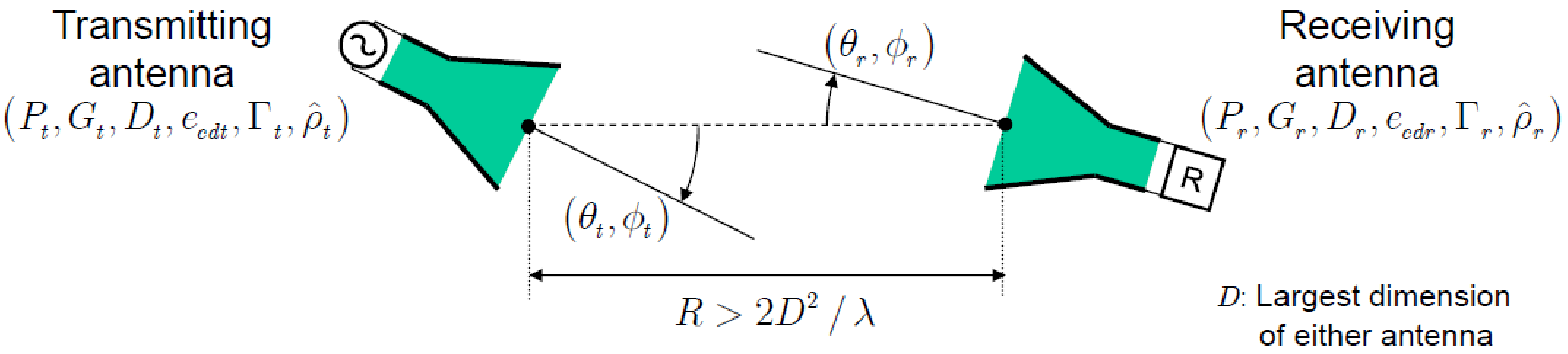
$$\boxed{D_0 \Longleftrightarrow A_{em}}$$

If reflection and polarization losses are included $A_{em} = e_{cd}(1 - |\Gamma|^2)\dfrac{\lambda^2}{4\pi} D_0 \left|\hat{\rho}_w \cdot \hat{\rho}_a\right|^2$

Friis Transmission Equation – *How power given to transmitting antenna relates to "useful" power finally available at reception*

$$P_t \Longleftrightarrow P_r$$

Friis Transmission Equation relates the power delivered to the load of the receiving antenna to the input power of the transmitting antenna.



Transmission:

Power density from the transmitter at distance R

$$W_t = \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R^2} = e_{cdt} \frac{P_t D_t(\theta_t, \phi_t)}{4\pi R^2}$$

Reception:

Power collected by the receiver (taking into account polarization mismatch)

$$P_r = e_p W_t A_r = \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R^2} A_r \cdot |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

Note:
$A_r = A_r(\theta_r, \phi_r)$

The effective aperture is related to the gain by the following relation

$$A_r = \frac{\lambda^2}{4\pi} G_r(\theta_r, \phi_r)$$

We can then write $$P_r = \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R^2} \frac{\lambda^2}{4\pi} G_r(\theta_r, \phi_r) \cdot \left|\hat{\rho}_t \cdot \hat{\rho}_r\right|^2$$

$$\frac{P_r}{P_t} = e_{cdr} e_{cdt} \frac{\lambda^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{(4\pi R)^2} \cdot \left|\hat{\rho}_t \cdot \hat{\rho}_r\right|^2$$

at the terminals of the antennas

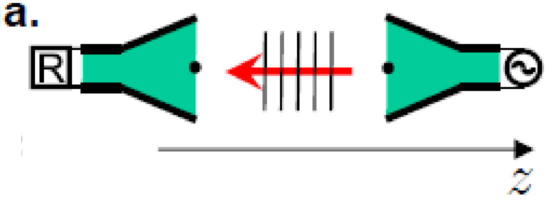
To consider the input and received powers, the mismatch of the lines must be taken into account

$$\boxed{\frac{P_r}{P_t} = e_{cdt} e_{cdr} \left(1 - |\Gamma_t|^2\right)\left(1 - |\Gamma_r|^2\right)\left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) \left|\hat{\rho}_t \cdot \hat{\rho}_r\right|^2}$$

For aligned, reflection and polarization matched antennas

$$\boxed{\frac{P_r}{P_t} = \underbrace{\left(\frac{\lambda}{4\pi R}\right)^2}_{\text{Free-space loss factor}} G_{0t} G_{0r}}$$

Example: A 30 dB right-hand circularly polarized antenna in a radio link radiates 5 W of power at 2 GHz. The receiving antenna has an impedance mismatch at its terminals, which leads to a VSWR of 2. The receiving antenna is about 95% efficient and has a field pattern near the beam maximum given by $\vec{E}_R = (2\hat{a}_x + j\hat{a}_y)\vec{F}_R(\theta,\phi)$ (definition towards +z). The distance between the two antennas is 4000 km, and the receiving antenna is required to deliver 10^{-14} W to the receiver. **Determine the maximum effective aperture of the receiving antenna.**



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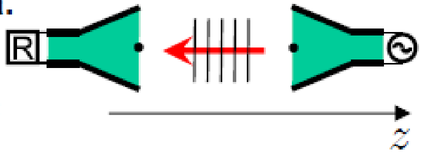


Solution:

$$\underbrace{P_r}_{10^{-14}\,\mathrm{W}} = \underbrace{P_t e_{cdt}\left(1-|\Gamma_t|^2\right)}_{5\mathrm{W}} \underbrace{e_{cdr}}_{0.95} \underbrace{\left(1-|\Gamma_r|^2\right)}_{} \left(\frac{\lambda}{4\pi \underbrace{R}_{4000\,\mathrm{km}}}\right)^2 D_t \underbrace{D_r}_{\text{find}} \left|\hat{\rho}_t \cdot \hat{\rho}_r\right|^2$$

$$\Rightarrow A_r = \frac{\lambda^2}{4\pi} D_r$$

$$|\Gamma_r| = \left|\frac{VSWR-1}{VSWR+1}\right| = \frac{2-1}{2+1} = 1/3 \quad \Rightarrow \left(1-|\Gamma_r|^2\right) = 8/9$$

$$D_t = 30\,\mathrm{dB} = 10^3\,, \qquad \lambda = c/f = 0.15\,\mathrm{m}$$

$$\left.\begin{aligned} \hat{\rho}_t &= \left(\hat{a}_x + j\hat{a}_y\right)/\sqrt{2} \quad \text{RHCP (towards -z)} \\ \hat{\rho}_r &= \left(2\hat{a}_x + j\hat{a}_y\right)/\sqrt{5} \end{aligned}\right\} \quad \left|\hat{\rho}_t \cdot \hat{\rho}_r\right|^2 = \left|\frac{1}{\sqrt{2}}\begin{pmatrix}1 & j\end{pmatrix} \cdot \frac{1}{\sqrt{5}}\begin{pmatrix}2 \\ j\end{pmatrix}\right|^2 = 0.1$$

$$\Rightarrow D_r = \frac{10^{-14}}{5 \cdot 0.95 \cdot 8/9 \cdot (0.15/4\cdot\pi\cdot 4\cdot 10^6)10^3 \cdot 0.1} = 2.66$$

Since losses (e_{cdr}) have already been included,

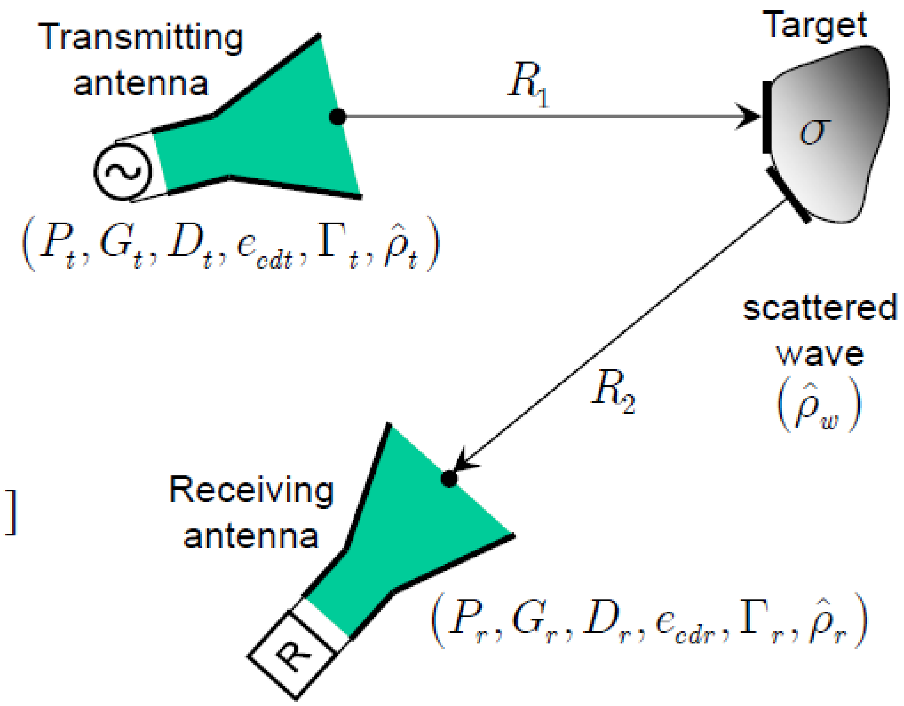
$$A_r = \frac{\lambda^2}{4\pi} D_r = \frac{0.15^2\,\mathrm{m}^2}{4\pi} 2.66 \quad \Rightarrow A_r = 0.00476\,\mathrm{m}^2$$

Radar Range Equation – *How good is a radar system at throwing and collecting power from a target*

$$P_t \Longleftrightarrow \sigma \Longleftrightarrow P_r$$

12. Radar range equation

$$P_t \Longleftrightarrow \sigma \Longleftrightarrow P_r$$



- Target echo area σ
  (radar cross section, RCS)

$$\sigma = \lim_{R\to\infty}\left[4\pi R^2 \frac{W_s}{W_i}\right] = \lim_{R\to\infty}\left[4\pi R^2 \frac{|E^s|^2}{|E^i|^2}\right] \quad [\mathrm{m}^2]$$

Index i: Incident
Index s: Scattered

The RCS is the area that is necessary to intercept the amount of power that - when radiated **isotropically** - produces at the receiver a density equal to that scattered by the target.

- Power captured by target $\quad P_c = \sigma W_t = \sigma \dfrac{P_t G_t(\theta_t, \phi_t)}{4\pi R_1^2} = e_{cdt}\sigma \dfrac{P_t D_t(\theta_t, \phi_t)}{4\pi R_1^2}$

- Scattered power density at the receiver $\quad W_s = \dfrac{P_c}{4\pi R_2^2} = e_{cdt}\sigma \dfrac{P_t D_t(\theta_t, \phi_t)}{(4\pi R_1 R_2)^2}$

  (Power P_c reradiated isotropically)

- Power collected by load at receiver $P_r = A_r W_s = e_{cdt} e_{cdr} \sigma \dfrac{P_t D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \left(\dfrac{\lambda}{4\pi R_1 R_2} \right)^2$

Including reflection and polarization losses, we can write the radar range equation

$$\boxed{\frac{P_r}{P_t} = e_{cdt} e_{cdr} \left(1 - |\Gamma_t|^2\right)\left(1 - |\Gamma_r|^2\right) \sigma \frac{D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2 \left| \hat{\rho}_w \cdot \hat{\rho}_r \right|^2}$$

If the two antennas are aligned and matched, this reduces to

$$\boxed{\frac{P_r}{P_t} = \sigma \frac{G_{0t} G_{0r}}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2}$$

Example: The maximum radar cross section of a resonant $\lambda/2$ dipole is approximately $0.86\lambda^2$. For a monostatic system (i.e., transmitter and receiver at the same location), find the received power if the transmitted power is 100 W, the distance of the dipole from the transmitting and receiving antennas is 100 m, the gain of the transmitting and receiving antennas is 15 dB each, and the frequency of operation is 3 GHz. Assume a polarization loss factor of –1 dB.

Solution:

$$P_r = P_t \sigma \frac{G_{0t} G_{0r}}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2 \underbrace{\left| \hat{\rho}_t \cdot \hat{\rho}_r \right|^2}_{-1\text{dB} \equiv 0.7943}$$

$$G_{0t} = G_{0r} = 15\text{dB} = 31.6228\ , \quad \lambda = c/f = 0.1m$$

$$\Rightarrow P_r = \ldots = 34.4\ \text{pW}$$

Questions?? Thoughts??

EE 328
Wave Propagation and Antennas

with

Dr. Naveed R. Butt

@

Jouf University

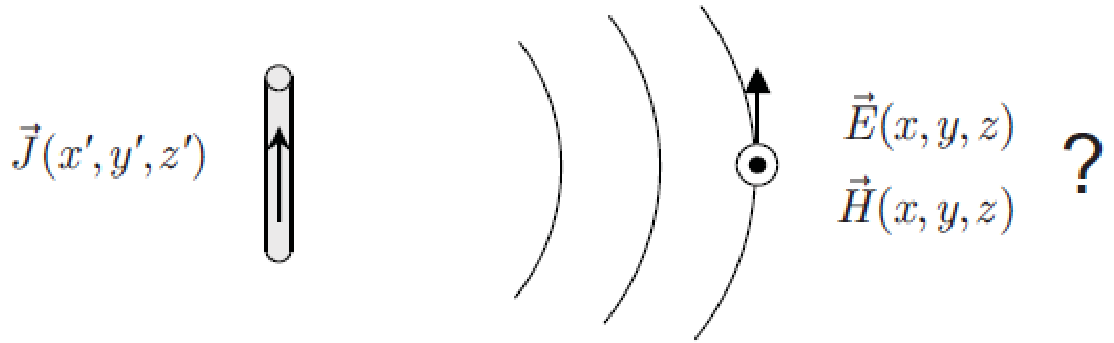
Previously we have discussed in general terms …

- **Formal (mathematical) definitions** for characterizing an antenna’s radiation pattern, like
  - Radiation Power Density
  - Radiation Intensity
  - Average Radiated Power
  - Directivity
  - Efficiency
  - Gain

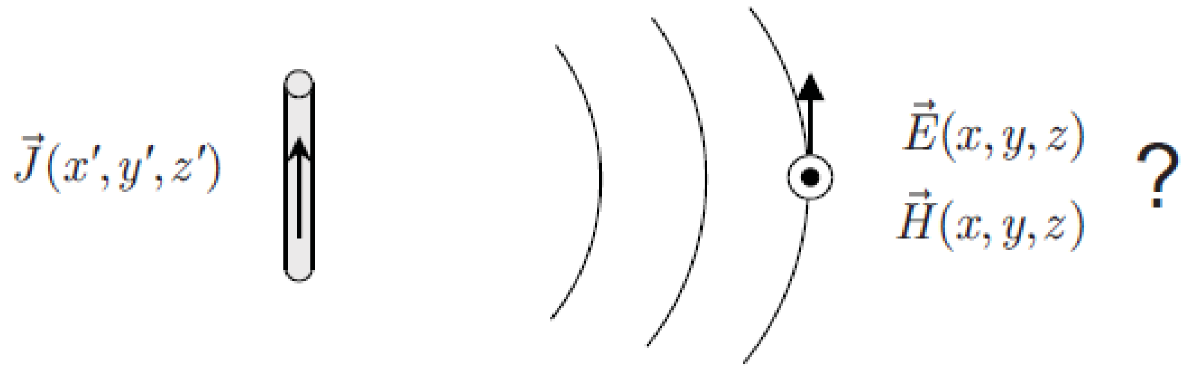
Next …

- We will discuss these properties for various kinds of antennas in common use. These include
  - Wire Antennas (dipole, loop) and Antenna Arrays
- But before that, we will do a quick revision of Maxwell's Equations, and discuss some strategies for calculating electric and magnetic fields

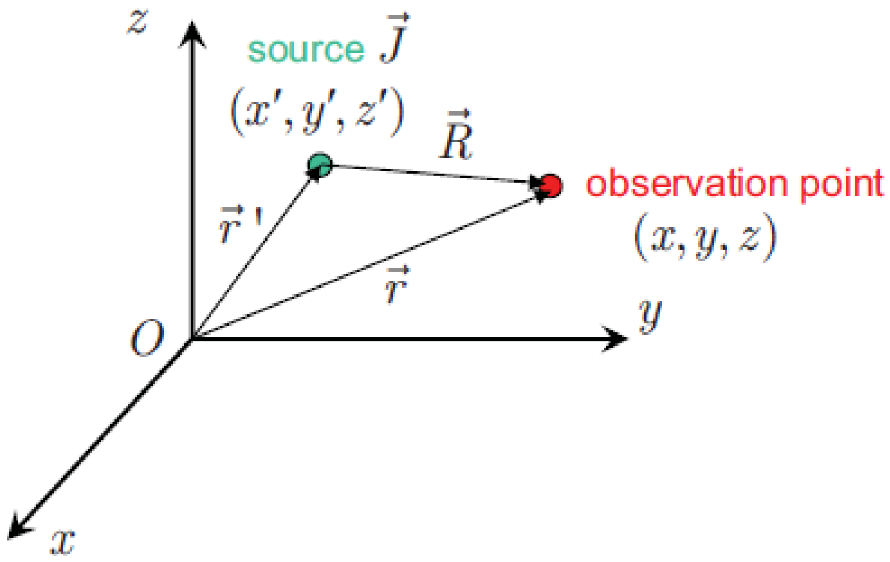
Main Objective - *Evaluating Fields Radiated by a Current Source*



The general problem is to find the fields radiated by a current source



We will use the following arrangement and notations



$\vec{r}\,'$: vector from origin to source

$\vec{r}$: vector from origin to observation point

$\vec{R}$: vector from source to observation point

Maxwell's equations

The physics of the fields radiated by an antenna are described by Maxwell's equations. For harmonic variations of the fields $(e^{j\omega t})$, we can write

$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E} + \vec{J} \qquad (1) \qquad \text{with } \vec{J} \begin{cases} \neq 0 & \text{in source region} \\ = 0 & \text{elsewhere} \end{cases}$$

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \qquad (2)$$

$$\nabla \cdot \vec{D} = \varepsilon \nabla \cdot \vec{E} = \rho \qquad (3)$$

$$\nabla \cdot \vec{B} = \mu \nabla \cdot \vec{H} = 0 \qquad (4)$$

$E,\ H$: Electric and magnetic field
D: Dielectric displacement
B: Magnetic flux (induction)
J: Electric source current density
ρ: Charge density

$$\boxed{\begin{aligned} \nabla \times \vec{V} &= \operatorname{curl} \vec{V} \\ \nabla \cdot \vec{V} &= \operatorname{div} \vec{V} \\ \nabla \phi &= \operatorname{grad} \phi \end{aligned}}$$

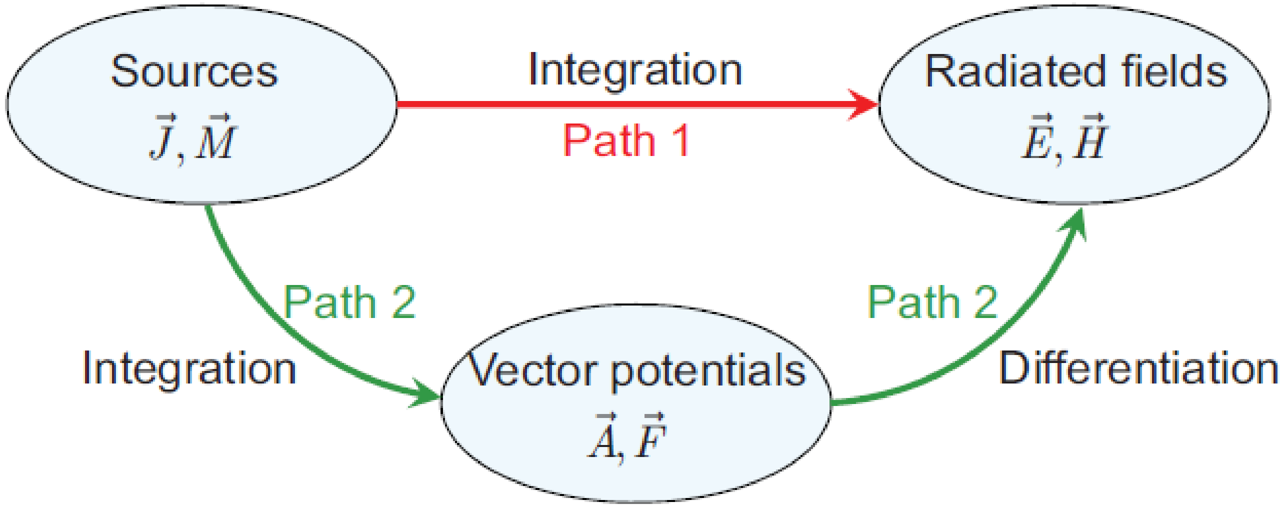
Note that by using the vector identity $\nabla \cdot \left(\nabla \times \vec{V}\right) \equiv 0$, we can show that (4) follows from (2).

The continuity equation $\nabla \cdot \vec{J} = -j\omega\rho \qquad (5)$

can be derived from (1) and (3).

Vector potentials

To analyze the fields radiated by sources, it is common practice to introduce auxiliary functions known as **vector potentials**, which will aid in the solution of Maxwell's equations.



- The two-step procedure usually involves simpler integrations than the direct path.
- All field functions can be retrieved from the vector potentials through differentiations.
- Other auxiliary functions may be used (Hertz potentials): $\vec{\Pi}_e, \vec{\Pi}_h$
- The use of vector potentials basically permits to reduce the number of unknowns.

Summary of the analysis procedure

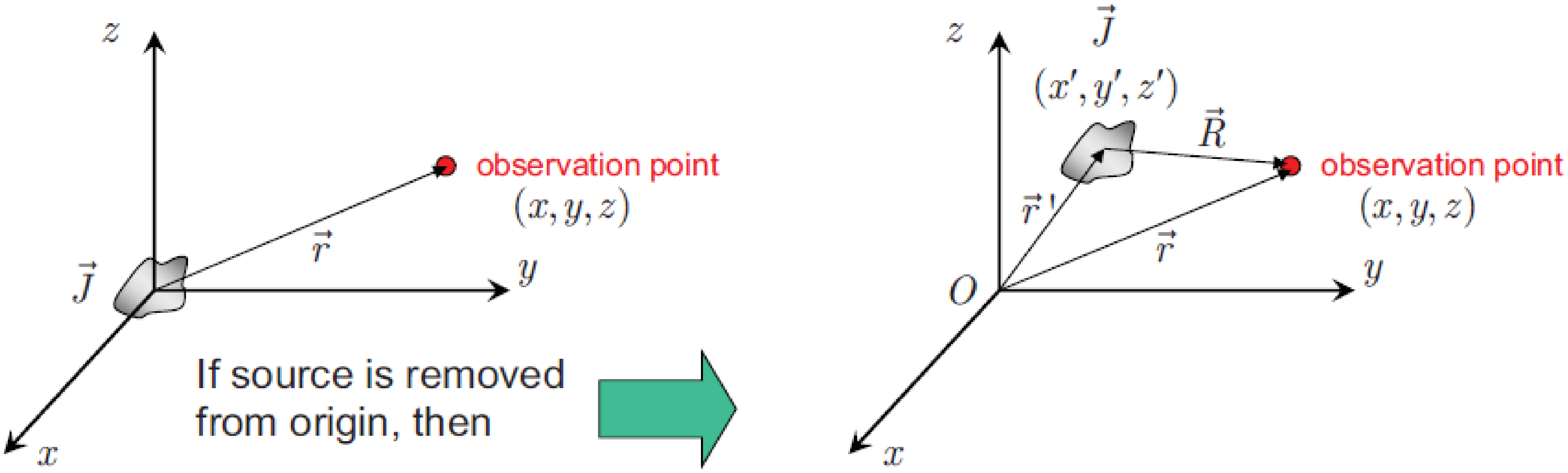
1) Specify the sources $\vec{J}$ and $\vec{M}$

2) Find the vector potential $\vec{A}$ and $\vec{F}$

3) Find the field contributions $\vec{E}_F$ and $\vec{H}_A$

4) The contributions $\vec{E}_A$ and $\vec{H}_F$ can be found either directly using $\vec{A}$ and $\vec{F}$ or through application of Maxwell's curl equations on $\vec{E}_F$ and $\vec{H}_A$

- <u>General solution of the vector potential equation:</u>



$$\boxed{\begin{aligned}\vec{A}(x,y,z) &= \frac{\mu}{4\pi}\iiint_V \vec{J}(x',y',z')\frac{e^{-jkR}}{R}dv' \\ \vec{F}(x,y,z) &= \frac{\varepsilon}{4\pi}\iiint_V \vec{M}(x',y',z')\frac{e^{-jkR}}{R}dv'\end{aligned}}$$

The total fields

The radiated fields will be the superposition of two contributions:

- fields generated by electric current sources $\vec{J}$
- fields generated by magnetic current sources $\vec{M}$

The total fields from these two contributions can be written in terms of $\vec{A}$ and $\vec{F}$

$$\boxed{\begin{aligned} \vec{E} = \vec{E}_A + \vec{E}_F &= -j\omega\vec{A} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot\vec{A}) - \frac{1}{\varepsilon}\nabla\times\vec{F} &&= \frac{1}{j\omega\varepsilon}\nabla\times\vec{H}_A - \frac{1}{\varepsilon}\nabla\times\vec{F} \\ \vec{H} = \vec{H}_A + \vec{H}_F &= -\frac{1}{\mu}\nabla\times\vec{A} - j\omega\vec{F} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot\vec{F}) &&= -\frac{1}{\mu}\nabla\times\vec{A} - \frac{1}{j\omega\mu}\nabla\times\vec{E}_F \end{aligned}}$$

using Maxwell's curl equations

Far-field approximation

Fields radiated by antennas of finite dimensions are spherical waves in the far-field. Therefore, we use spherical coordinates to write a general form of the vector potential

$$\vec{A} = \vec{a}_r A_r(r,\theta,\phi) + \vec{a}_\theta A_\theta(r,\theta,\phi) + \vec{a}_\phi A_\phi(r,\theta,\phi)$$

Amplitude variations in r for each component are of the form $1/r^n$.

electric sources:

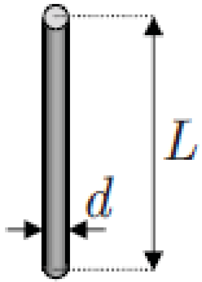
$$\boxed{\begin{aligned} &\vec{E}_A \approx -j\omega\vec{A} \qquad (E_r \simeq 0) \\ &\vec{H}_A \approx \vec{a}_r \times \frac{\vec{E}_A}{\eta} = -j\frac{\omega}{\eta}\left(\vec{a}_r \times \vec{A}\right) \end{aligned}}$$

magnetic sources:

$$\boxed{\begin{aligned} &\vec{H}_F \approx -j\omega\vec{F} \qquad (H_r \simeq 0) \\ &\vec{E}_F \approx -\eta\left(\vec{a}_r \times \vec{H}_F\right) = j\omega\eta\left(\vec{a}_r \times \vec{F}\right) \end{aligned}}$$

$\eta \simeq 120\pi\ \Omega$ in free space

Infinitesimal Dipole – *Radiated Fields*

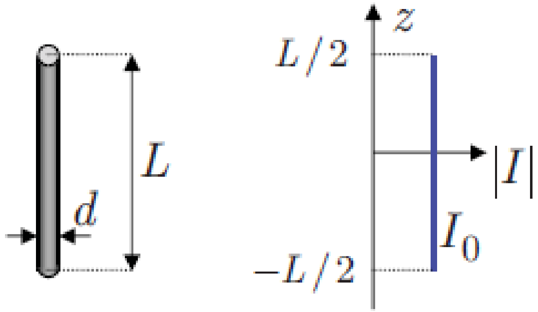


$$L \ll \lambda, \;\; d \ll \lambda$$

The Hertzian dipole (or infinitesimal dipole) is an infinitesimal electric current source. Infinitesimal dipoles are used as building blocks of more complex geometries.

We consider a very thin and very short wire:

$$L \ll \lambda, \;\; d \ll \lambda$$

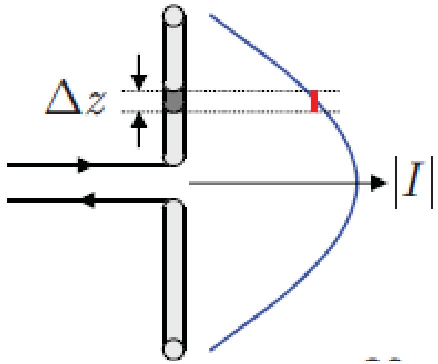


On this small piece of wire, the current distribution can be assumed as constant.

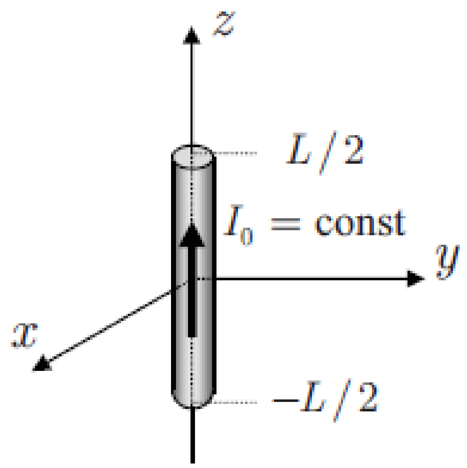
Notes:

- The concept of the infinitesimal dipole is useful to approximate the fields of electrically small dipoles.

- Pieces of wire antennas can be approximated as constant elements if $\Delta z \ll \lambda$. The total field from the wire can then be predicted by superposition.



Radiated fields of the infinitesimal dipole



The vector potential of a current source is

$$\vec{A}(x,y,z) = \frac{\mu}{4\pi} \iiint_V \vec{J}(x',y',z') \frac{e^{-jkR}}{R} dv'$$

For an infinitesimal current source along z, we write the current distribution as

$$\begin{cases} \vec{J}(x',y',z') = 0 \cdot \hat{a}_x + 0 \cdot \hat{a}_y + I_0\, \delta(x')\delta(y') \cdot \hat{a}_z & \text{for } -L/2 \leq z' \leq L/2 \\ \vec{J}(x',y',z') = \vec{0} & \text{elsewhere} \end{cases}$$

For an infinitesimal current source placed at the origin, we have in addition

$$x' = y' = z' = 0 \quad \text{since} \quad L << \lambda$$

$$R = \sqrt{x^2 + y^2 + z^2} = r$$

Therefore, the integration of the vector potential yields

$$\vec{A}(x,y,z) = \frac{\mu}{4\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-L/2}^{L/2} \hat{a}_z I_0 \delta(x')\delta(y')\frac{e^{-jkr}}{r}dx'dy'dz' = \hat{a}_z \frac{I_0\mu}{4\pi r}e^{-jkr}\int_{-L/2}^{L/2} dz' = \hat{a}_z \frac{I_0 \mu L}{4\pi r}e^{-jkr}$$

The result is now transformed into spherical coordinates (better adapted to the physics of the problem)

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix}\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$A_x = A_y = 0 \Rightarrow \begin{cases} A_r = A_z\cos\theta = \dfrac{\mu I_o L}{4\pi r}e^{-jkr}\cos\theta \\ A_\theta = -A_z\sin\theta = -\dfrac{\mu I_o L}{4\pi r}e^{-jkr}\sin\theta \\ A_\phi = 0 \end{cases}$$

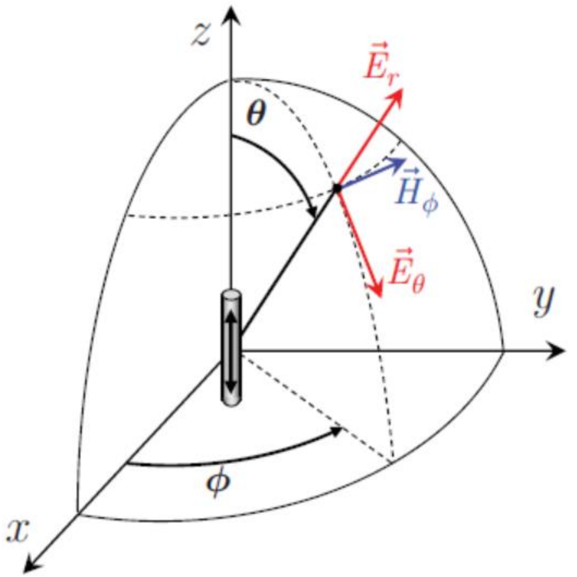
$$H_r = H_\theta = 0$$

$$H_\phi = \frac{1}{\mu r}\left[jk\frac{\mu I_0 L}{4\pi}e^{-jkr}\sin\theta + \frac{\mu I_0 L}{4\pi r}e^{-jkr}\sin\theta\right] = j\frac{kI_0 L\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$E_r = \eta\frac{I_0 L\cos\theta}{2\pi r^2}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$E_\theta = j\eta\frac{kI_0 L\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]e^{-jkr}$$

$$E_\phi = 0$$



In the far-field: r large $\Rightarrow \frac{1}{r^3} << \frac{1}{r^2} << \frac{1}{r}$ ⇒ Components with $1/r$ dependence dominate

$$E_\theta = j\eta \frac{kI_0 L e^{-jkr}}{4\pi r} \sin\theta \qquad E_r, E_\phi = 0$$

$$kr >> 1$$

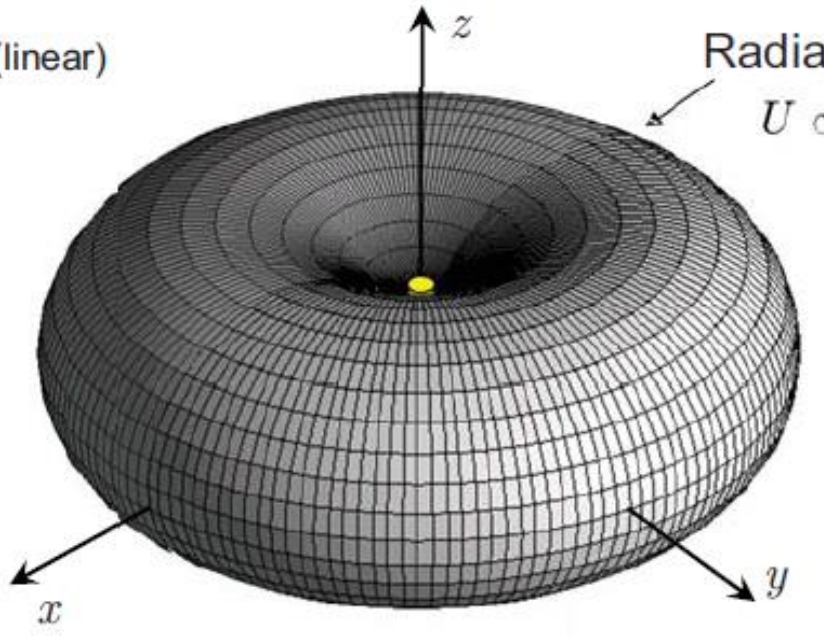
$$H_\phi = j \frac{kI_0 L e^{-jkr}}{4\pi r} \sin\theta \qquad H_r, H_\theta = 0$$

Wave impedance:

$$Z_w = \frac{E_\theta}{H_\phi} = \eta$$

($\simeq 120\pi\ \Omega$ in free space)

(linear)



Radiation pattern $U \propto \sin^2\theta$

Radiation intensity U is derived from power density

$$\Rightarrow U \propto E_\theta^2$$

Principal planes

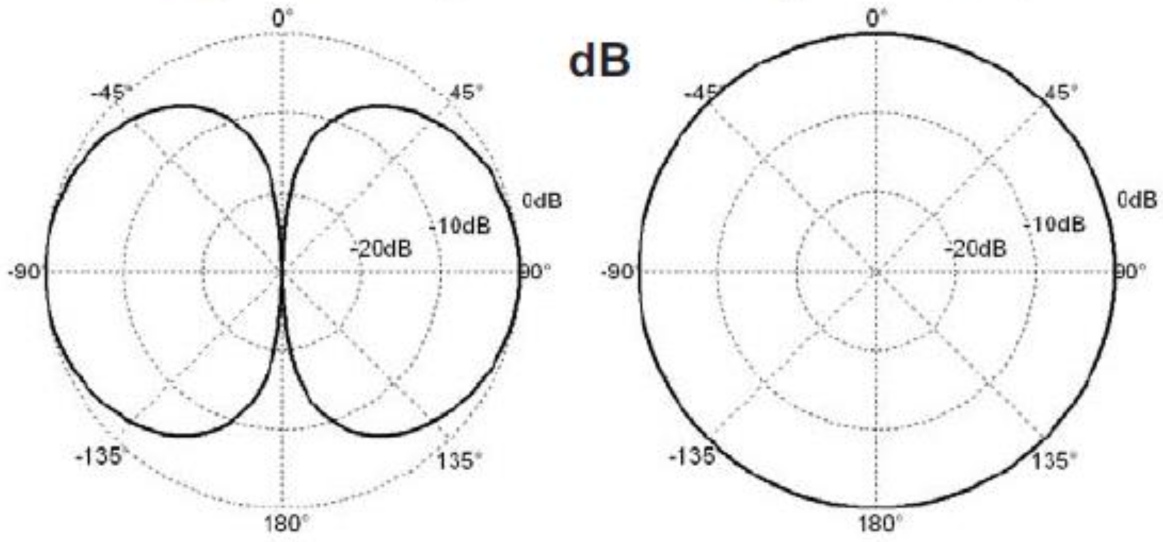
The radiation pattern of a dipole is omnidirectional

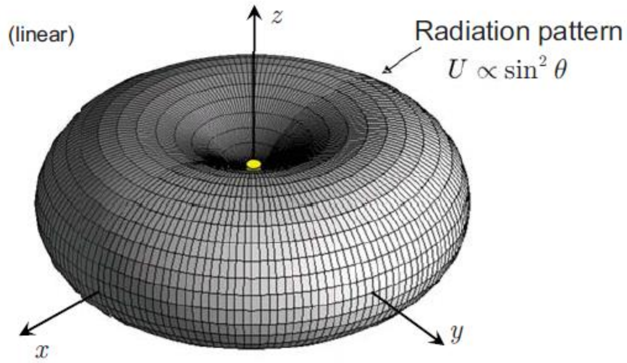
E-plane pattern

$E_\theta(\theta\ ,\phi = 90^0)$

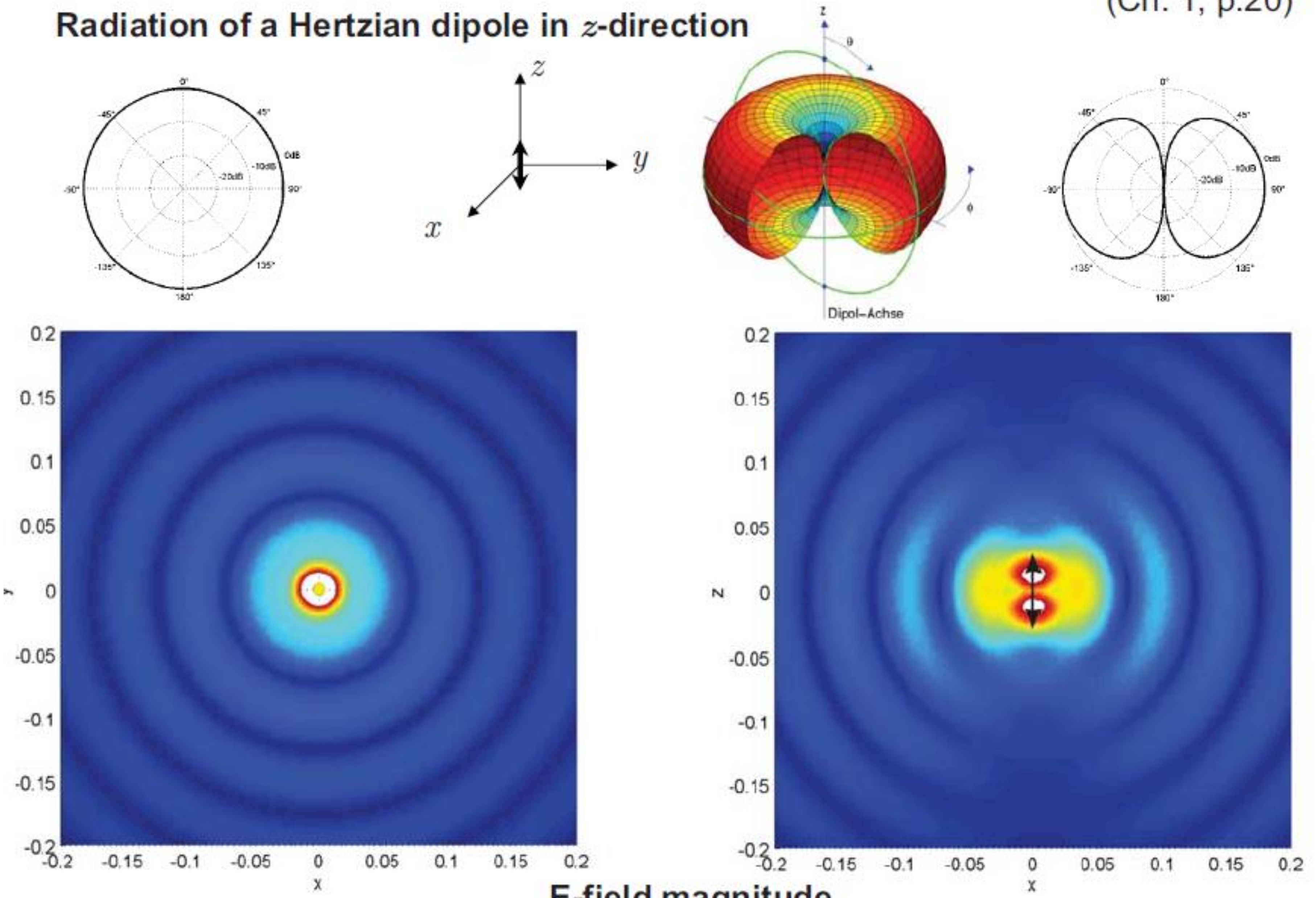
H-plane pattern

$E_\theta(\theta = 90^0,\ \phi)$





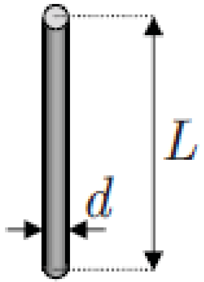
Radiation of a Hertzian dipole in z-direction



E-field magnitude

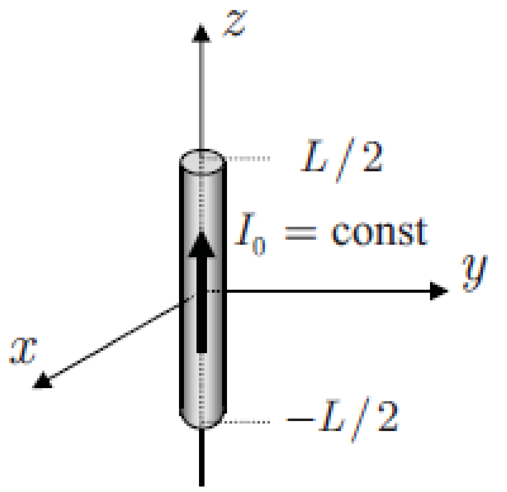
Top view (xy-plane) Side view (xz-plane)

Infinitesimal Dipole – Field Characteristics



$L \ll \lambda, \ d \ll \lambda$

As seen in the previous chapter, the infinitesimal dipole is an electric current source with an infinitesimal length $L \ll \lambda$.



The far-field components have been computed as

$$E_\theta = j\eta \frac{kI_0 L e^{-jkr}}{4\pi r} \sin\theta \qquad E_r, E_\phi = 0$$

$$H_\phi = j \frac{kI_0 L e^{-jkr}}{4\pi r} \sin\theta \qquad H_r, H_\theta = 0$$

Next, we discuss properties of the radiate field, such as

- ***Radiation Power Density***
- ***Radiated Power***
- ***Radiation Resistance***
- ***Directivity***

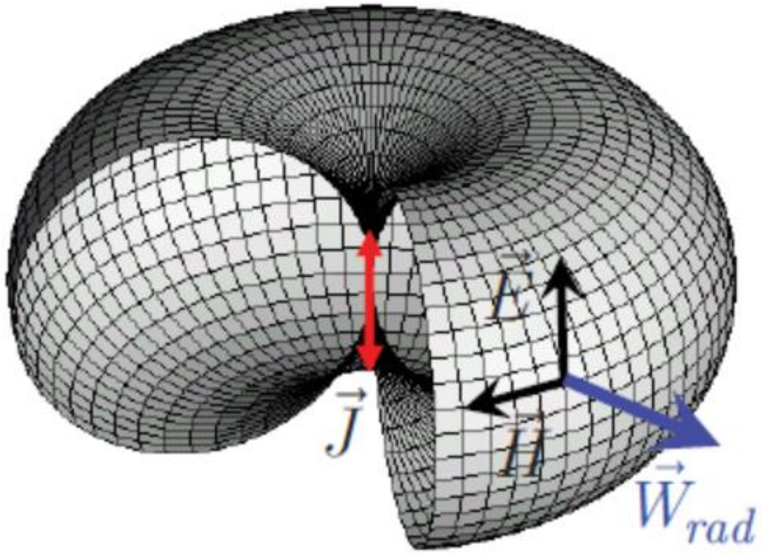
Radiation power density

$$\vec{W}_{rad} = W_{rad} \cdot \hat{a}_r = \frac{1}{2}\mathrm{Re}\left[\underline{\vec{E}} \times \underline{\vec{H}}^*\right] \quad \left[\mathrm{W/m^2}\right]$$

$$= \frac{1}{2} E_\theta H_\phi^* \cdot \hat{a}_r$$

$$W_{rad} = \frac{1}{2}\left(j\eta \frac{kI_0 L e^{-jkr}}{4\pi r}\sin\theta\right)\left(-j \frac{kI_0 L e^{+jkr}}{4\pi r}\sin\theta\right)$$

$$= \frac{\eta}{2}\left(\frac{kI_0 L}{4\pi r}\sin\theta\right)^2$$



Radiated power

To find the total radiated power, we integrate the radiation (i.e. far-field) power density over a sphere with (large) radius r

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} W_{rad}\, r^2 \sin\theta\, d\theta d\phi = \int_0^{2\pi}\int_0^{\pi} \frac{\eta}{2}\left(\frac{kI_0L}{4\pi r}\sin\theta\right)^2 r^2 \sin\theta\, d\theta d\phi$$

$$= \frac{\eta}{2}\left(\frac{kI_0L}{4\pi}\right)^2 2\pi \underbrace{\int_0^{\pi} \sin^3\theta d\theta}_{4/3} = \frac{\eta}{12\pi}(kI_0L)^2 = \eta\frac{\pi}{3}\left(\frac{I_0L}{\lambda}\right)^2$$

Radiation resistance $$P_{rad} = \frac{1}{2}|I_0|^2 R_{rad}$$

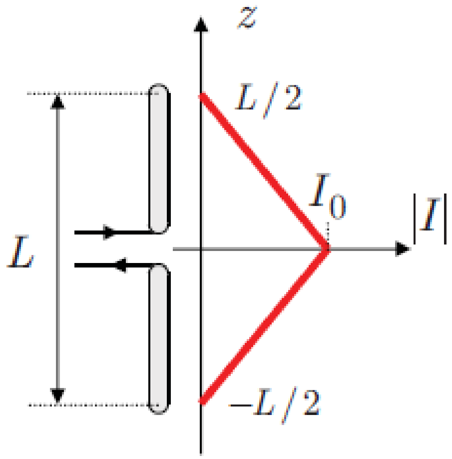
$$\mathrm{P}_{rad} = \eta \frac{\pi}{3}\left(\frac{I_0 L}{\lambda}\right)^2 \triangleq \frac{1}{2}|I_0|^2 R_{rad} \Rightarrow \boxed{R_{rad} = \eta \frac{2\pi}{3}\left(\frac{L}{\lambda}\right)^2 \cong 80\pi^2\left(\frac{L}{\lambda}\right)^2 \Omega}$$

Directivity

$$U_{\max} = \max\left\{r^2 \frac{1}{2}\mathrm{Re}\left\{\vec{E}\times\vec{H}^*\right\}\right\} = \max\left\{\frac{\eta}{2}\left(\frac{kI_0 l}{4\pi}\right)^2 \sin^2\theta\right\} = \frac{\eta}{2}\left(\frac{kI_0 l}{4\pi}\right)^2 = \frac{\eta}{2}\left(\frac{I_0 l}{2\lambda}\right)^2$$

$$D_0 = 4\pi\frac{U_{\max}}{P_{rad}} = 4\pi\left[\frac{\eta}{8}\left(\frac{I_0 l}{\lambda}\right)^2\right] \Big/ \left[\frac{\eta\pi}{3}\left(\frac{I_0 l}{\lambda}\right)^2\right] \Rightarrow \boxed{D_0 = \frac{3}{2}}$$

Small Dipole – Radiated Fields & Characteristics



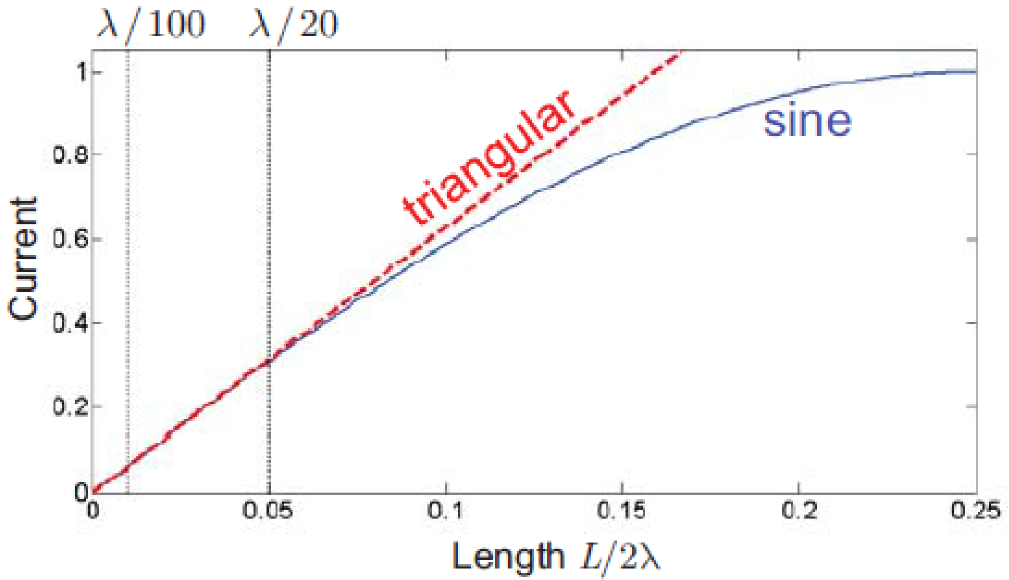
$$\lambda/50 \leq L \leq \lambda/10$$

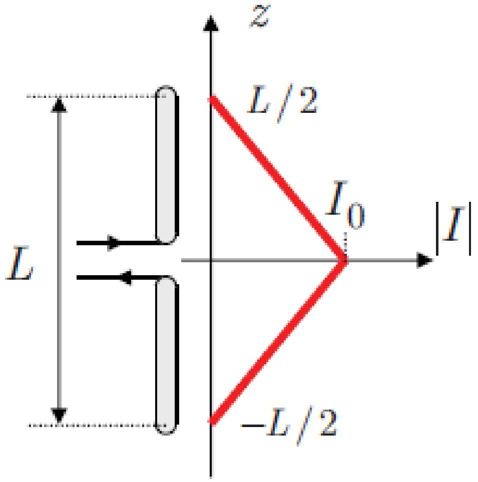
The current distribution for the infinitesimal dipole has been considered constant over the length of the wire. This approximation is usually considered valid for very short dipole lengths, i.e. $L \leq \lambda / 50$.

A better approximation of the current distribution on short dipoles with lengths

$$\lambda / 50 \leq L \leq \lambda / 10$$

is the triangular approximation.



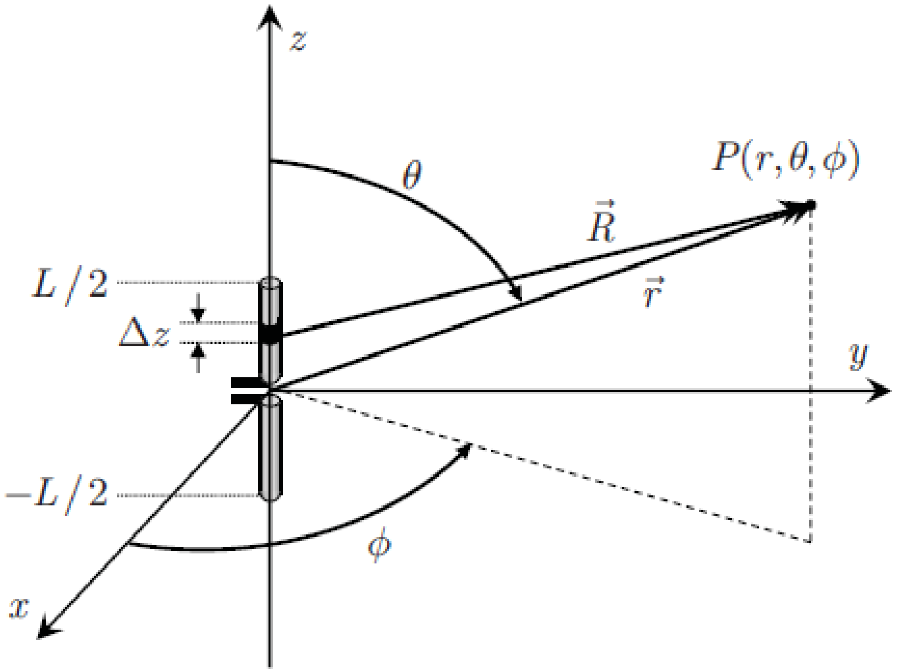


For a small dipole along z:

$$\vec{I}_e(x', y', z') = \vec{a}_z I_0 \begin{cases} 1 - \dfrac{2}{L} z' & 0 \leq z' \leq L/2 \\ 1 + \dfrac{2}{L} z' & -L/2 \leq z' \leq 0 \end{cases}$$

Following the procedure established in the previous chapter, we first compute the electric vector potential

$$\vec{A}(x,y,z) = \frac{\mu}{4\pi}\int_{V'} \vec{J}(x',y',z')\frac{e^{-jkR}}{R}dV'$$



Since the dipole is small, we can make the following approximation

$$R \approx r$$

For a very thin wire, the volume integration of the vector potential is reduced to a line integral (c.f. infinitesimal dipole)

$$\vec{A} = \frac{\mu}{4\pi}\vec{a}_z I_0\left[\int_{-L/2}^{0}\left(1+\frac{2}{L}z'\right)\frac{e^{-jkr}}{r}dz' + \int_{0}^{L/2}\left(1-\frac{2}{L}z'\right)\frac{e^{-jkr}}{r}dz'\right]$$

Performing the integrations, we get $\vec{A} = \frac{1}{2}\frac{\mu I_0 L}{4\pi r} e^{-jkr}\vec{a}_z$

which is one-half of that obtained for the infinitesimal dipole $\vec{A} = \frac{1}{2}\vec{A}_{\text{(infinitesimal dipole)}}$

Far-zone fields

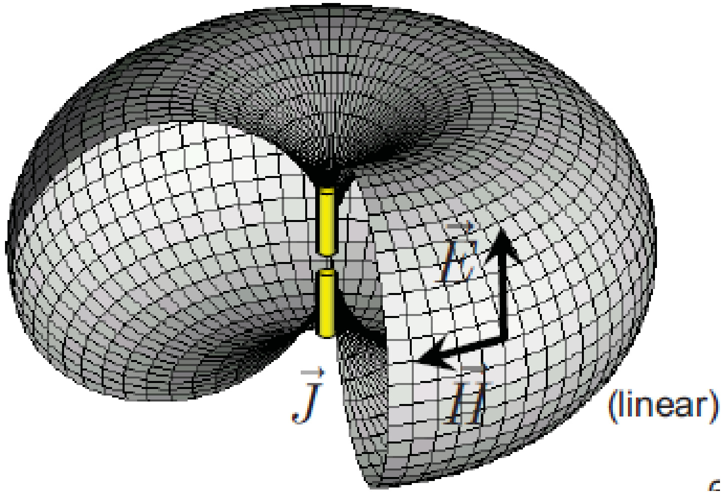
From the results of the infinitesimal dipole, we can therefore find the far-zone fields of the small dipole very simply (factor 1/2):

$$E_\theta = j\eta \frac{kI_0 L e^{-jkr}}{8\pi r}\sin\theta \qquad E_r, E_\phi = 0$$

$$H_\phi = j \frac{kI_0 L e^{-jkr}}{8\pi r}\sin\theta \qquad H_r, H_\theta = 0$$

→ Same pattern as infinitesimal dipole

→ Same directivity as infinitesimal dipole



Radiation resistance

Since the current is ½ of that of the infinitesimal dipole, the radiated power is ¼ of that of the infinitesimal dipole. Therefore

$$R_{rad} = 20\pi^2 \left(\frac{L}{\lambda}\right)^2 \Omega$$

Questions?? Thoughts??

EE 328
Wave Propagation and Antennas

with

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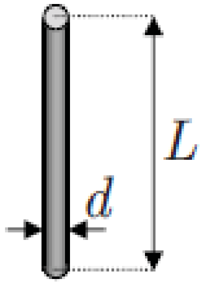
@

Jouf University

Finite Length Dipole – *Radiated Fields & Characteristics*

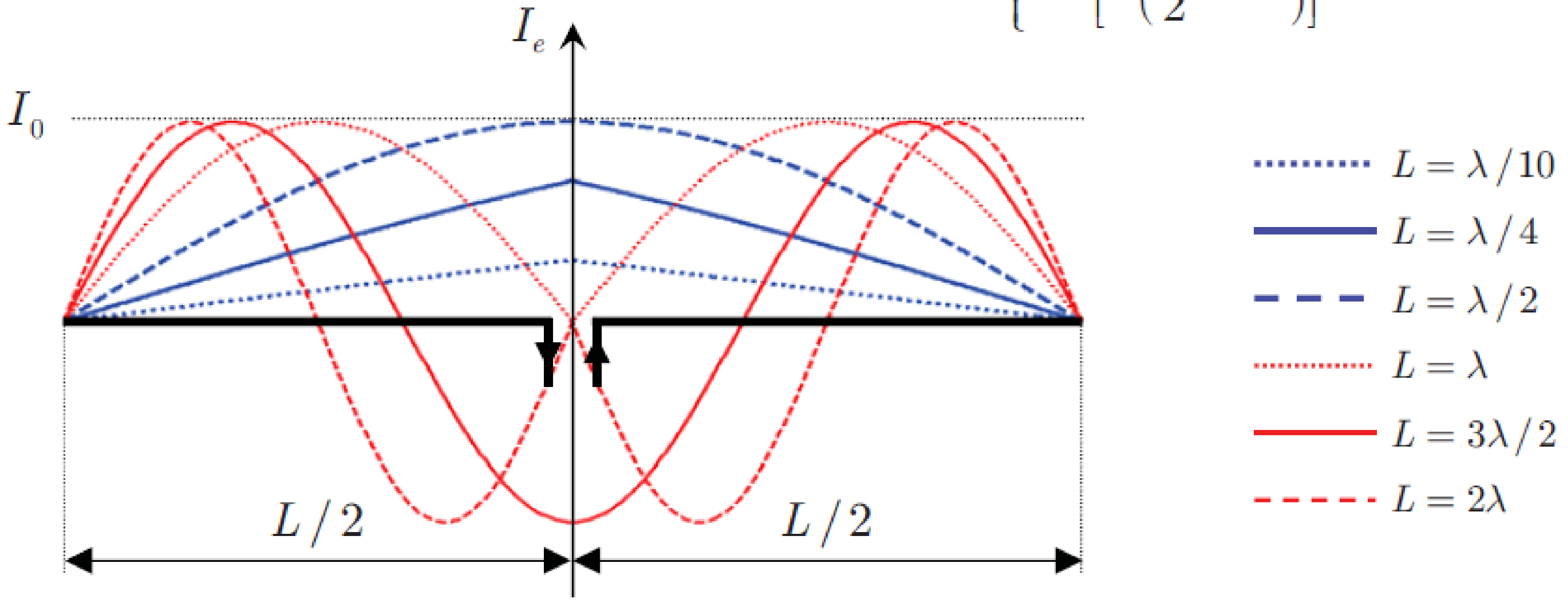


$L > \lambda/10 \quad d \ll \lambda$

a) Current distribution

For a very thin center-fed dipole along z, the current distribution is sinusoidal in a good approximation.

$$\vec{I}_e = \vec{a}_z I_e(x'=0, y'=0, z') = \vec{a}_z I_0 \begin{cases} \sin\left[k\left(\frac{L}{2} - z'\right)\right] & 0 \le z' \le L/2 \\ \sin\left[k\left(\frac{L}{2} + z'\right)\right] & -L/2 \le z' \le 0 \end{cases}$$

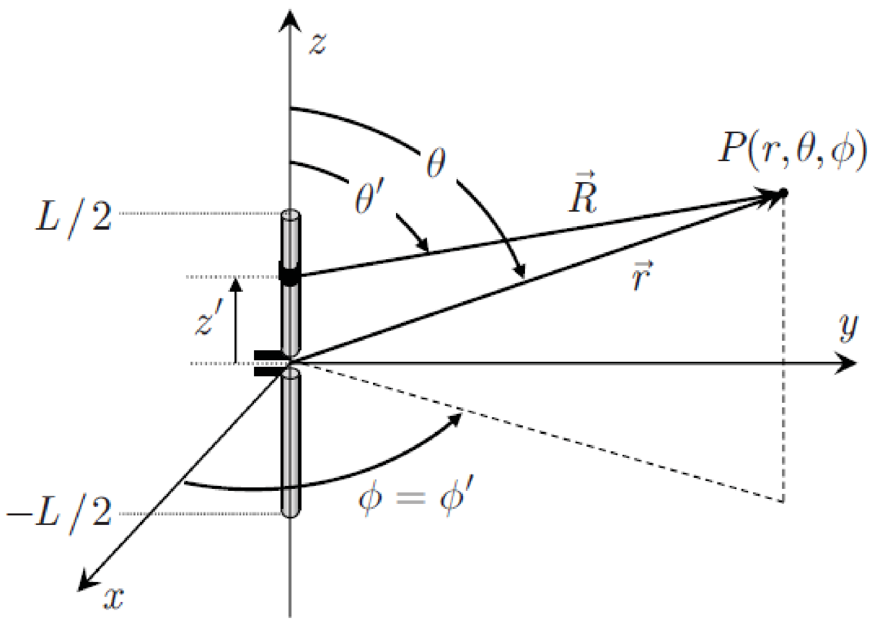


b) Far-field approximation

For finite length dipoles with $L > \lambda/10$, the approximation $R \approx r$ is no longer valid.

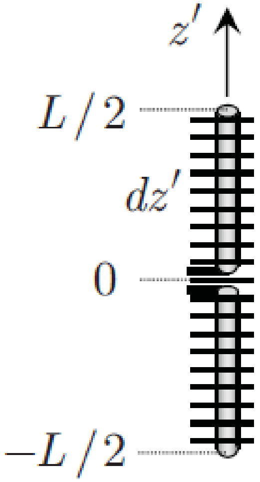
Instead, we use $\boxed{R \approx r - z' \cos\theta}$

$$\vec{A}(x,y,z) = \mu \frac{e^{-jkr}}{4\pi r} \int_C \vec{I}_e(x',y',z')\, e^{jkz'\cos\theta}\, dz'$$



This approximation is valid as long as $r \geq D^2/\lambda$
where D is the maximum dimension of the antenna.

c) Field computation



Assume that the dipole is subdivided into an infinite number of infinitesimal dipoles with length dz'.

The far-field contribution of each of these infinitesimal dipoles can then be written as (see previous chapter)

$$dE_\theta = j\eta \frac{kI_e(x',y',z')e^{-jkR}}{4\pi R}\sin\theta\, dz' = \eta\, dH_\phi$$

$$dE_\phi = dE_r = dH_\theta = dH_r = 0$$

Using the far-field approximation from the previous page gives

$$dE_\theta \cong j\eta \frac{kI_e(x',y',z')e^{-jkr}}{4\pi r}\sin\theta\, e^{+jkz'\cos\theta}\, dz'$$

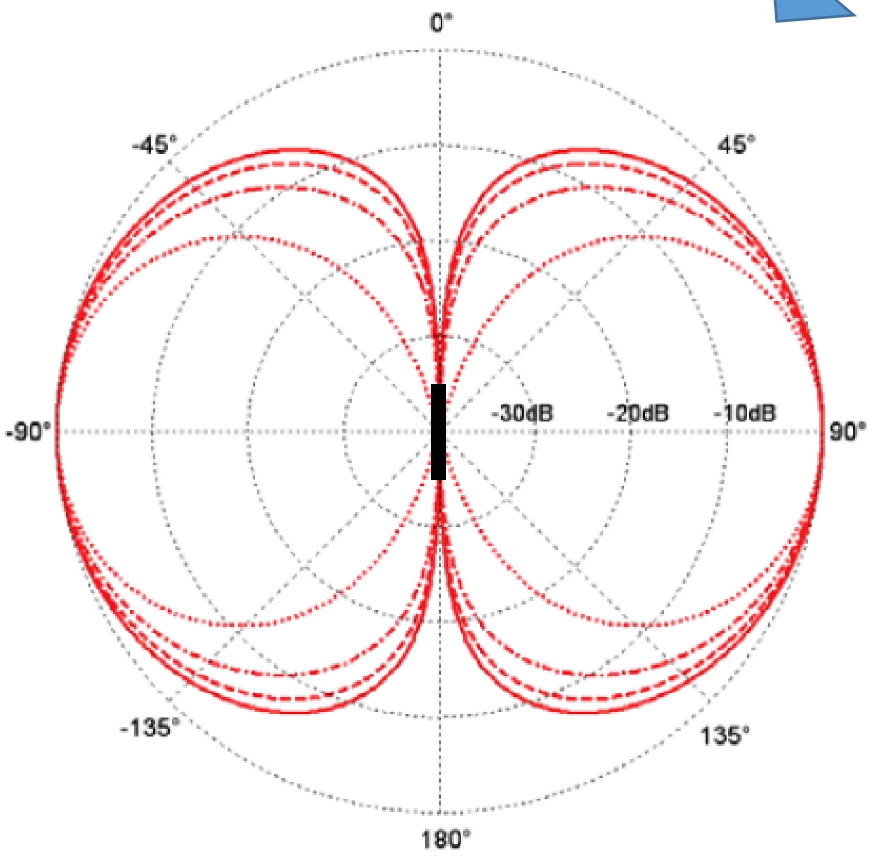
The total radiated field is the summation of all the contributions from the infinitesimal dipoles. The summation reduces to an integration in the limit $dz' \to 0$

$$E_\theta = \int_{-L/2}^{L/2} dE_\theta = j\eta \frac{k\, e^{-jkr}}{4\pi r}\sin\theta \underbrace{\int_{-L/2}^{L/2} I_e(x',y',z')e^{jkz'\cos\theta}dz'}_{N_z}$$

$$E_\theta = j\eta \frac{k e^{-jkr}}{r} \sin\theta \; N_z$$

N_z

$$\boxed{E_\theta = j\eta \frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(k\frac{L}{2}\cos\theta\right) - \cos\left(k\frac{L}{2}\right)}{\sin\theta}\right] = \eta H_\phi}$$



Elevation patterns for thin dipoles with sinusoidal current distribution

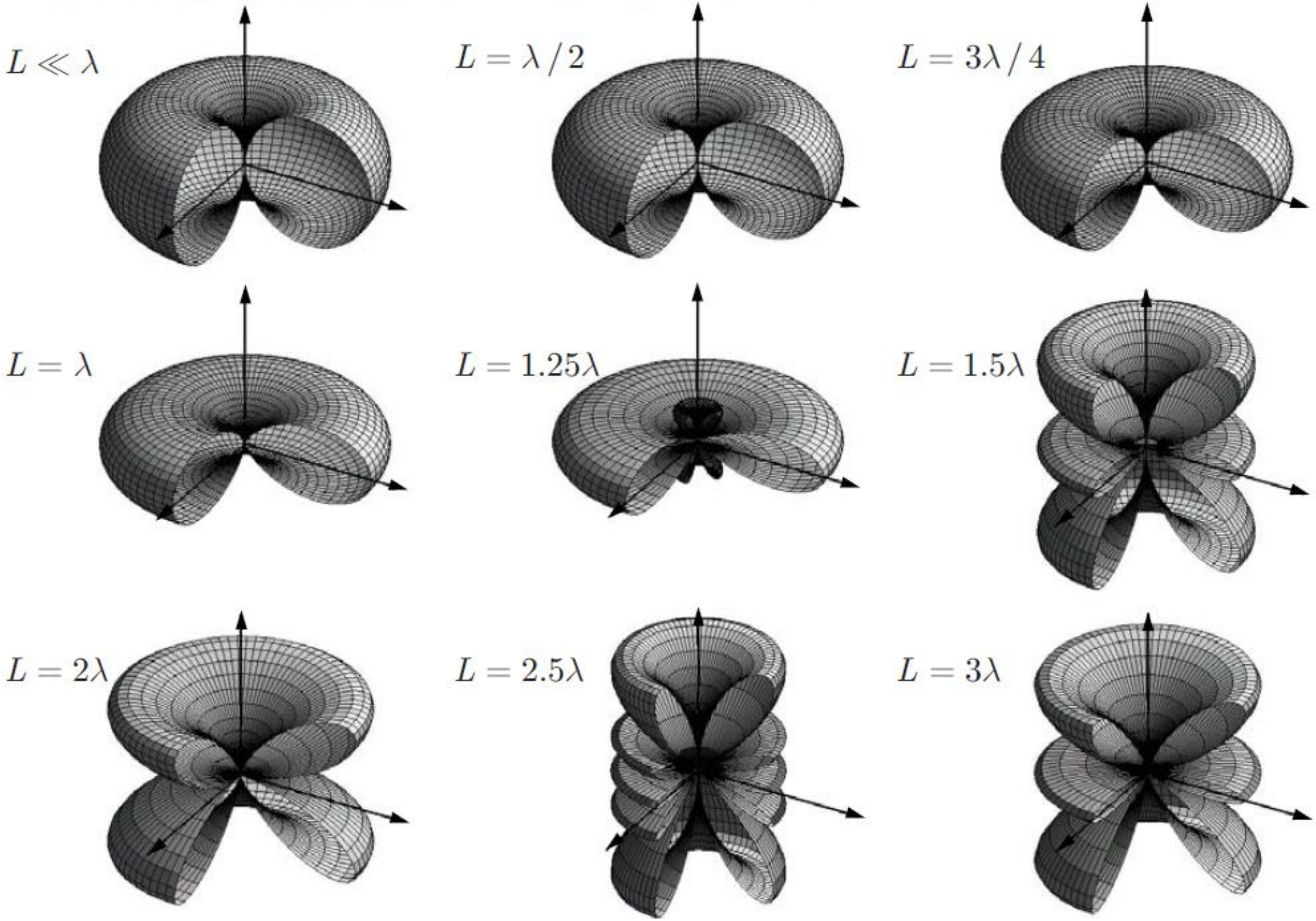
$L \ll \lambda$

$L = \lambda / 2$

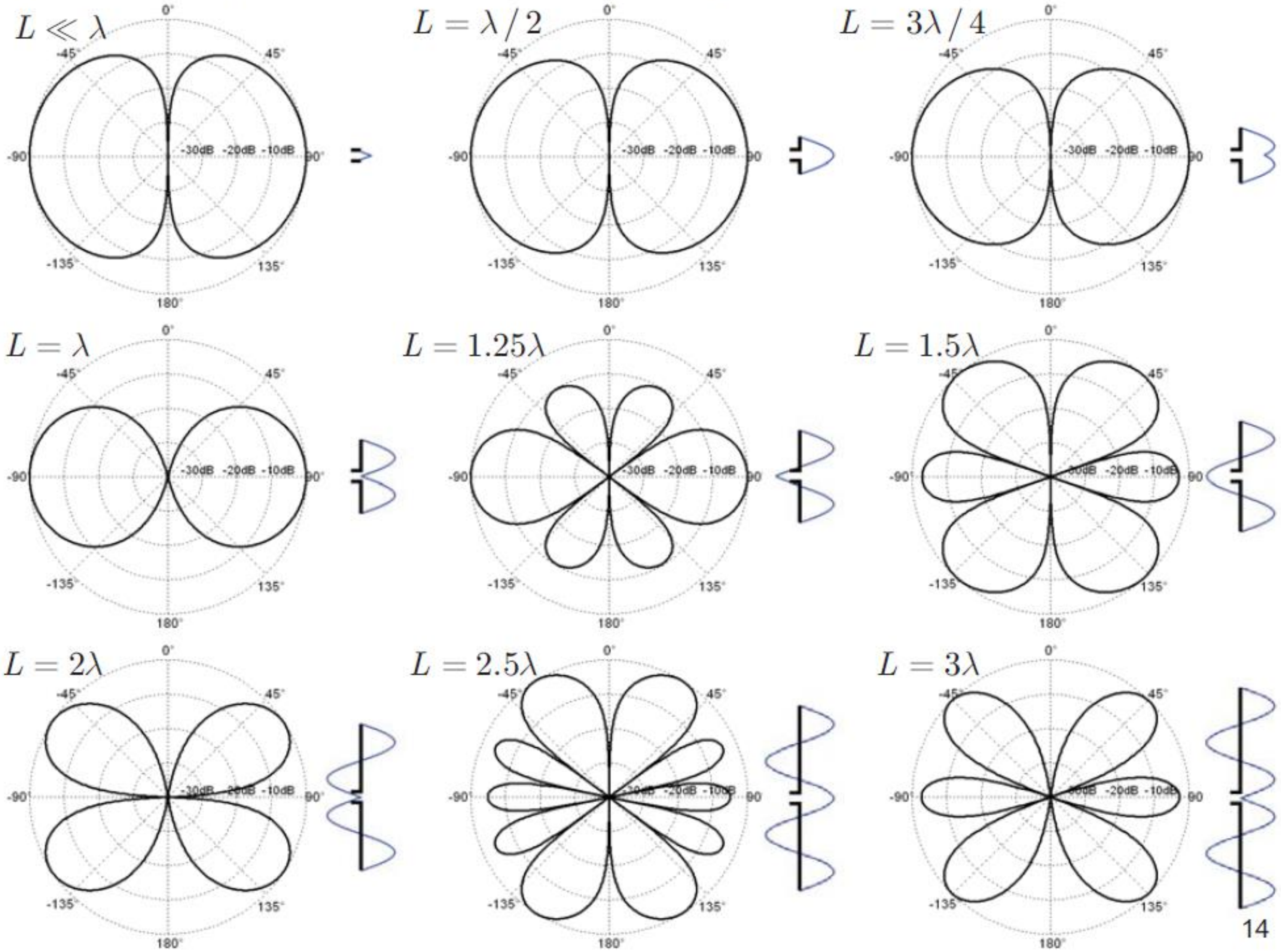
$L = 3\lambda / 4$

$L = \lambda$

Dipole amplitude patterns in linear representation



Dipole elevation patterns (dB) and corresponding current distributions



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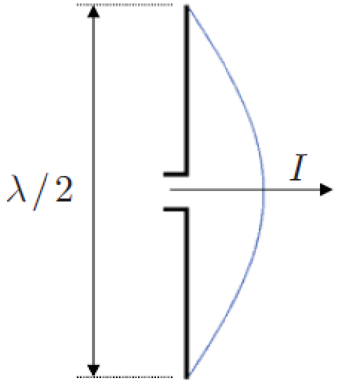
Half-Wave Dipole – Radiated Fields & Characteristics

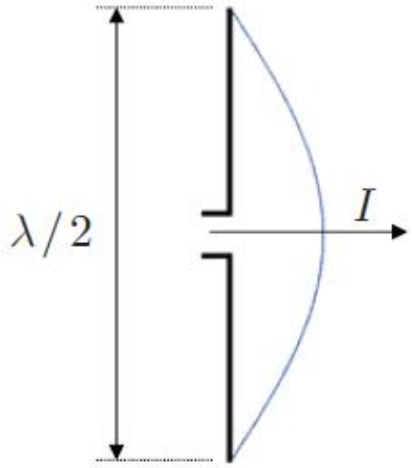


$$L = \lambda/2$$

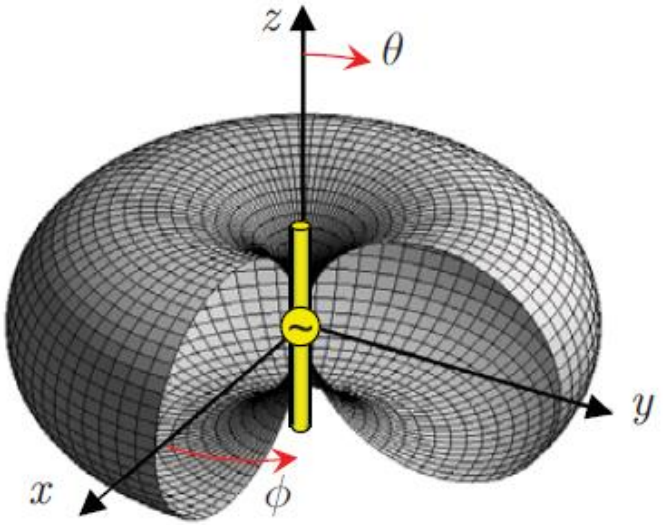
d) Half-wave dipole

The half-wave dipole ($L = \lambda/2$) is one of the most commonly used antennas.

The radiated field components can be written as



$$E_\theta = j\eta \frac{I_0 e^{-jkr}}{2\pi r} \frac{\cos\left(k\frac{L}{2}\cos\theta\right) - \cos\left(k\frac{L}{2}\right)}{\sin\theta} \Bigg|_{L=\lambda/2}$$



$$\cos\left(k\frac{L}{2}\right) = \cos\left(\frac{2\pi}{\lambda}\frac{\lambda}{4}\right) = \cos\frac{\pi}{2} = 0$$

$$\boxed{E_\theta = j\eta \frac{I_0 e^{-jkr}}{2\pi r} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} = \eta H_\phi}$$

Radiation intensity
$$U = r^2 W_{rad} = \eta \frac{|I_0|^2}{8\pi^2} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right]^2 \approx \eta \frac{|I_0|^2}{8\pi^2} \sin^3\theta$$
$$W_{rad} = \frac{1}{2} E_\theta H_\phi^*$$

Radiated power
$$P_{rad} = \eta \frac{|I_0|^2}{4\pi} \int_0^\pi \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta \approx \eta \frac{|I_0|^2}{8\pi} 2.435$$

Directivity
$$D_0 = 4\pi \frac{U_{\max}}{P_{rad}} = 4\pi \frac{\eta \frac{|I_0|^2}{8\pi^2}}{\eta \frac{|I_0|^2}{8\pi} 2.435} = \frac{4}{2.435} \approx 1.643 \triangleq 2.156\,\text{dB}$$

Radiation resistance
$$R_{rad} = \frac{2P_{rad}}{|I_0|^2} = \eta \frac{2.435}{4\pi} \approx 73\ \Omega$$

Input Resistance – Dipole of Arbitrary Length

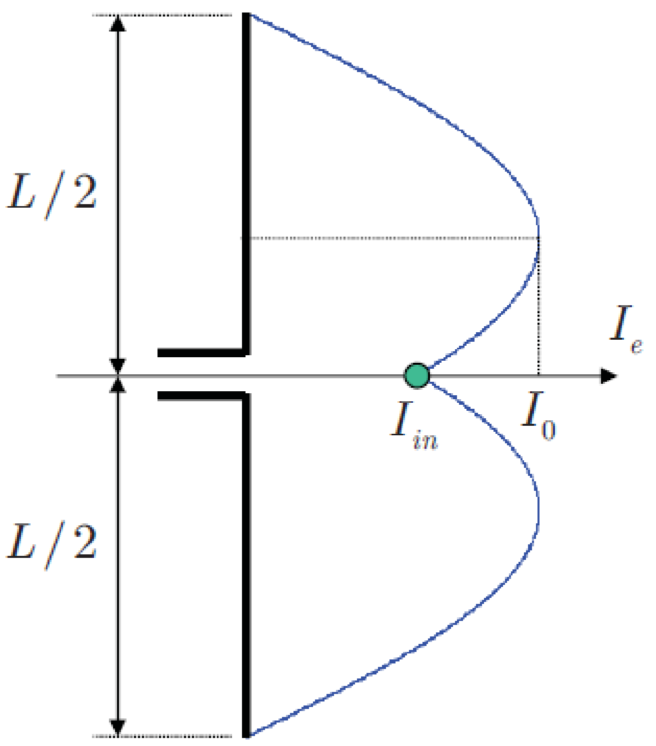
e) Input resistance of a dipole for arbitrary length

We consider a lossless antenna, i.e. $P_{in} = P_{rad}$

For the sinusoidal current distribution, the maximum current does not necessarily occur at the input terminals.

The radiation resistance is then referred to the input terminals of the antenna according to

$$\frac{|I_{in}|^2}{2} R_{in} = \frac{|I_0|^2}{2} R_{rad} \qquad \begin{cases} R_{in} : & \text{Input resistance} \\ R_{rad} : & \text{Radiation resistance} \end{cases}$$



$$\Rightarrow \quad R_{in} = \left(\frac{I_0}{I_{in}}\right)^2 R_{rad}$$

$$\boxed{R_{in} = \frac{R_{rad}}{\sin^2\left(k\frac{L}{2}\right)}}$$

For a dipole of length L, the current at the terminal is related to the current maximum according to

$$I_{in} = I_0 \sin\left(k\frac{L}{2}\right)$$

Half-wave dipole: $L=\dfrac{\lambda}{2} \Rightarrow R_{in}=R_{rad} \quad \left(X_{in}=42.5\ \Omega\right)$

Arbitrary length dipole:

$$P_{rad}=\int_0^{2\pi}\int_0^{\pi} W_{rad} r^2 \sin\theta \, d\theta \, d\phi = \eta \frac{|I_0|^2}{4\pi}\int_0^{\pi}\frac{\left[\cos\left(k\frac{L}{2}\cos\theta\right)-\cos\left(k\frac{L}{2}\right)\right]^2}{\sin\theta}d\theta$$

$$R_{rad}=\frac{2P_{rad}}{|I_0|^2}$$

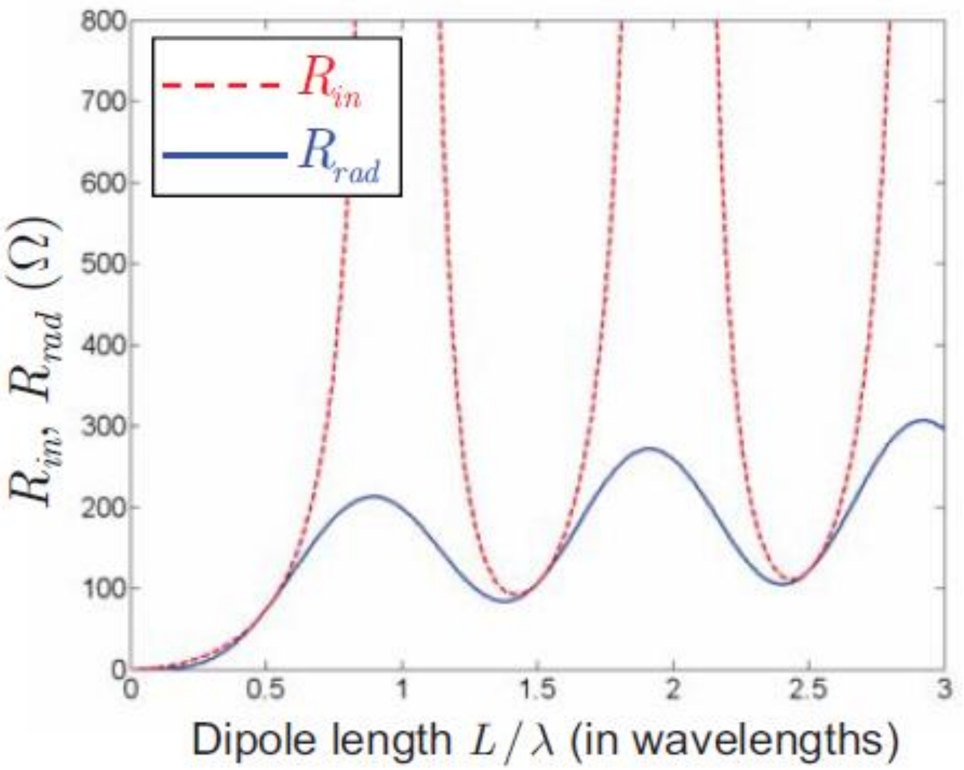
...

$$R_{rad}=\frac{\eta}{2\pi}\begin{cases} C+\ln(kL)-C_i(kL) \\ +\frac{1}{2}\sin(kL)\left[S_i(2kL)-2S_i(kL)\right] \\ +\frac{1}{2}\cos(kL)\left[C+\ln\left(k\frac{L}{2}\right)+C_i(2kL)-2C_i(kL)\right] \end{cases}$$

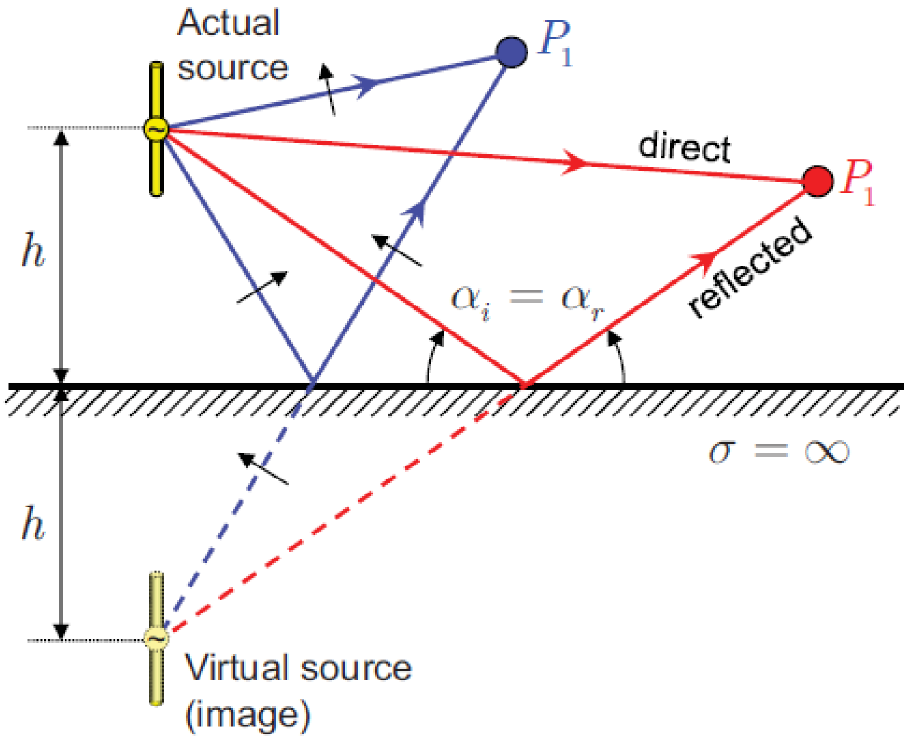
$C=0.5772$ Euler constant

$C_i(x)=-\int_x^{\infty}\frac{\cos y}{y}dy=\int_{\infty}^{x}\frac{\cos y}{y}dy$ Cosine integral

$S_i(x)=\int_0^{x}\frac{\sin y}{y}dy$ Sine integral



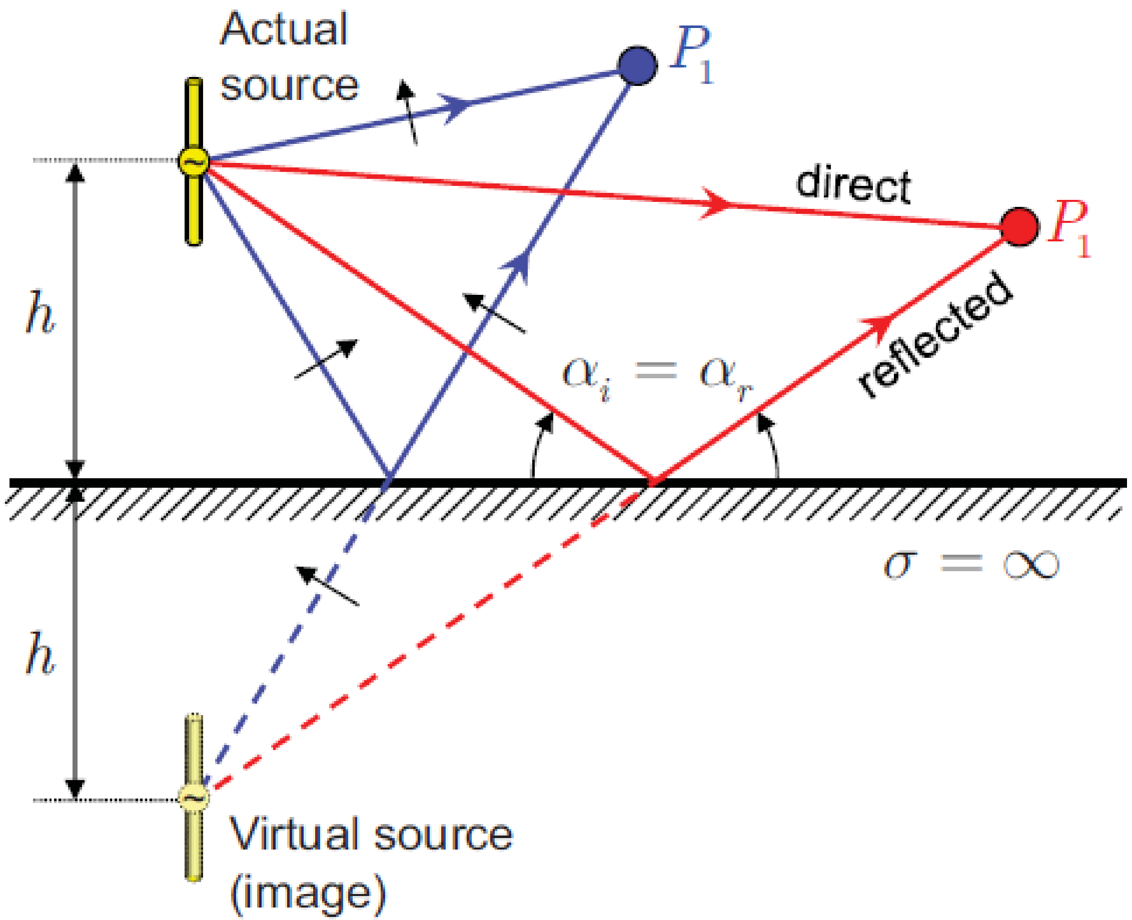
Wire Antennas Above Perfect Conducting Ground



The presence of material near the radiating element can significantly alter its radiation properties.

The most common situation considers the influence of the ground on the radiation properties.

In general, the ground is a lossy medium that can be considered a good conductor. To simplify the analysis, we assume that the ground is a perfect electric conductor, flat and infinite in extent.



Reflected path is accounted for by an image source below the ground plane.

<u>Equivalent problems:</u>

Source + Ground plane	Source + Image

↓

Same radiated fields in the upper half-space

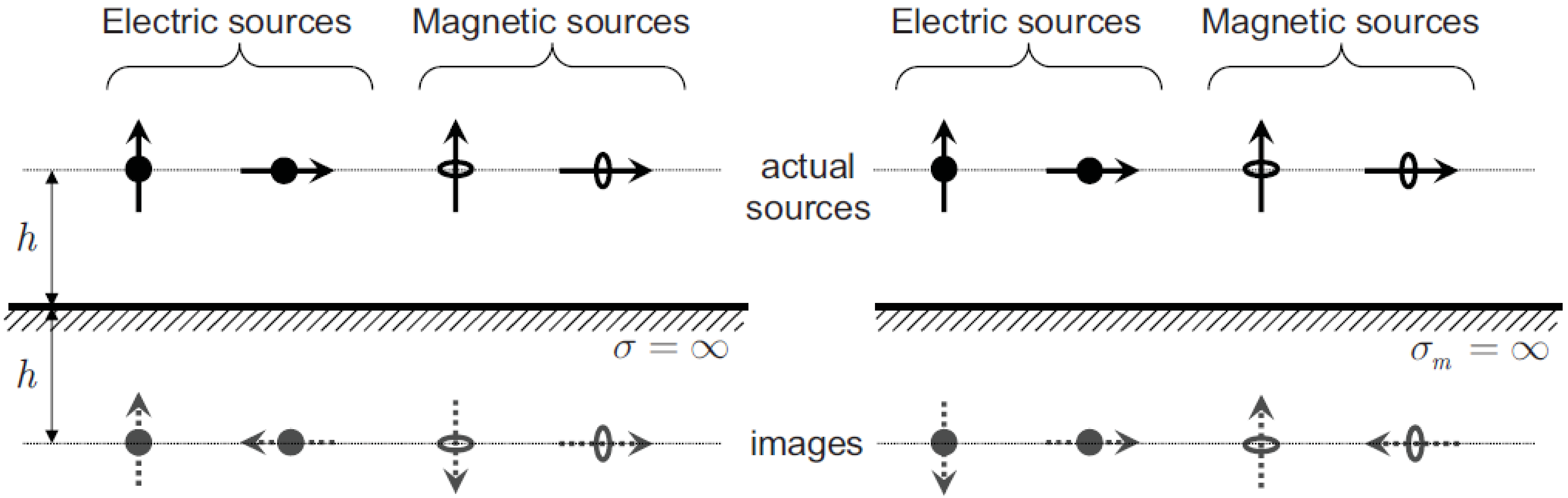
The polarity of the image is specified by the boundary condition of the conductor.

<u>Perfect electric conductor</u>
(PEC)

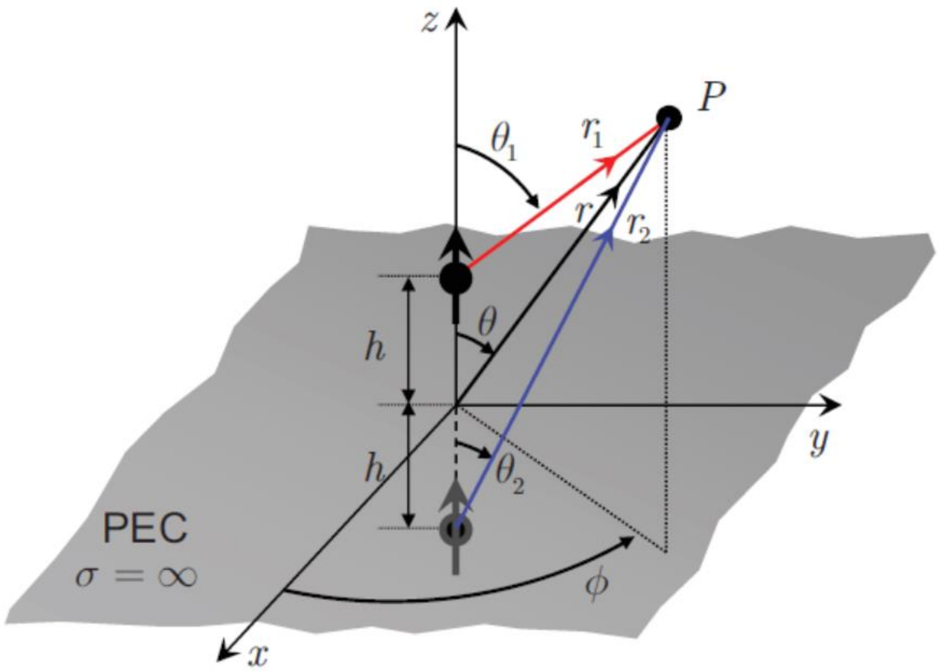
$E_t = 0$ on surface

<u>Perfect magnetic conductor</u>
(PMC)

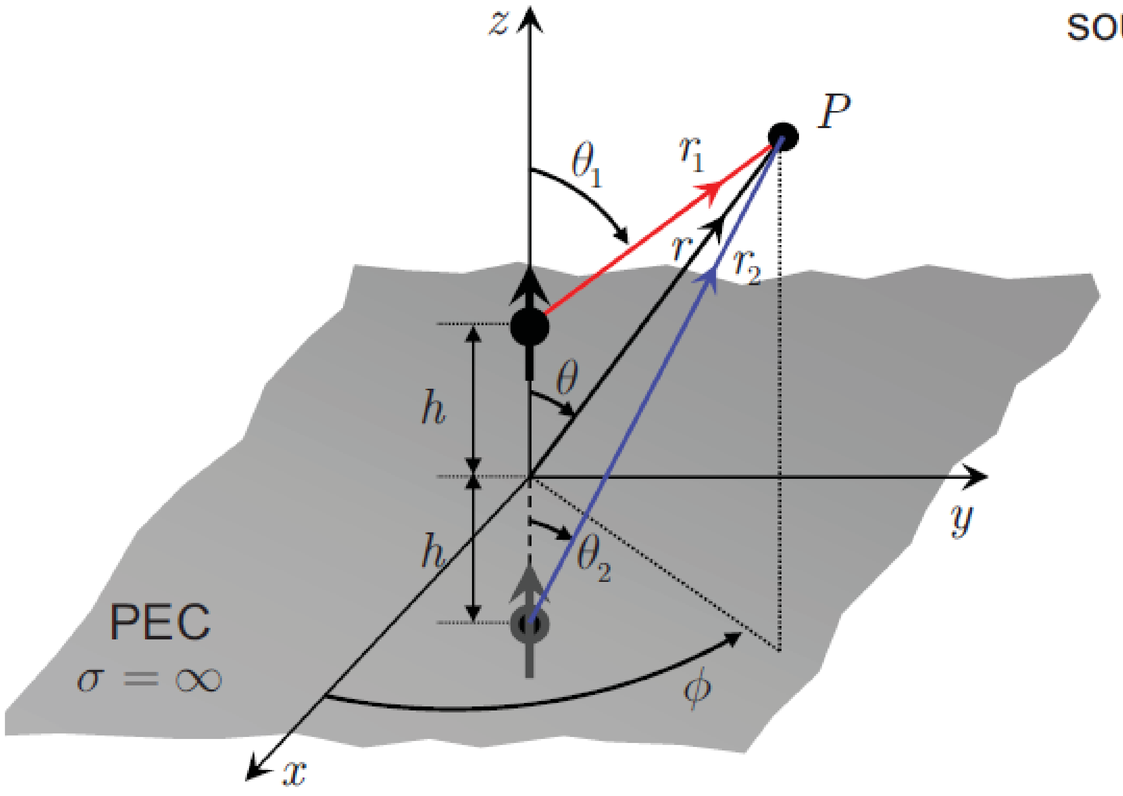
$H_t = 0$ on surface



Vertical Dipole Above Perfect Conducting Ground



To simplify the analysis, we consider a vertical infinitesimal dipole above an infinite ground plane.



From image theory, we can find the radiated field in the far-zone as the superposition of the fields from the actual and virtual sources

$$E_\theta = E_\theta^d + E_\theta^r$$

with

$$E_\theta^d = j\eta \frac{kI_0Le^{-jkr_1}}{4\pi r_1}\sin\theta_1$$

$$E_\theta^r = j\eta \frac{kI_0Le^{-jkr2}}{4\pi r_2}\sin\theta_2$$

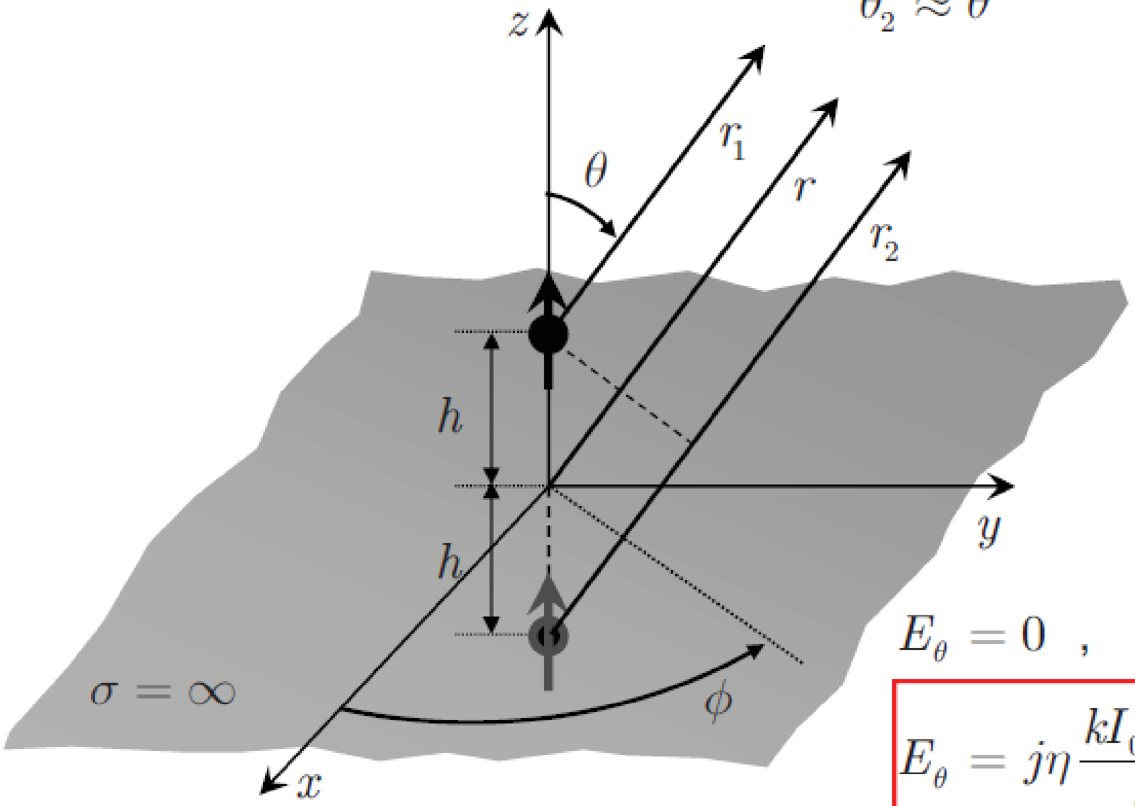
d: direct
r: reflected

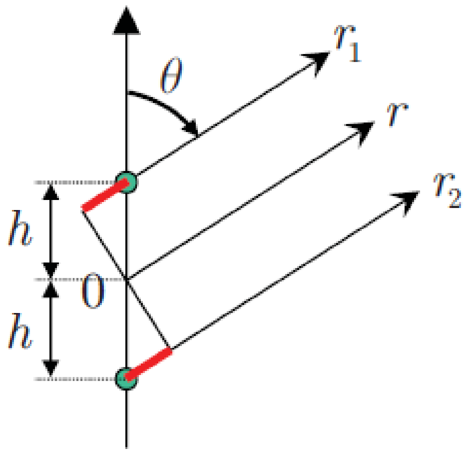
Far-field approximation

$$\theta_1 \approx \theta \qquad \theta_2 \approx \theta$$

$$\left.\begin{aligned} r_1 &\approx r - h\cos\theta \\ r_2 &\approx r + h\cos\theta \end{aligned}\right\} \quad \text{for phase}$$

$$r_1 \approx r_2 \approx r \quad \text{for amplitude}$$





$$E_\theta = 0 \quad , \quad z < 0$$

$$\boxed{E_\theta = \underbrace{j\eta \frac{kI_0Le^{-jkr}}{4\pi r}\sin\theta}_{\text{element factor}} \underbrace{\left[2\cos\left(kh\cos\theta\right)\right]}_{\text{array factor}} \quad , \quad z \geq 0}$$

interference effects dependent on h and θ

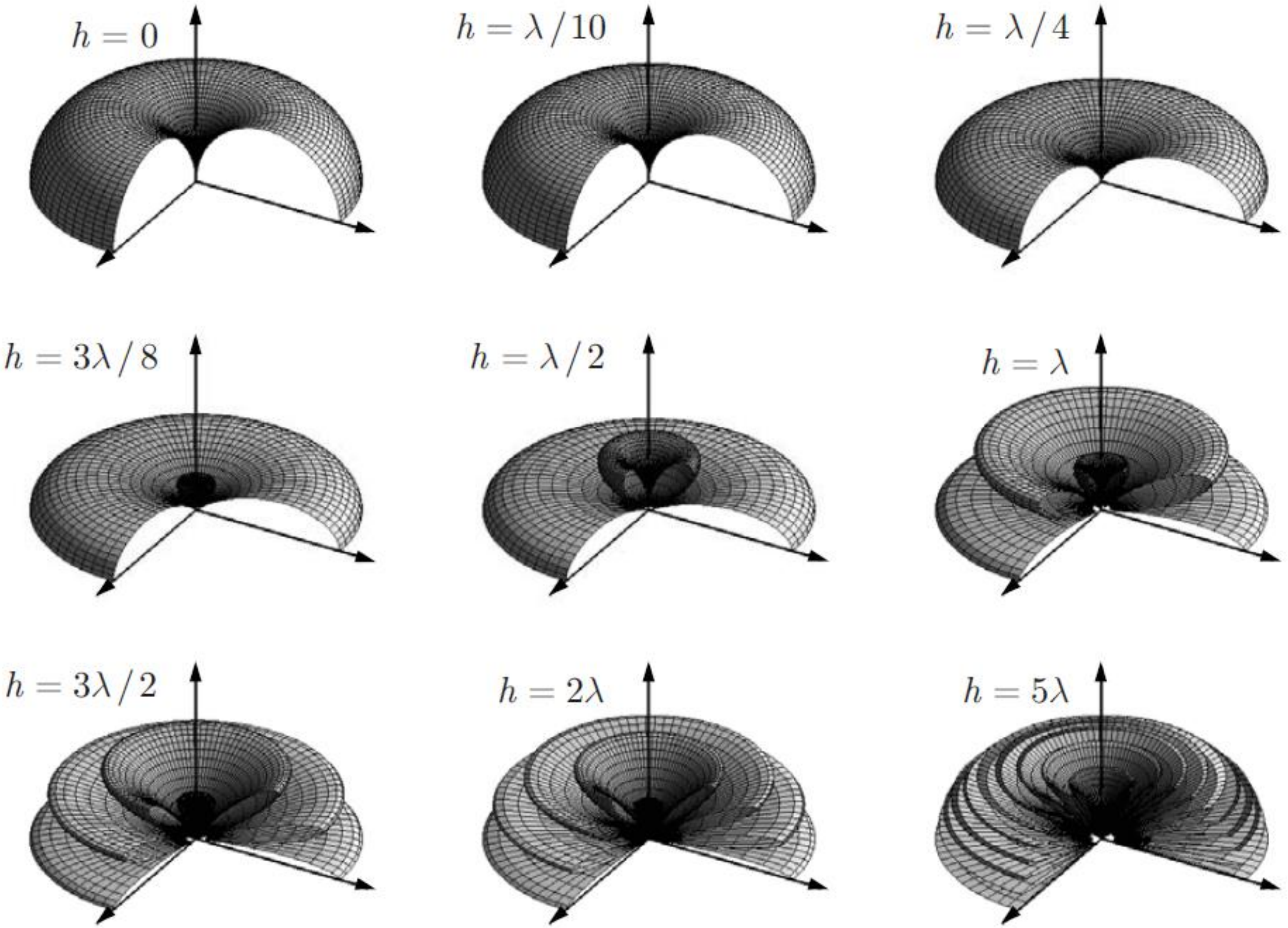
Directivity

$$D_0 = \frac{2}{\left[\frac{1}{3} - \frac{\cos(2kh)}{(2kh)^2} + \frac{\sin(2kh)}{(2kh)^3}\right]}$$

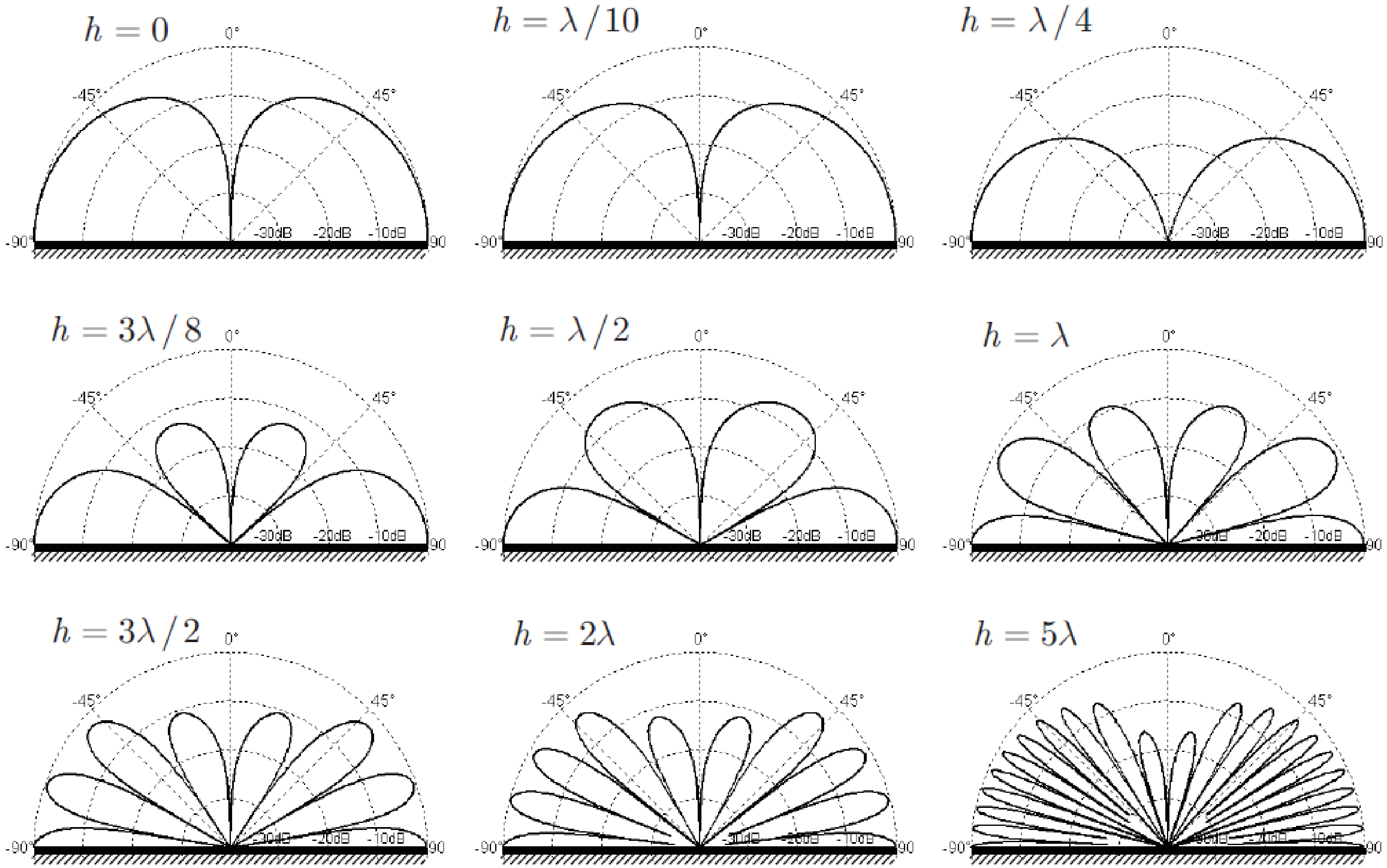
Radiation resistance

$$R_{rad} = 2\pi\eta\left(\frac{L}{\lambda}\right)^2 \left[\frac{1}{3} - \frac{\cos(2kh)}{(2kh)^2} + \frac{\sin(2kh)}{(2kh)^3}\right]$$

Vertical dipole above a ground plane: 3D amplitude patterns (linear representation)



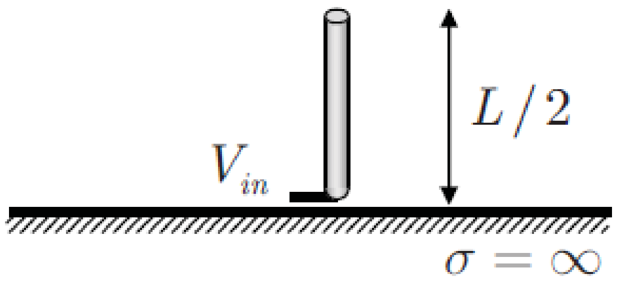
Vertical dipole above a ground plane: Elevation patterns (dB)



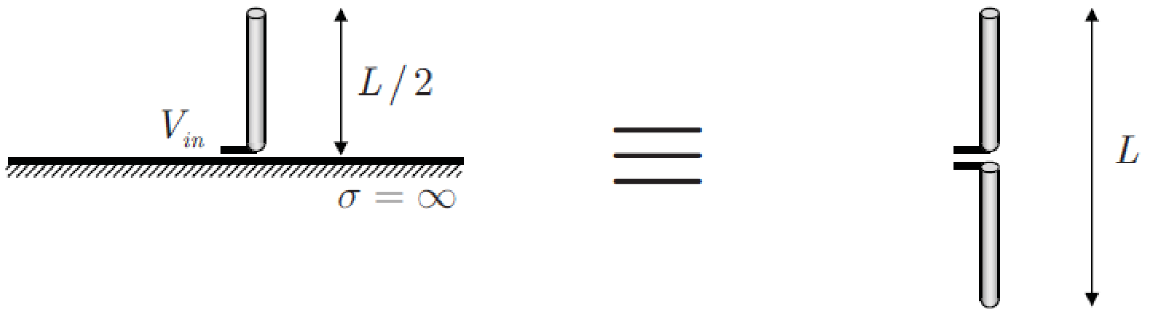
Omnidirectional patterns.

Number of lobes: integer closest to $2h/\lambda + 1$

Monopole Above Perfect Conducting Ground



Monopoles mounted above a ground plane are very commonly used antennas.



Monopole

Equivalent dipole

Radiation Pattern:

- A monopole of length $\frac{L}{2}$ above perfect conducting ground can be considered equivalent to a dipole of length L in free-space.
- The fields thus calculated will be valid only for the half-space above the ground

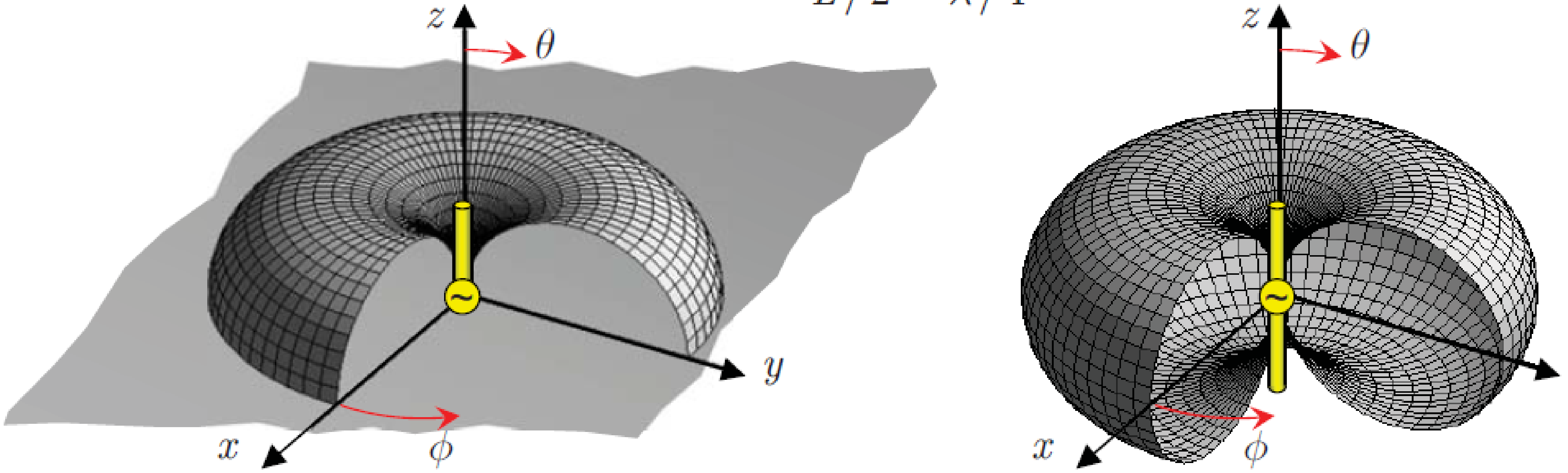
Input impedance: $$Z_{in\ ,\ mono} = \frac{V_{in,\ mono}}{I_{in,\ mono}} = \frac{\frac{1}{2}V_{in,\ dipole}}{I_{in,\ dipole}} = \frac{1}{2}Z_{in,\ dipole}$$

Radiation resistance: $$R_{rad,\ mono} = \frac{1}{2}R_{rad,\ dipole}$$

Directivity $$D_{mono} = \frac{4\pi}{\Omega_{A,\ mono}} = \frac{4\pi}{\frac{1}{2}\Omega_{A,\ dipole}} = 2D_{dipole}$$

A widely used configuration is the quarter-wave monopole

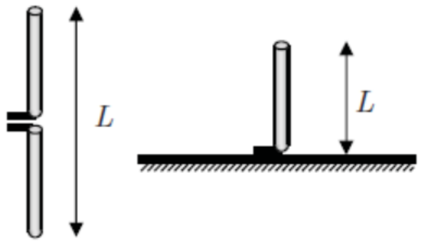
$$L/2 = \lambda/4$$



Amplitude pattern (linear) of the quarter-wave monopole and the corresponding half-wave dipole

Approximate formulas for rapid calculations and design of dipoles and monopoles:

L = total dipole or monopole length



$$K \triangleq \begin{cases} \dfrac{kL}{2} & \text{dipole} \\ kL & \text{monopole} \end{cases}$$

$0 < K < \pi / 4$:

$$R_{in,\ dip\max} = 2R_{in,\ mono\max} = 12.337\,\Omega$$

$$R_{in,\ dip} = 20\,K^2\,\Omega \qquad (0 < L < \lambda/4)$$

$$R_{in,\ mono} = 10\,K^2\,\Omega \qquad (0 < L < \lambda/8)$$

$\pi / 4 \le K \le \pi / 2$:

$$R_{in,\ dip\max} = 2R_{in,\ mono\max} = 76.383\,\Omega$$

$$R_{in,\ dip} = 24.7\,K^{2.5}\,\Omega \qquad (\lambda/4 \le L < \lambda/2)$$

$$R_{in,\ mono} = 12.35\,K^{2.5}\,\Omega \qquad (\lambda/8 \le L < \lambda/4)$$

$\pi / 2 \le K < 2$:

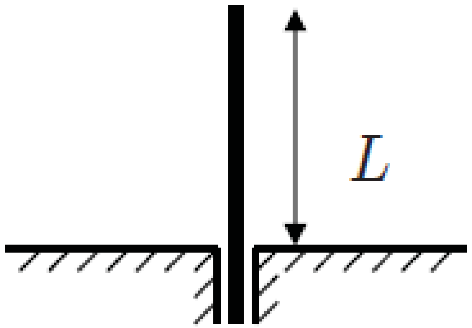
$$R_{in,\ dip\max} = 2R_{in,\ mono\max} = 200.53\,\Omega$$

$$R_{in,\ dip} = 11.14\,K^{4.17}\,\Omega \qquad (\lambda/2 < L < 0.6366\lambda)$$

$$R_{in,\ mono} = 5.57\,K^{4.17}\,\Omega \qquad (\lambda/4 < L < 0.3183\lambda)$$

Example:

Find the lengths of a dipole and a monopole with $R_{in} = 50\ \Omega$



$50\,\Omega$

Dipole

$$R_{in} = 50\,\Omega = 24.7\,K^{2.5}\Omega$$

$$K = 1.326 = \frac{kL}{2} = \pi\frac{L}{\lambda}$$

$$L = 0.422\ \lambda$$

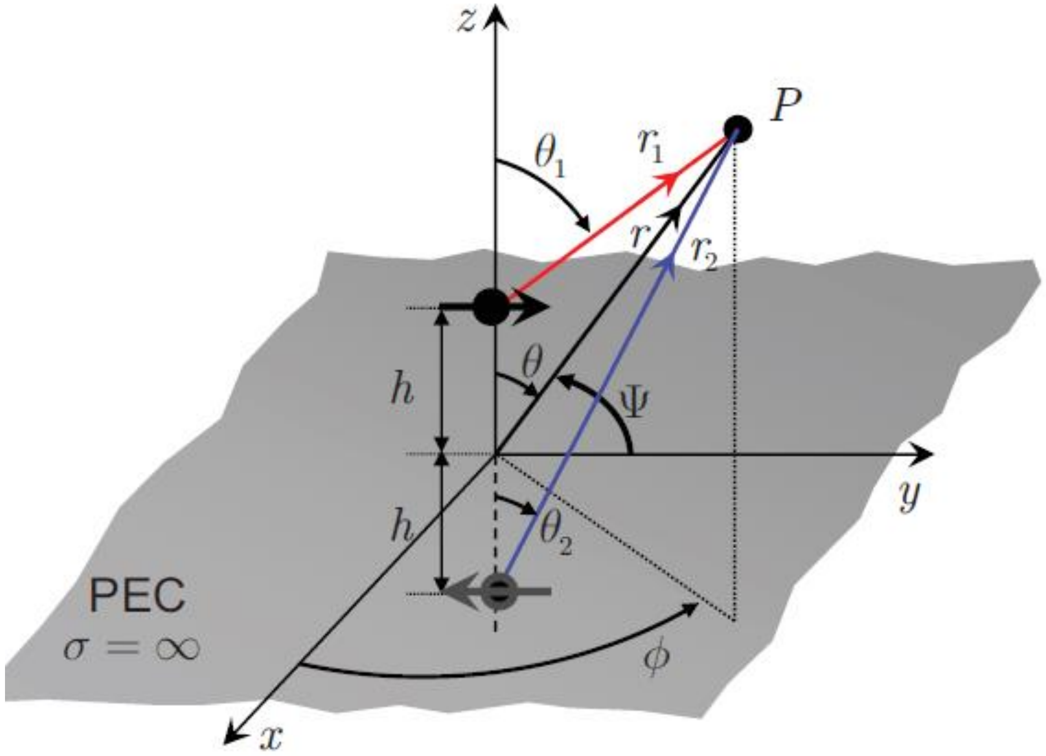
Monopole

$$R_{in} = 50\,\Omega = 5.57\,K^{4.17}\Omega$$

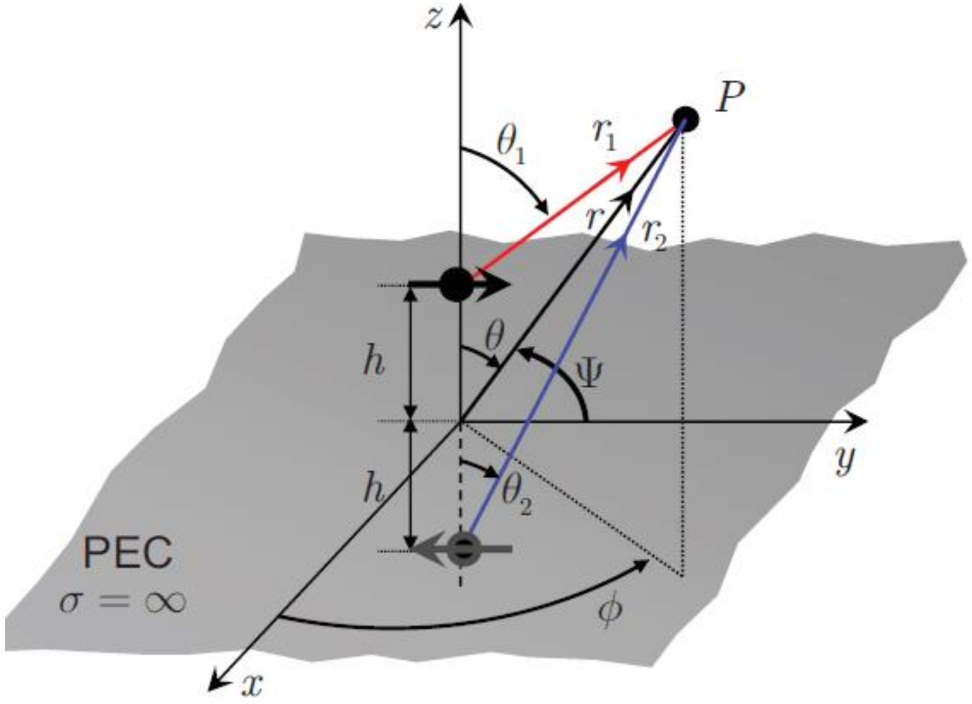
$$K = 1.693 = kL = 2\pi\frac{L}{\lambda}$$

$$L = 0.269\ \lambda$$

Horizontal Dipole Above Perfect Conducting Ground



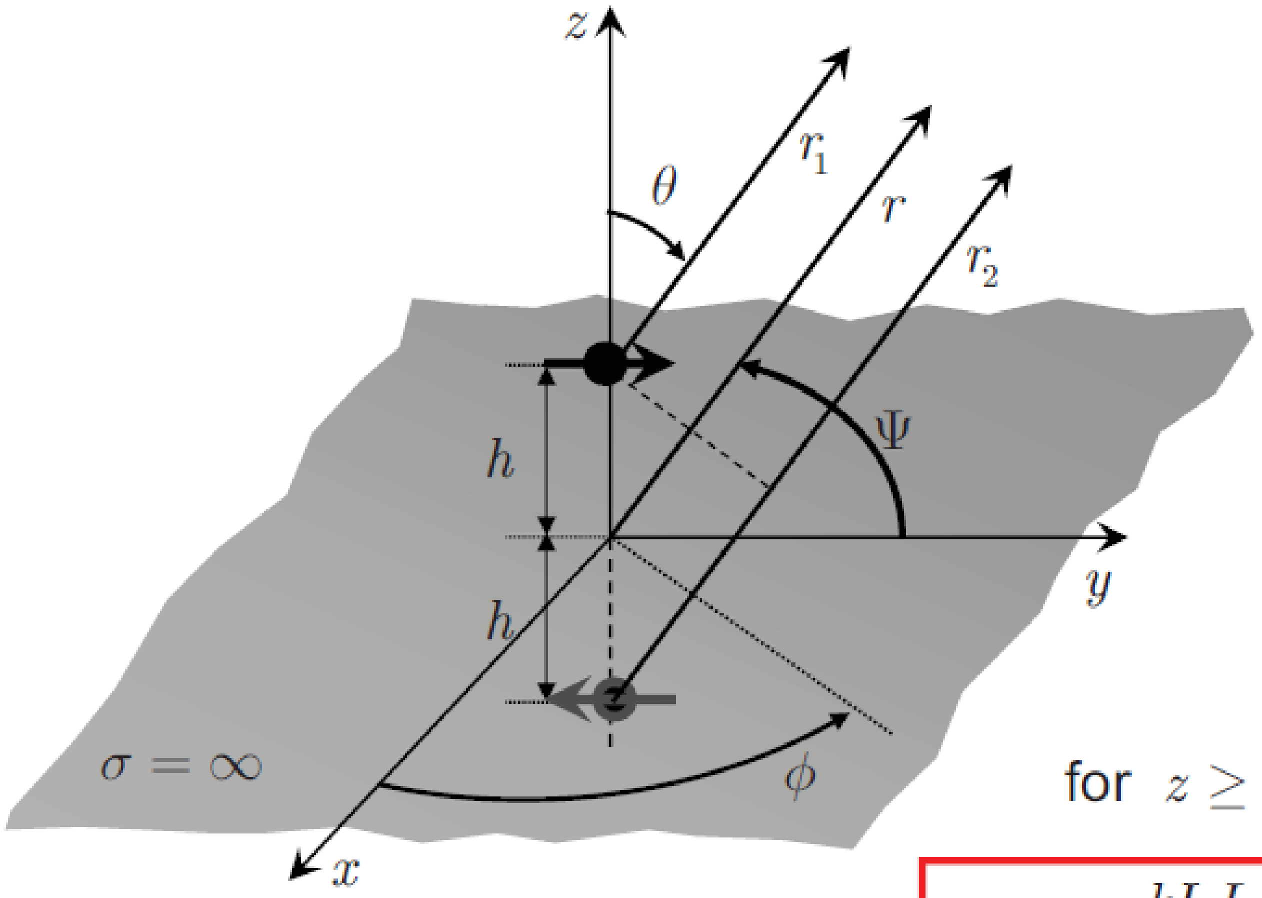
$$\sin\Psi = \sqrt{1-\cos^2\Psi} = \sqrt{1-\sin^2\theta\sin^2\phi}$$



The direct and reflected components can be written as

$$E_\Psi^d = j\eta\frac{kI_0Le^{-jkr_1}}{4\pi r_1}\sin\Psi$$

$$E_\Psi^r = -j\eta\frac{kI_0Le^{-jkr2}}{4\pi r_2}\sin\Psi$$



Far-field approximation $\begin{cases} \theta_1 \approx \theta \\ \theta_2 \approx \theta \end{cases}$

$$\left.\begin{aligned} r_1 &\approx r - h\cos\theta \\ r_2 &\approx r + h\cos\theta \end{aligned}\right\} \quad \text{for phase}$$

$$r_1 \approx r_2 \approx r \quad \text{for amplitude}$$

$$E_\Psi = E_\Psi^d + E_\Psi^r$$

for $z \geq 0 \quad (0 \leq \theta \leq \pi/2$ and $0 \leq \phi \leq 2\pi)$

$$\boxed{E_\Psi = \underbrace{j\eta \frac{kI_0 L e^{-jkr}}{4\pi r} \sqrt{1 - \sin^2\theta \sin^2\phi}}_{\text{element factor}} \underbrace{\left[2j\sin(kh\cos\theta)\right]}_{\text{array factor}}}$$

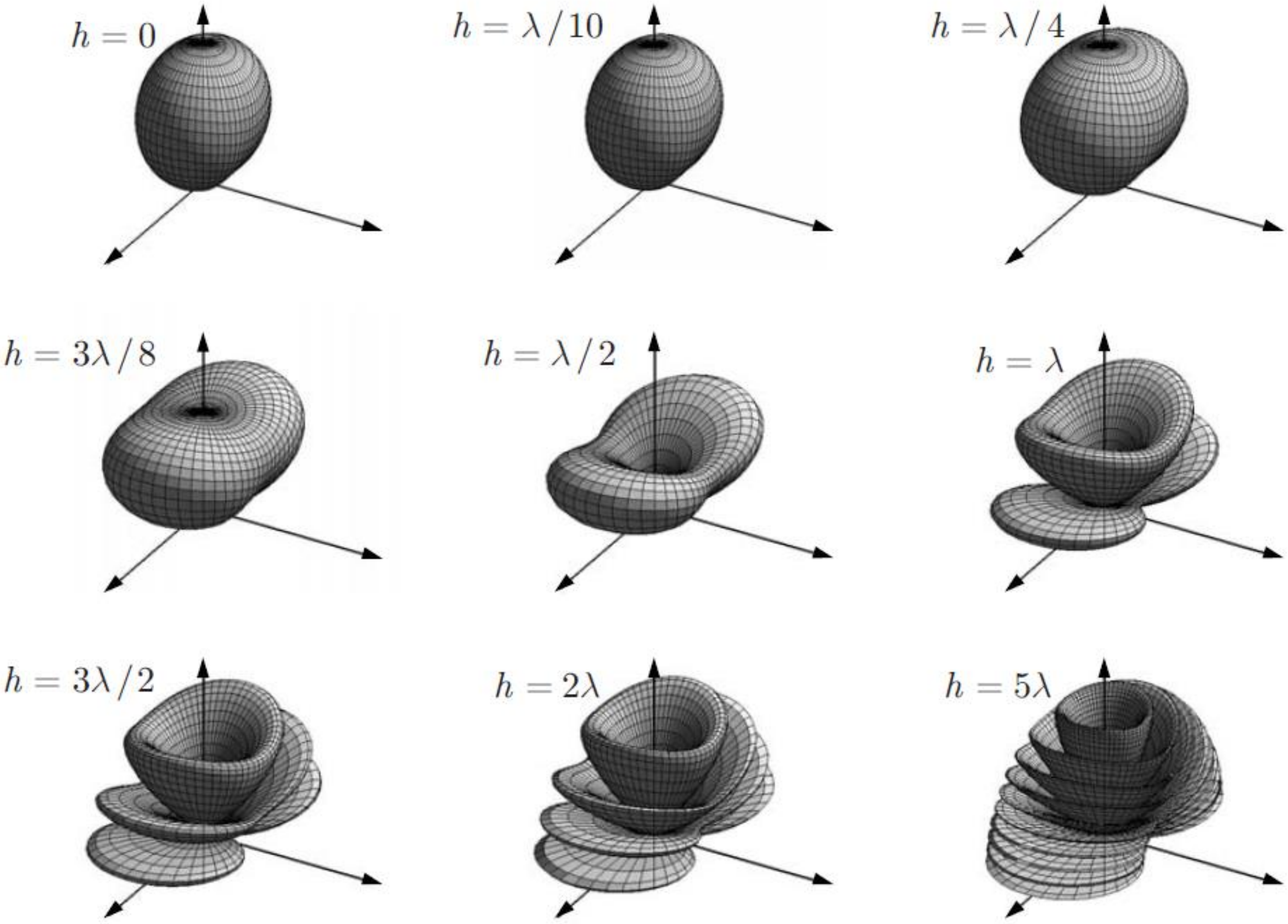
Directivity $D_0 = \begin{cases} 4\dfrac{\sin^2(kh)}{R(kh)} & kh \le \pi/2 \quad (h \le \lambda/4) \\ \dfrac{4}{R(kh)} & kh > \pi/2 \quad (h > \lambda/4) \end{cases}$

Radiation resistance

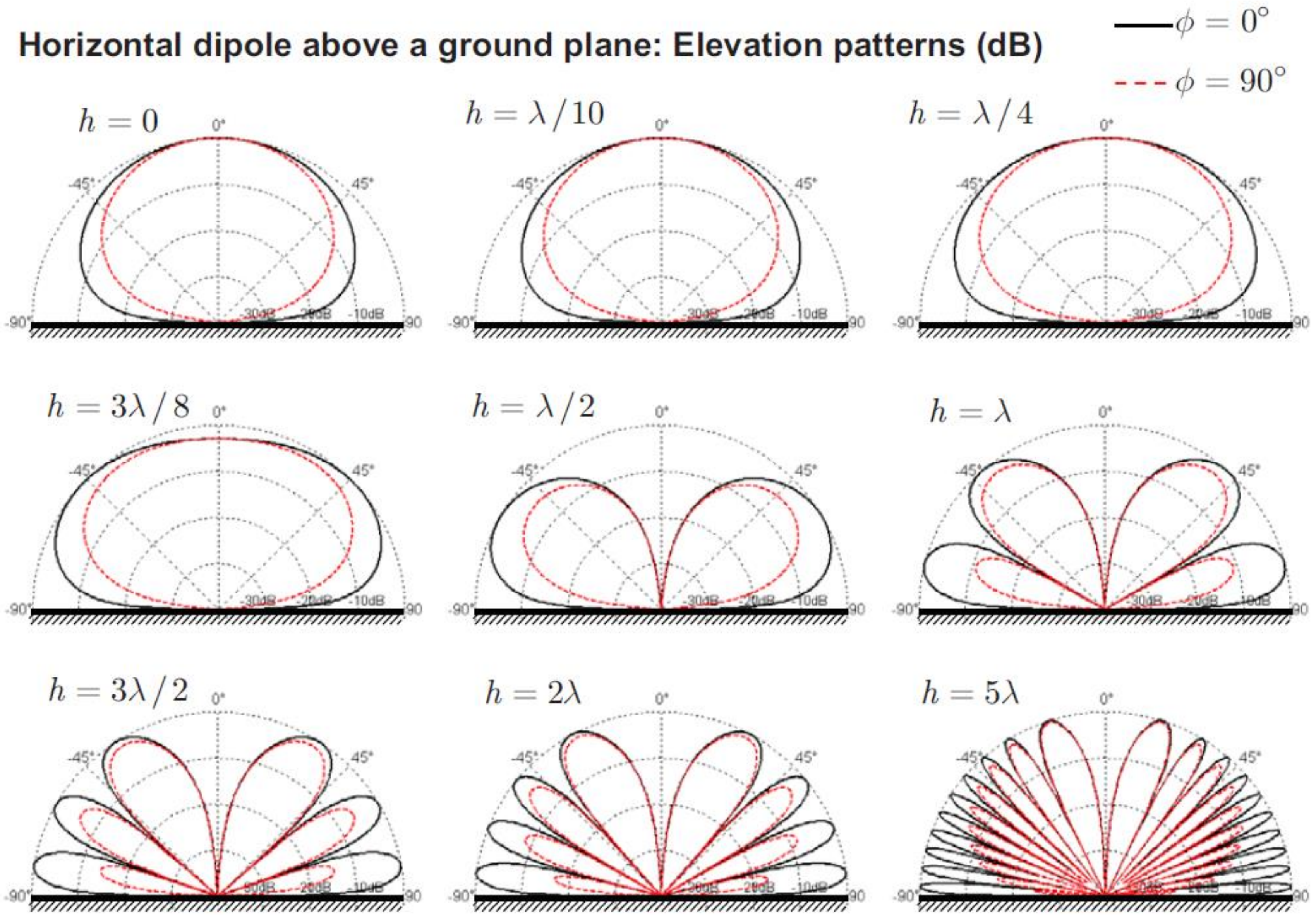
$$R_{rad} = \eta\pi\left(\frac{L}{\lambda}\right)^2 R(kh)$$

$$R(kh) = \left[\frac{2}{3} - \frac{\sin(2kh)}{2kh} - \frac{\cos(2kh)}{(2kh)^2} + \frac{\sin(2kh)}{(2kh)^3}\right]$$

Horizontal dipole above a ground plane: 3D amplitude patterns (linear represent.)



Horizontal dipole above a ground plane: Elevation patterns (dB)



Number of lobes: integer closest to $2h/\lambda$

Questions?? Thoughts??

EE 328
Wave Propagation and Antennas

with

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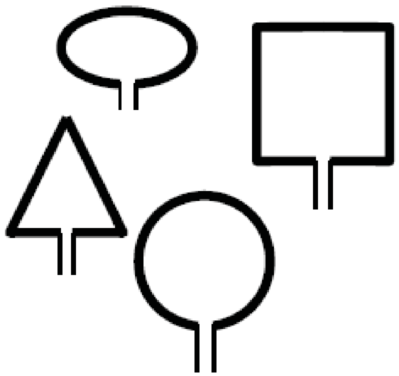
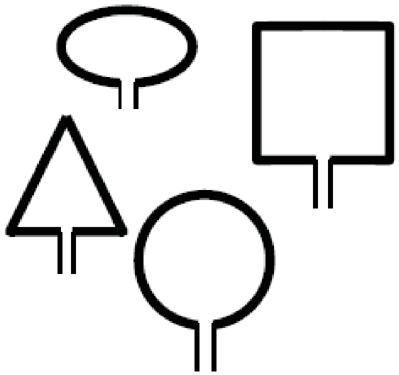
Loop Antennas



Various Shapes

Loop antennas can take many different forms such as a rectangle, square, triangle, ellipse, circle,… We will consider here only the **circular loop**, the most popular one, because of its simplicity of analysis and construction.

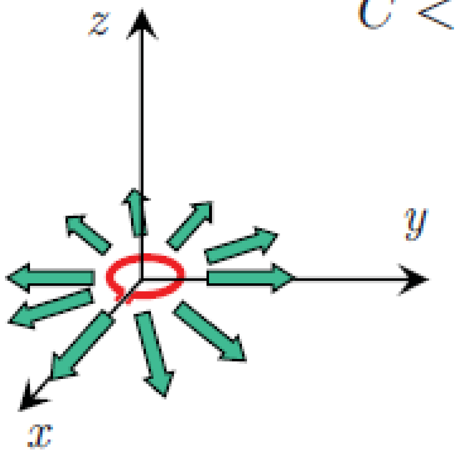


Two Categories

We distinguish two categories of loop antennas according to their circumference C :

Electrically small

$$C < \lambda / 10$$

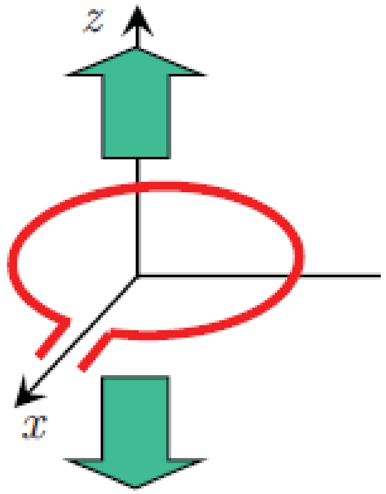


Low efficiency $R_{rad} < R_L$
Poor radiator
→ Receiving mode
→ pager, field probes

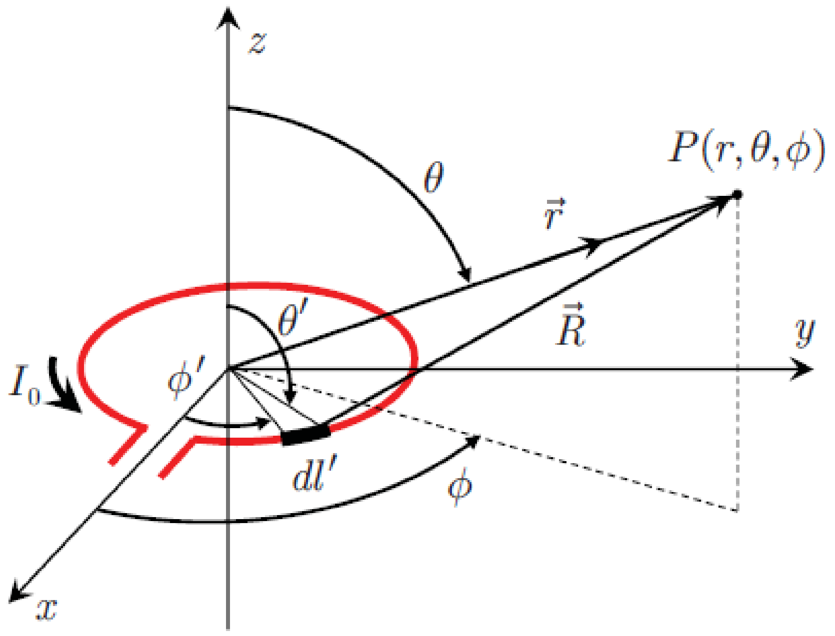
Electrically large

$$C \sim \lambda$$

Directional
Used in array form:
→ Helical antenna
→ Yagi-Uda



a) Analysis of circular loop with constant current



We consider a loop with radius a, positioned in the xy-plane.

The wire is assumed to be very thin and the current distribution on the wire is considered constant

$$I_\phi = I_0$$

This assumption is accurate only for small loops, but it simplifies the mathematical treatment.

- Vector potential: $\vec{A}(x,y,z) = \frac{\mu}{4\pi}\int_C \vec{I}_e(x',y',z')\frac{e^{-jkR}}{R}dl'$

$$\vec{I}_e = I_\phi \hat{a}_\phi$$

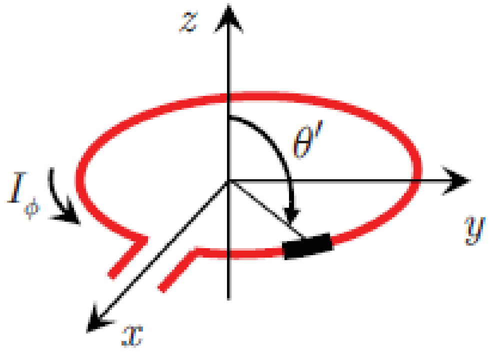
$$dl' = a\,d\phi'$$

$$\boxed{\vec{A} = A_\phi \hat{a}_\phi}$$

- We first need to write the current (in the integral) in Cartesian coordinates

$$\begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix} = \begin{bmatrix} \sin\theta'\cos\phi' & \cos\theta'\cos\phi' & -\sin\phi' \\ \sin\theta'\sin\phi' & \cos\theta'\sin\phi' & \cos\phi' \\ \cos\theta' & -\sin\theta' & 0 \end{bmatrix} \begin{bmatrix} I_r \\ I_\theta \\ I_\phi \end{bmatrix}$$

$$\xrightarrow[\theta' = \pi/2,\ I_\phi = I_0,\ I_\theta = 0,\ I_r = 0]{} \begin{cases} I_x = -I_0 \sin\phi' \\ I_y = I_0 \cos\phi' \\ I_z = 0 \end{cases}$$



$$\Rightarrow \quad \vec{I}_e = -I_0 \sin\phi'\ \hat{a}_x + I_0 \cos\phi'\ \hat{a}_y$$

- We can now write the vector potential integrals

$$\begin{cases} A_x = \dfrac{\mu}{4\pi}\displaystyle\int_0^{2\pi} -I_0 \sin\phi' \frac{e^{-jkR}}{R}\, a\,d\phi' \\[2ex] A_y = \dfrac{\mu}{4\pi}\displaystyle\int_0^{2\pi} I_0 \cos\phi' \frac{e^{-jkR}}{R}\, a\,d\phi' \\[2ex] A_z = 0 \end{cases}$$

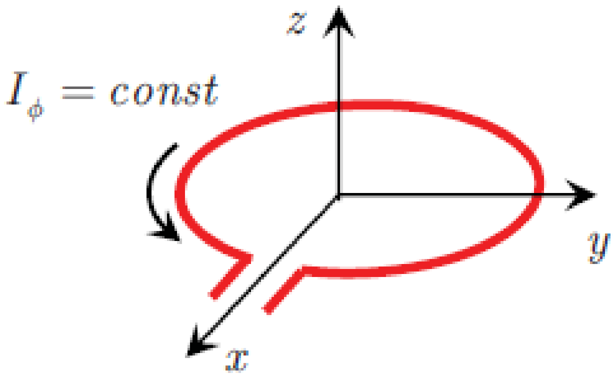
$$\vec{A} = A_\phi \hat{a}_\phi$$

Spherical coordinates

$$A_\phi = A_y \cos\phi - A_x \sin\phi$$

$$\begin{bmatrix} \cancelto{0}{A_r} \\ \cancelto{0}{A_\theta} \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ \cancelto{0}{A_z} \end{bmatrix}$$

- We consider now the cylindrical symmetry of the problem. For a constant current distribution, there will be no dependence on the azimuthal component ϕ.



Therefore, we can choose $\phi = 0$ for simplicity

$$\boxed{\rightarrow A_\phi = A_y = \frac{\mu}{4\pi}\int_0^{2\pi} I_0 \cos\phi' \frac{e^{-jkR}}{R} a d\phi'}$$

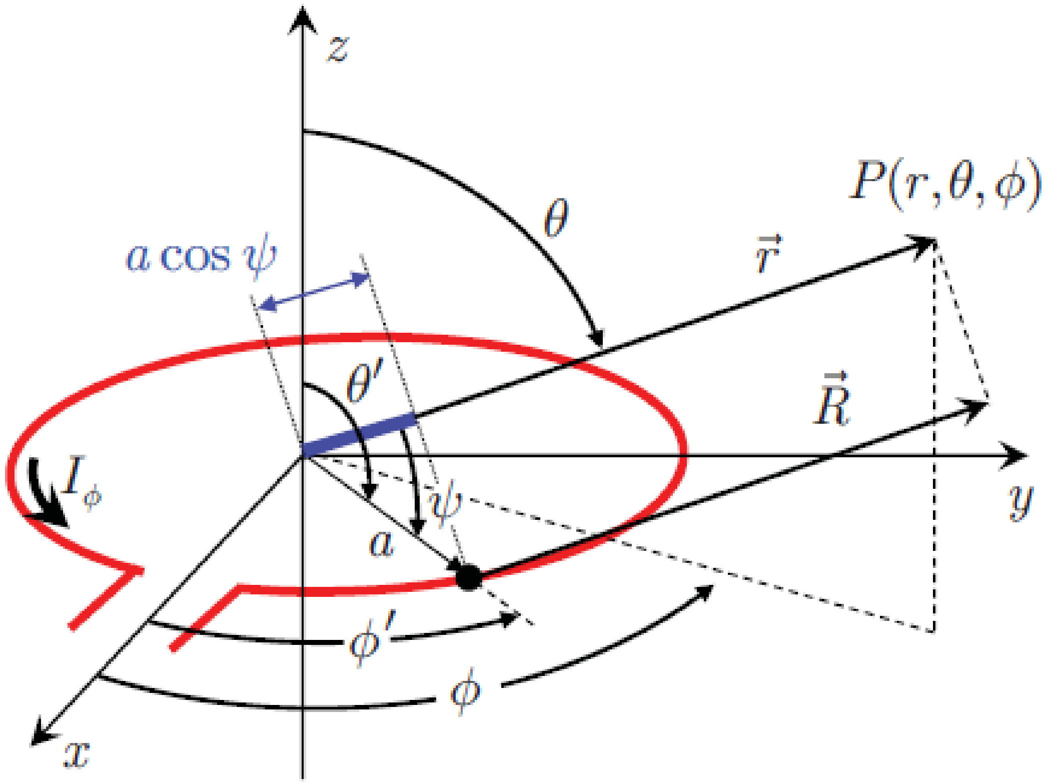
- Far-field approximation:

$$\begin{cases} R \approx r - a\cos\Psi & \text{for phase} \\ R \approx r & \text{for amplitude} \end{cases}$$

$$\cos\Psi = \vec{r}\cdot\vec{r}' = \sin\theta\cos\phi'$$

$\vec{r}$: $\hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$

$\vec{r}'$: $\hat{x}\cos\phi' + \hat{y}\sin\phi'$



$$\boxed{A_\phi = \frac{a\mu I_0}{4\pi}\frac{e^{-jkr}}{r}\int_0^{2\pi}\cos\phi'\, e^{+jka\sin\theta\cos\phi'}\,d\phi'}$$

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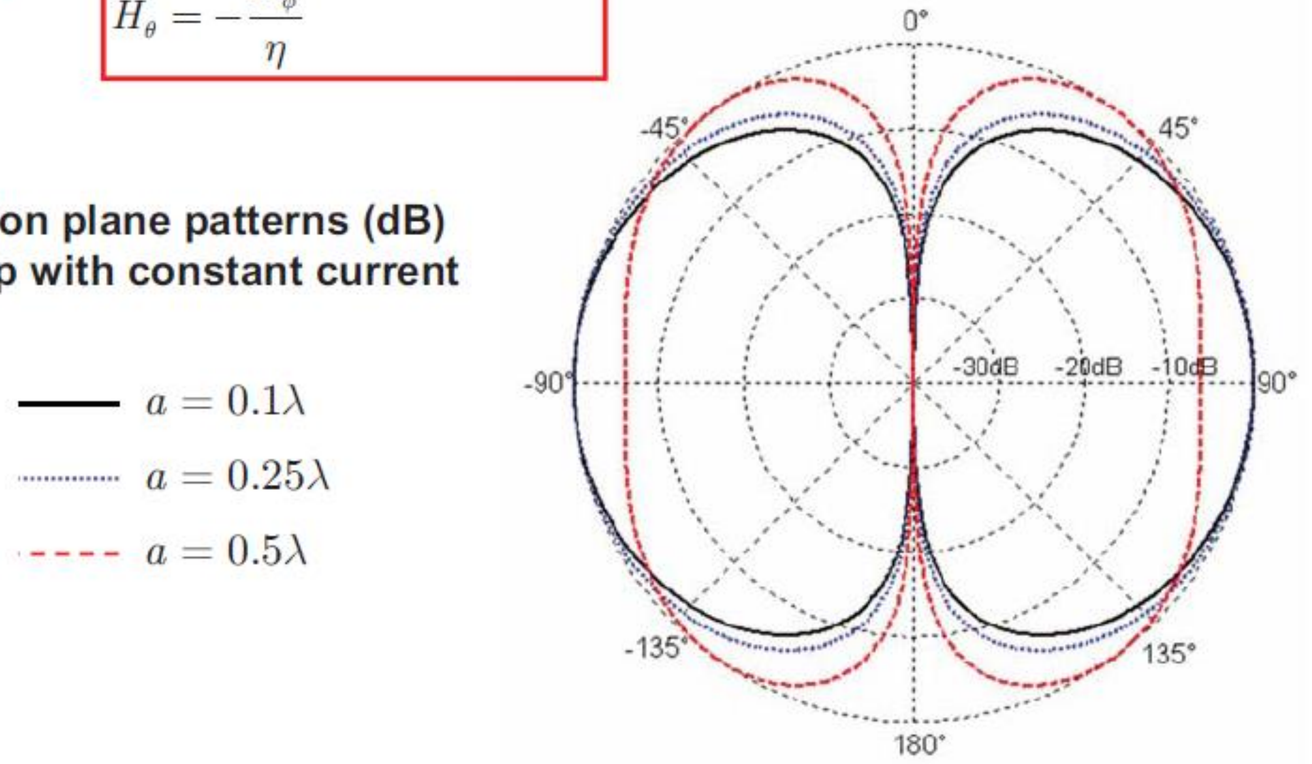
- The solution of the integral $A_\phi = \frac{a\mu I_0}{4\pi}\frac{e^{-jkr}}{r}\int_0^{2\pi}\cos\phi' \, e^{+jka\sin\theta\cos\phi'}d\phi'$

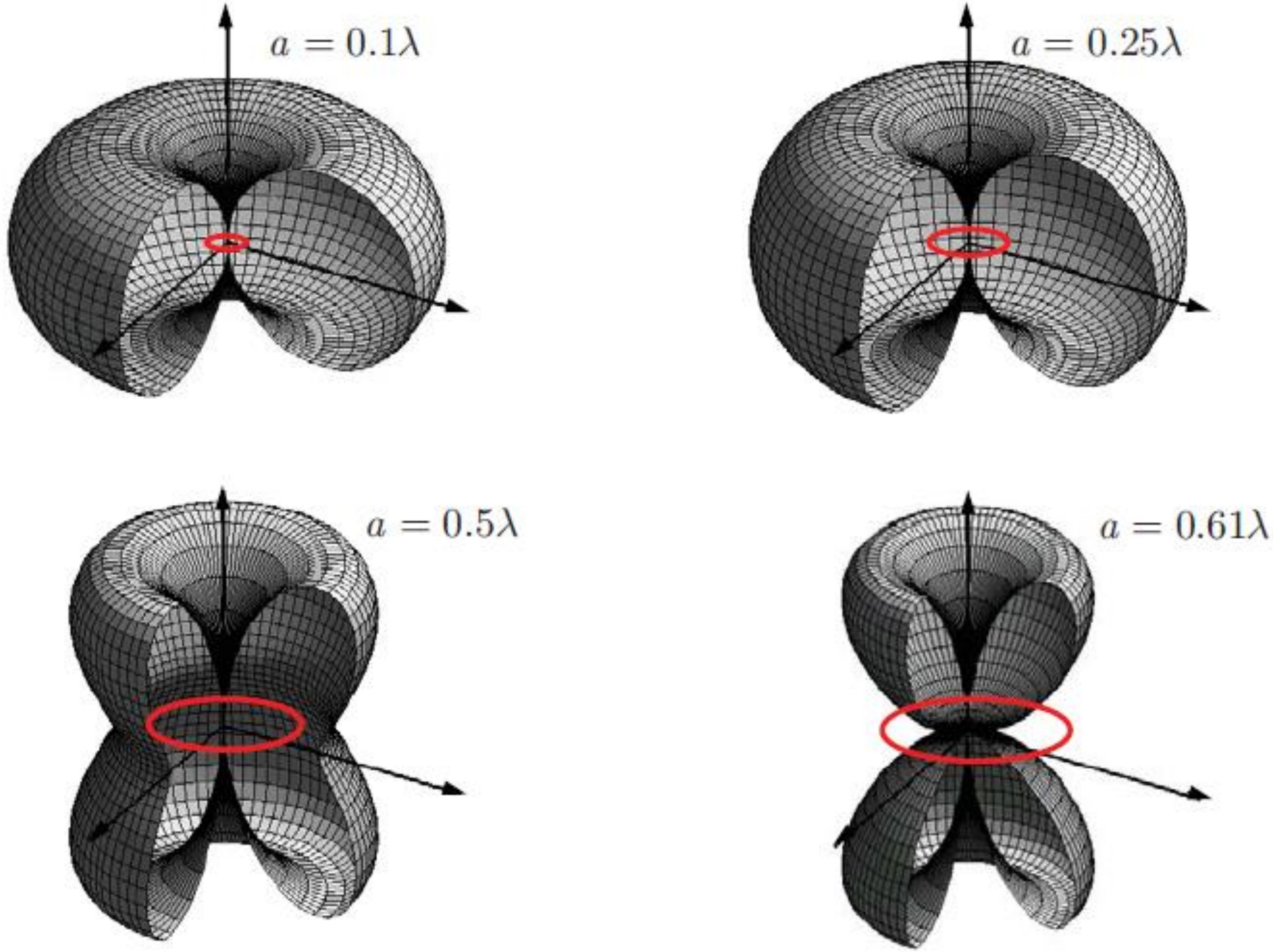
  involves Bessel functions J_n of the first kind $\int_0^{\pi}\cos(n\phi)\,e^{-jz\cos\phi}d\phi = \pi j^n J_n(z)$

$$\boxed{\begin{aligned} E_\phi &= \eta\frac{kaI_0}{2r}e^{-jkr}J_1(ka\sin\theta) \\ H_\theta &= -\frac{E_\phi}{\eta} \end{aligned}}$$

Elevation plane patterns (dB) for loop with constant current



3D amplitude patterns (linear)



Note: The constant current approximation used here is **not** valid for practical loops!

Radiation resistance and directivity for a circular loop of constant current

Directivity

$$D_0 \approx 0.682 \frac{C}{\lambda} \quad (a \geq \lambda / 2)$$

Radiation resistance

$$R_{rad} \approx 60\pi^2 \frac{C}{\lambda}$$

Maximum effective aperture $$A_{em} = \frac{\lambda^2}{4\pi} D_0 \approx 0.0543\ \lambda C \quad (a \geq \lambda / 2)$$

b) Electrically small loop antennas

We consider a very small loop antenna with $ka < \frac{1}{3} \quad \rightarrow \quad a < \frac{1}{6\pi}\lambda \approx 0.05\lambda$

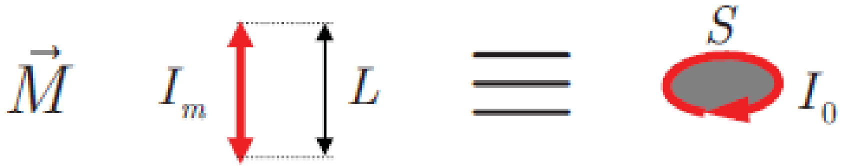
The Bessel function can then be approximated as $J_1(ka\sin\theta) \approx \frac{ka}{2}\sin\theta$

$$\boxed{\begin{aligned} E_\phi &= \eta\frac{k^2a^2I_0}{4r}e^{-jkr}\sin\theta = \eta\frac{\pi S I_0}{\lambda^2 r}e^{-jkr}\sin\theta \\ H_\theta &= -\frac{E_\phi}{\eta} \end{aligned}}$$

$S = \pi a^2$ loop area

$k = \frac{2\pi}{\lambda}$

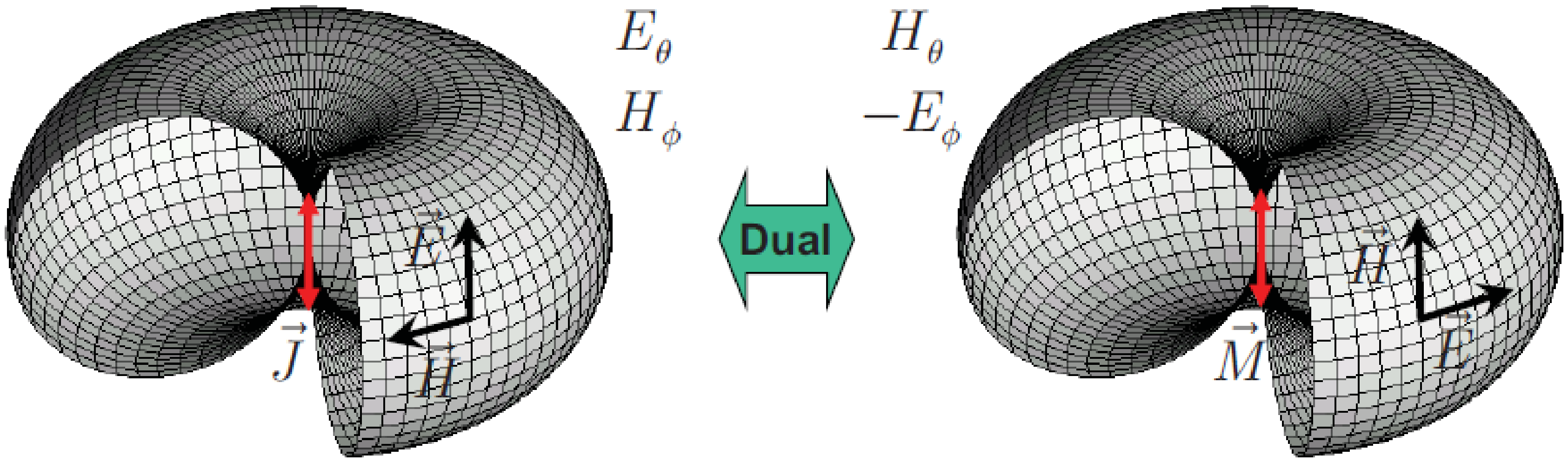
→ A magnetic dipole is equivalent to a small electric loop



$$\boxed{I_m L = jS\omega\mu I_0}$$

Duality

Because of the similarity of equations, electric and magnetic sources are called dual quantities. Duality permits in practice to quickly find mathematical solutions.



Radiation resistance	$R_{rad} = 20\pi^2 \left(\dfrac{C}{\lambda}\right)^4 \quad [\Omega]$	circumference $C = 2\pi a$
Directivity	$D_0 = 3/2$	(Same as infinitesimal dipole)
Maximum effective aperture	$A_{em} = \dfrac{\lambda^2}{4\pi} D_0 = \dfrac{3\lambda^2}{8\pi}$	

Arrays of small loops:

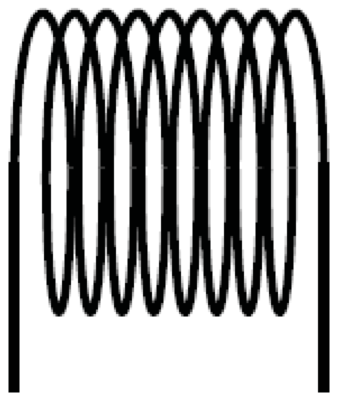
One single loop is not very effective since the radiation resistance is often smaller than the loss resistance of the loop.

That is why small loops are often constructed with multiple turns.

For N closely spaced loops the radiation resistance is

$$R_{rad} = 20\pi^2 \left(\frac{C}{\lambda}\right)^4 N^2 \quad [\Omega]$$

Example: $a = \dfrac{\lambda}{25}$



$$R_{rad}(\text{1 turn}) = 20\pi^2 \left(\frac{2\pi}{\lambda}\frac{\lambda}{25}\right)^4 = 0.7876\ \Omega$$

$$R_{rad}(\text{8 turn}) = R_{rad}(\text{1 turn}) \cdot 8^2 = 50.4\ \Omega$$

Note: The efficiency $e_{cd} = \dfrac{R_{rad}}{R_L + R_{rad}}$ increases with the number of turns since

$$\begin{cases} R_{rad} \propto N^2 \\ R_L \propto N \end{cases}$$

Questions?? Thoughts??

EE 328
Wave Propagation and Antennas

with

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Traveling Wave Antennas

Definition

- A traveling-wave antenna is a class of antenna that uses a traveling wave on a guiding structure as the main radiating mechanism.

- Their distinguishing feature is that the radio-frequency current that generates the radio waves travels through the antenna in one direction.

- This is in contrast to a resonant antenna, such as the monopole or dipole, in which the antenna acts as a resonator, with radio currents traveling in both directions, bouncing back and forth between the ends of the antenna

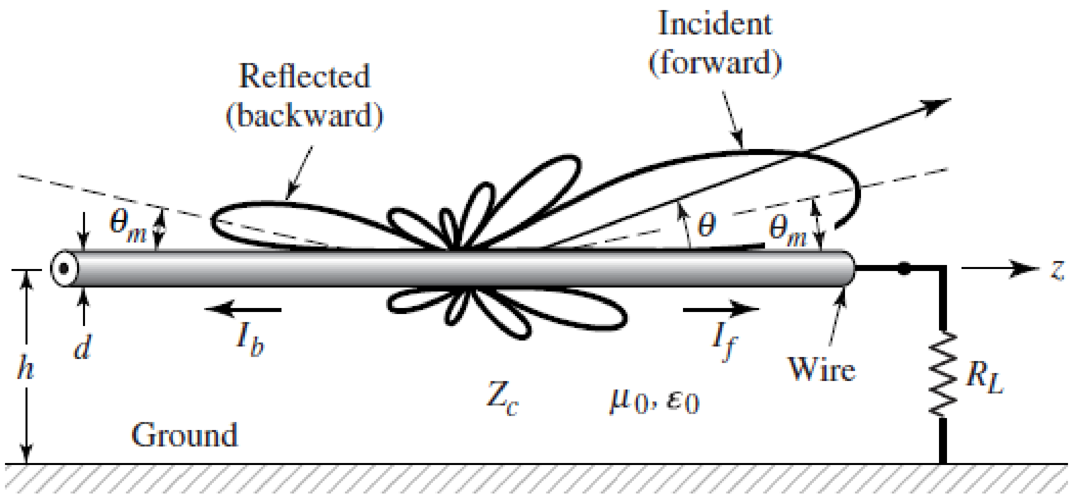
Advantage

- An advantage of traveling wave antennas is that since they are non-resonant they often have a wider bandwidth than resonant antennas.

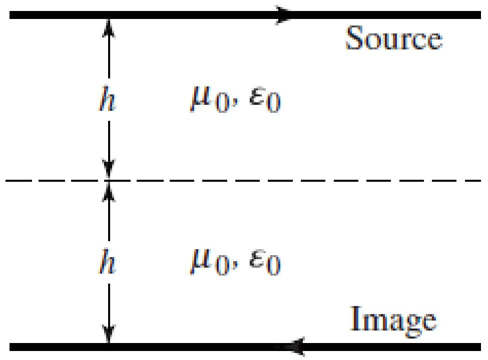
Beverage (Long Wire) Antenna

- A Beverage antenna consists of a horizontal wire from one-half to several wavelengths long suspended above the ground, with the feedline to the receiver attached to one end and the other terminated through a resistor to ground.

- The antenna has a unidirectional radiation pattern with the main lobe of the pattern at a shallow angle into the sky off the resistor-terminated end

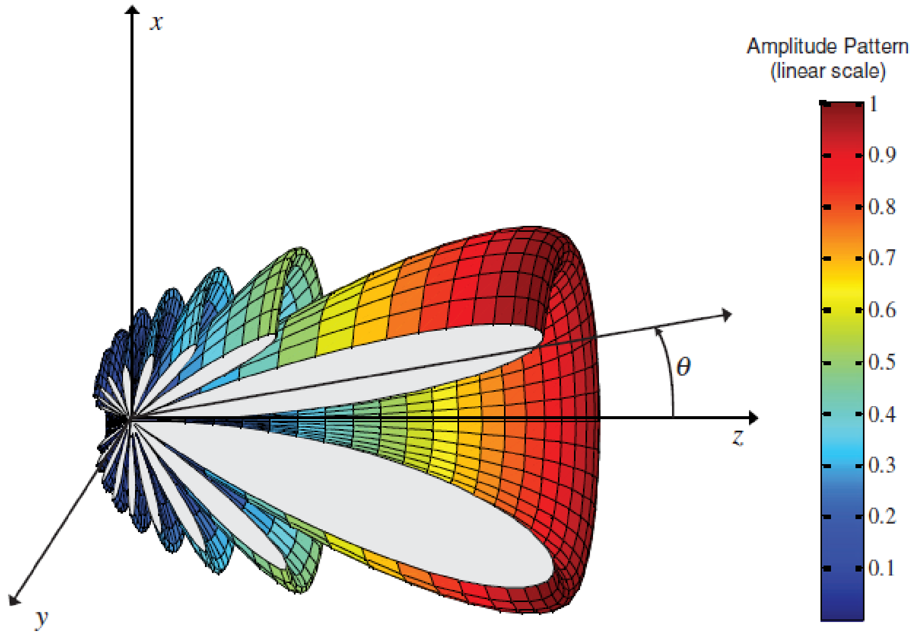


(a) Long wire above ground and radiation pattern

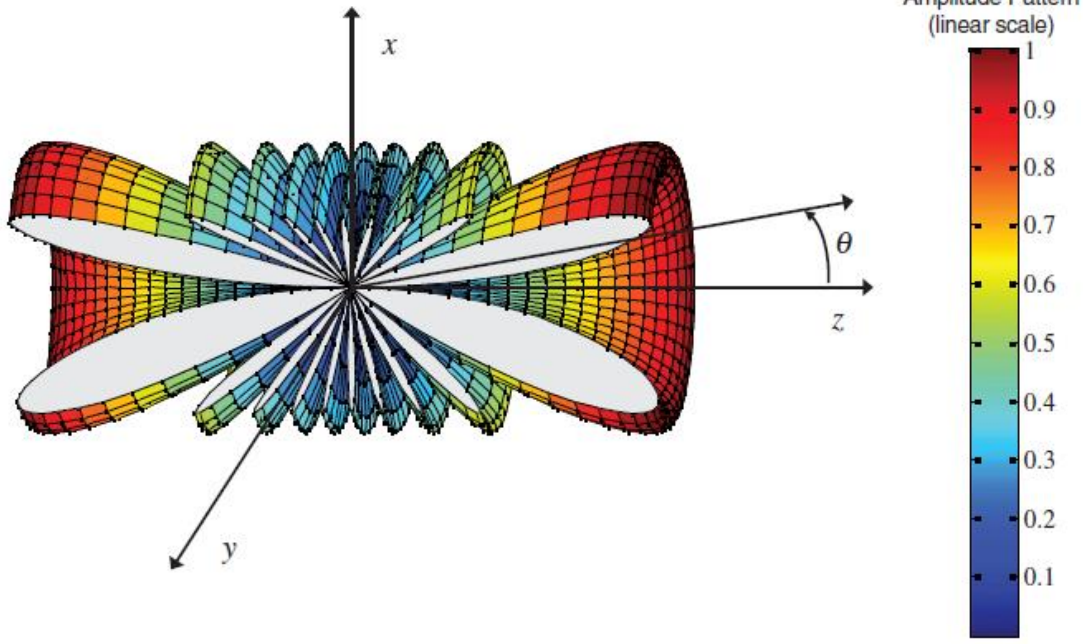


(b) Equivalent: Source and image

Radiation Pattern (standing vs. traveling wave)



(a) Traveling wave



(b) Standing wave

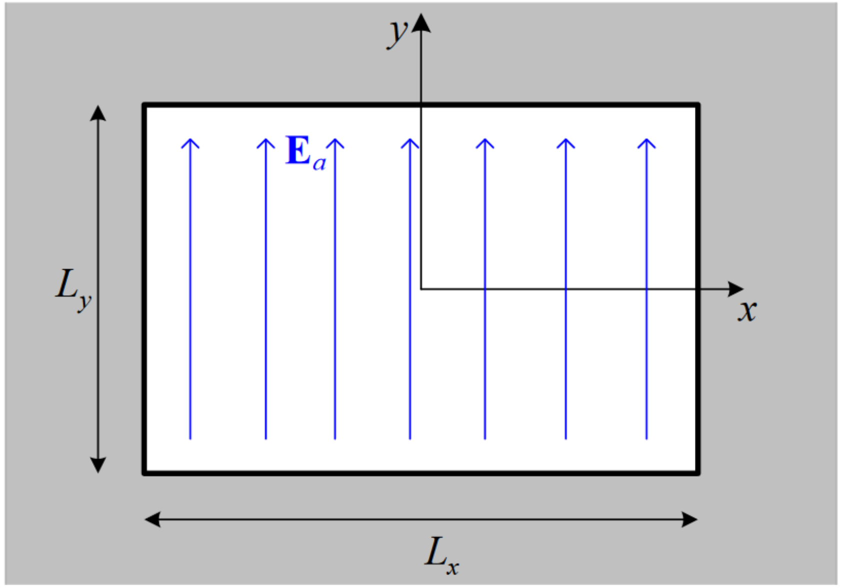
Aperture Antennas

Definition

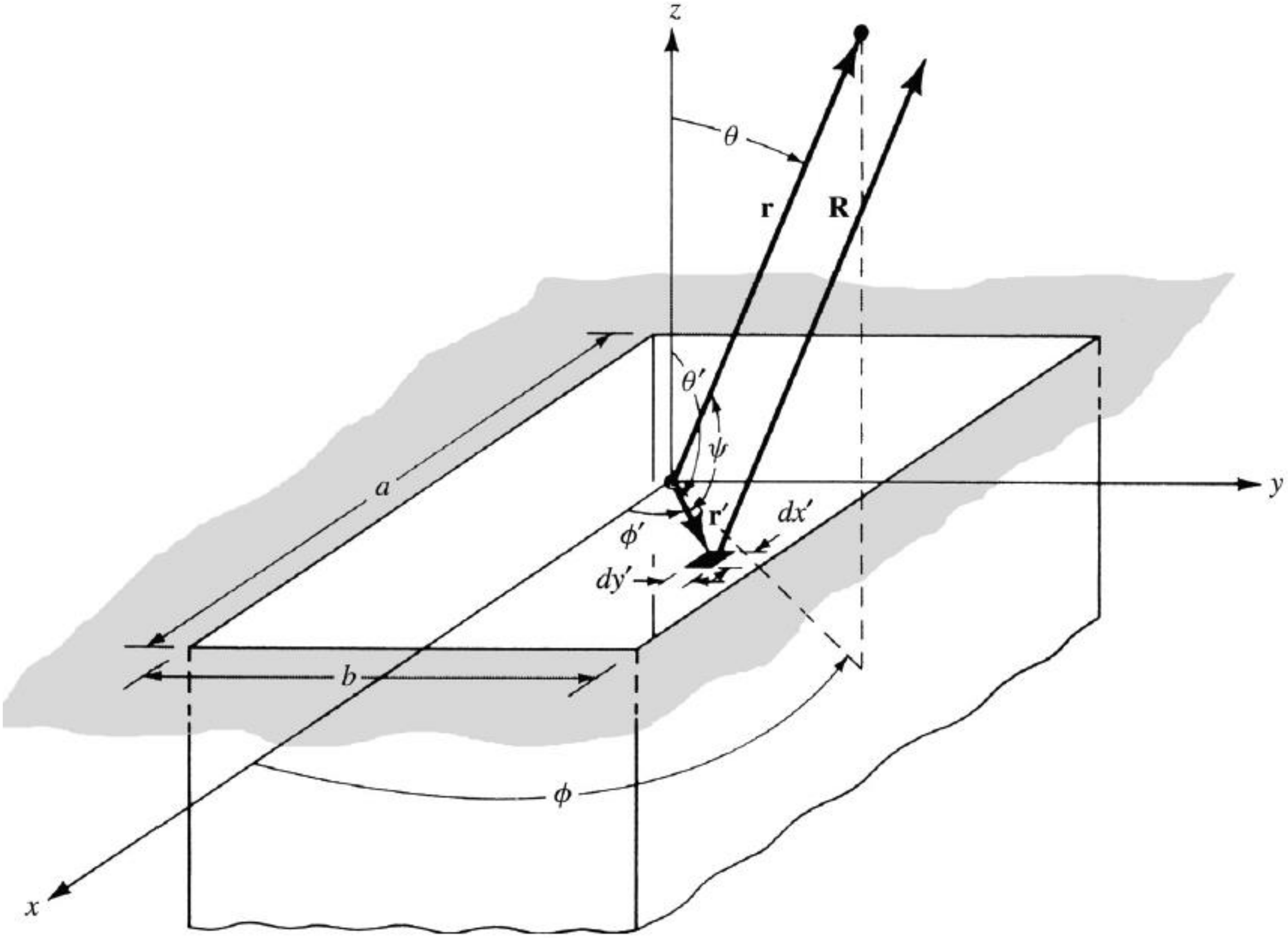
- Aperture antennas constitute a large class of antennas, which emit EM waves through an opening (or aperture).

Rectangular Aperture Antenna

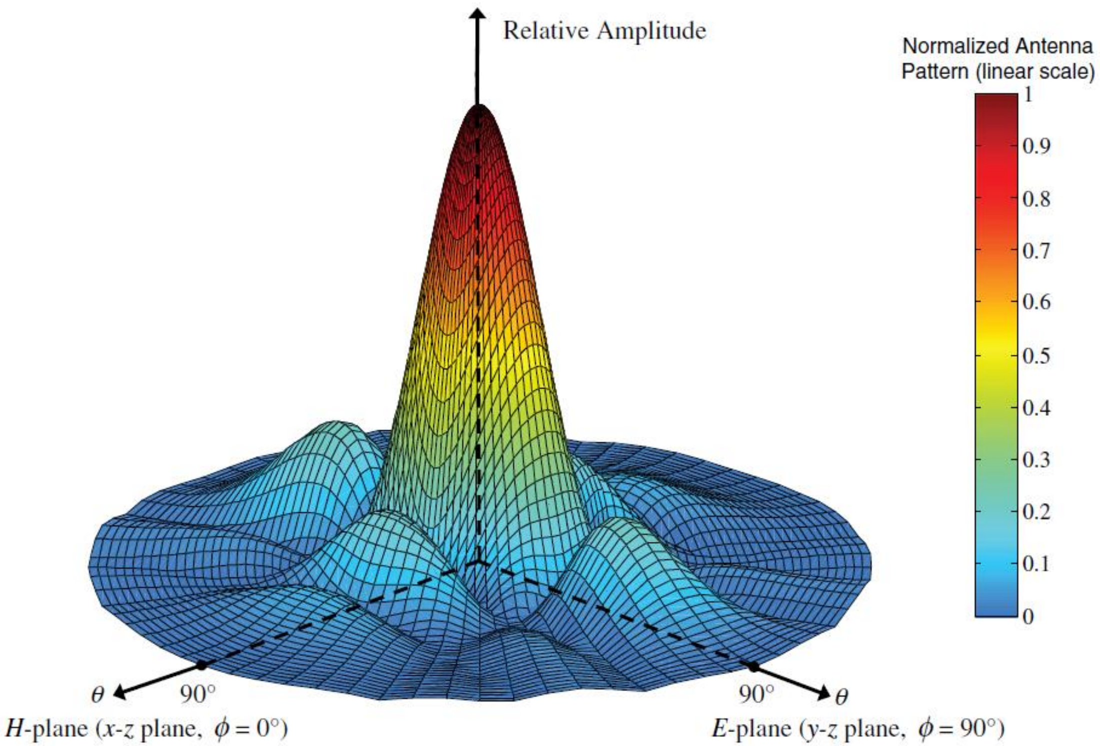
- A rectangular aperture is defined in the x-y plane as shown below.
- If the field is uniform in amplitude and phase across the aperture, it is referred to as a **uniform rectangular aperture**



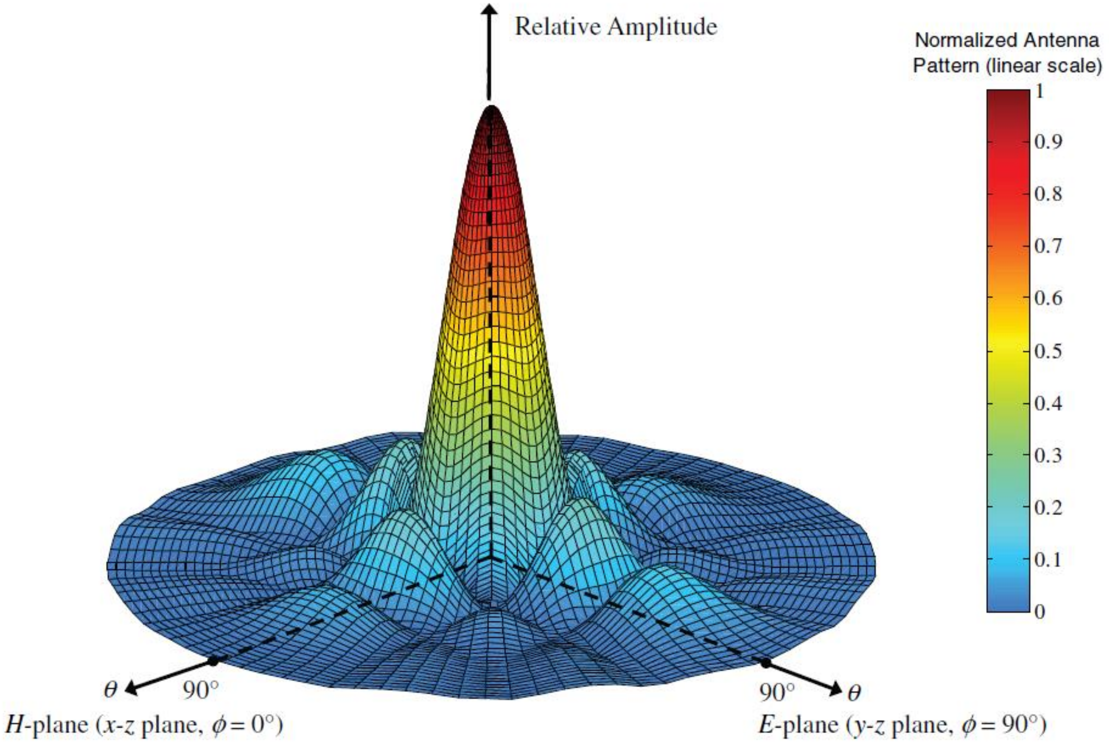
Rectangular Aperture on an Infinite Ground Plane



Radiation Pattern (Rectangular Aperture)



$a = 3\lambda, b = 2\lambda$

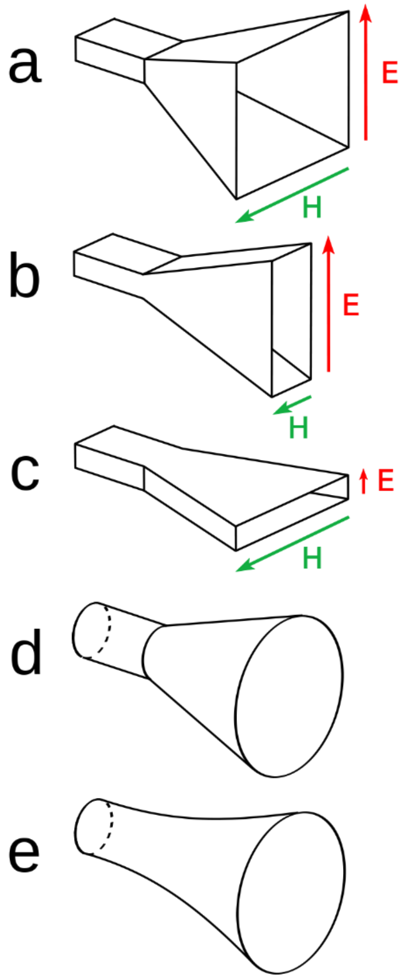


$a = b = 2\lambda$

Horn Antenna – Construction and Use

- A horn antenna or microwave horn is an antenna that consists of a hollow pipe of different cross sections, which has been tapered (flared) to a larger opening to direct radio waves in a beam.
- Horns are widely used as antennas at UHF and microwave frequencies, above 300 MHz
- The horn is widely used as a feed element for large radio astronomy, satellite tracking, and communication dishes found installed throughout the world. In addition to its utility as a feed for reflectors and lenses
- It is a common element of phased arrays and serves as a universal standard for calibration and gain measurements of other high-gain antennas.
- Its widespread applicability stems from its simplicity in construction, ease of excitation, versatility, large gain, and preferred overall performance.

Horn Antenna – Common Types



(a) Pyramidal horn – a horn antenna with the horn in the shape of a four-sided pyramid, with a rectangular cross section.

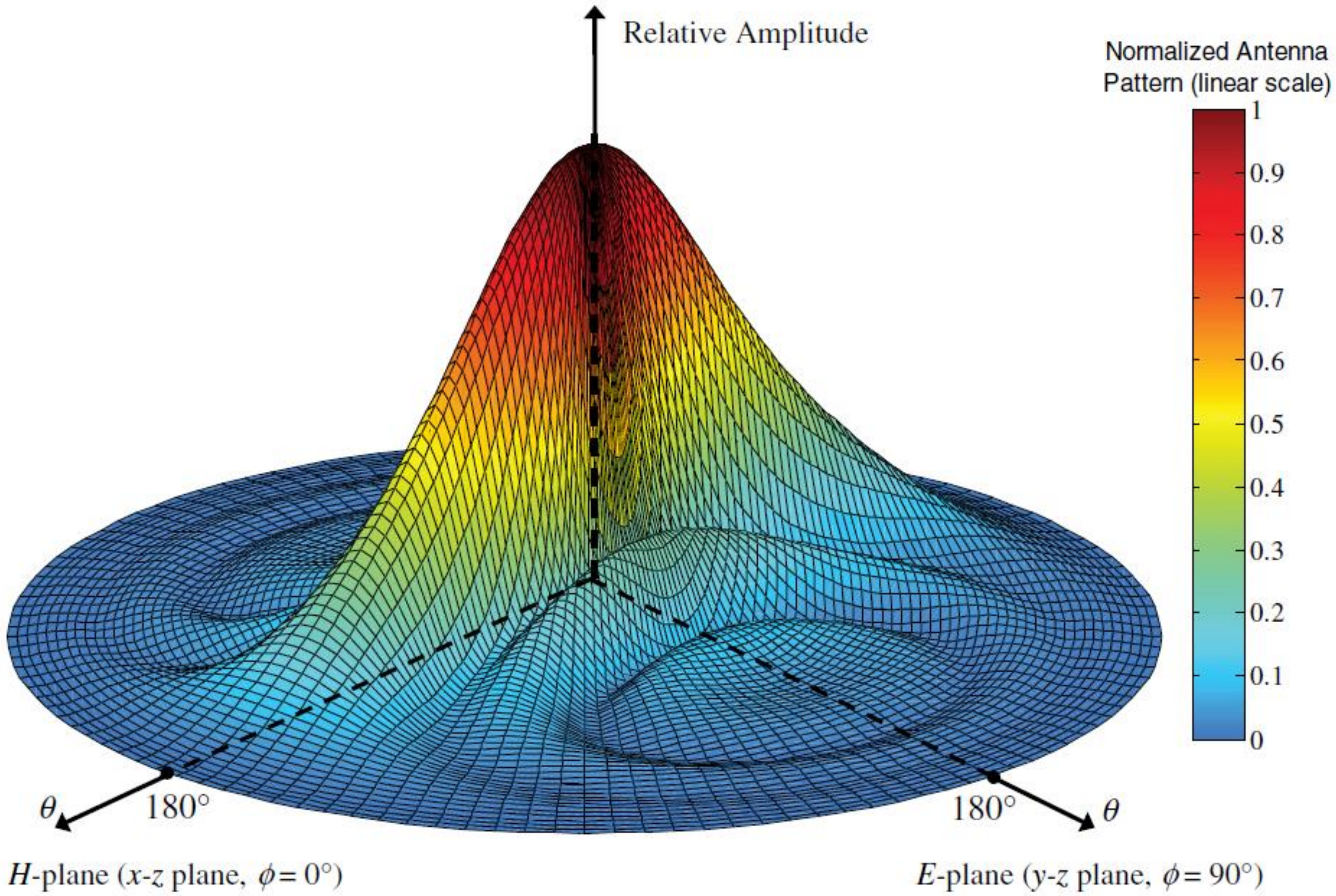
(b) E-plane Sectoral horn – A pyramidal horn with only one pair of sides (in the E-field direction) flared and the other pair parallel.

(c) H-plane Sectoral horn – A pyramidal horn with only one pair of sides (in the H-field direction) flared and the other pair parallel.

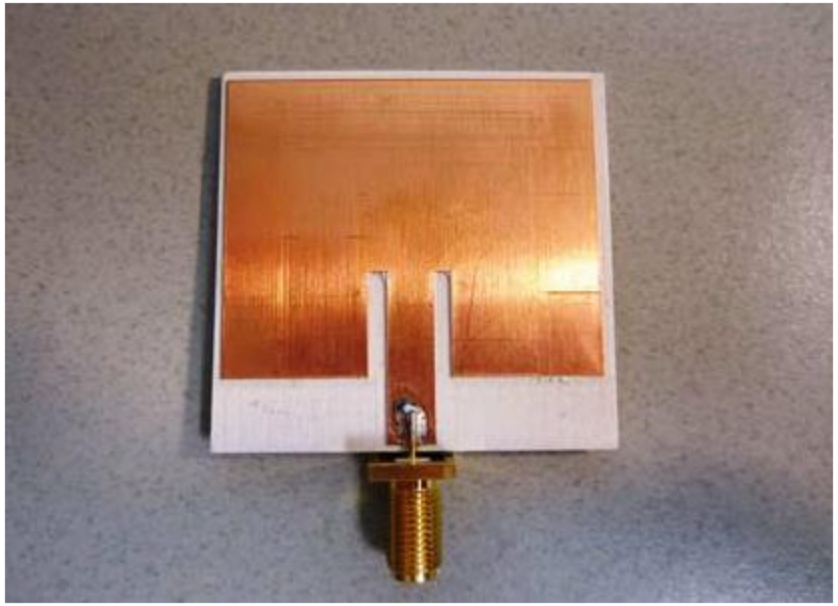
(d) Conical horn – A horn in the shape of a cone, with a circular cross section.

(e) Exponential horn – A horn with curved sides, in which the separation of the sides increases as an exponential function of length.

Radiation Pattern (E-Plane Sectoral Horn)



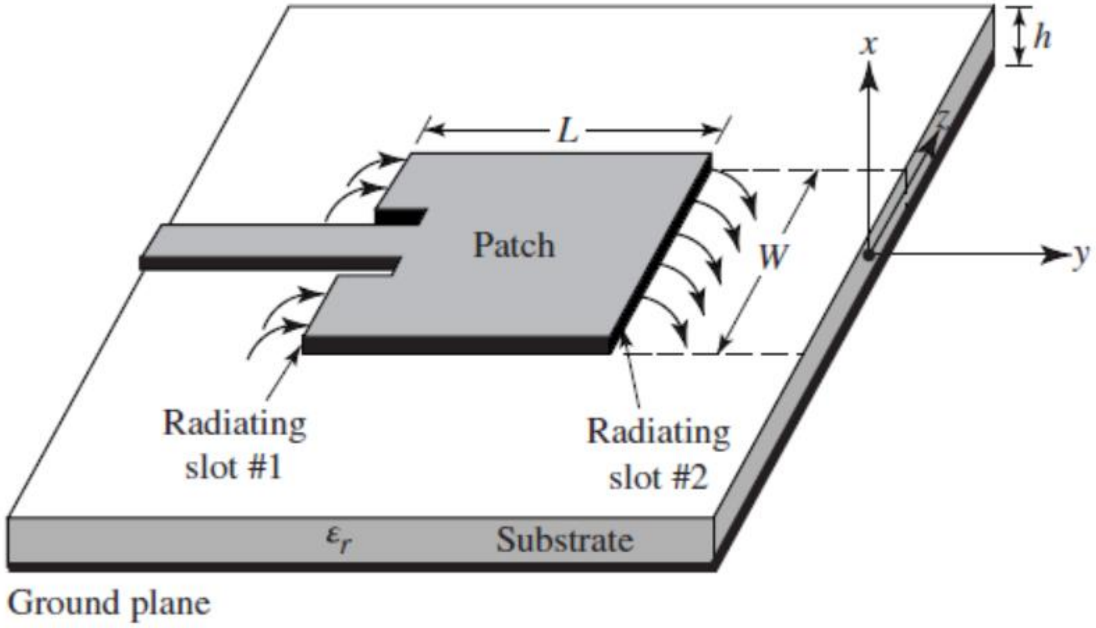
Microstrip (Patch) Antennas



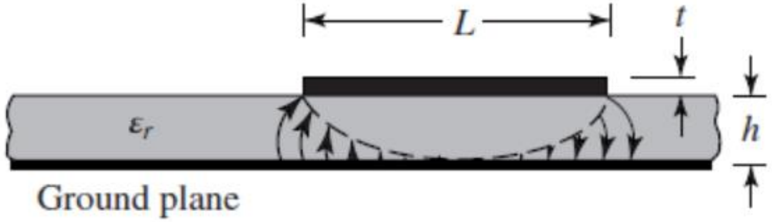
Construction

- A microstrip patch antenna consists of a “very thin” metallic strip (patch) placed a small fraction of a wavelength above a ground plane.
- The antenna is usually connected to the transmitter or receiver through foil microstrip transmission lines.
- The radio frequency current is applied between the antenna and ground plane.
- The microstrip patch is designed so its pattern maximum is normal to the patch (broadside radiator)

Construction



(a) Microstrip antenna

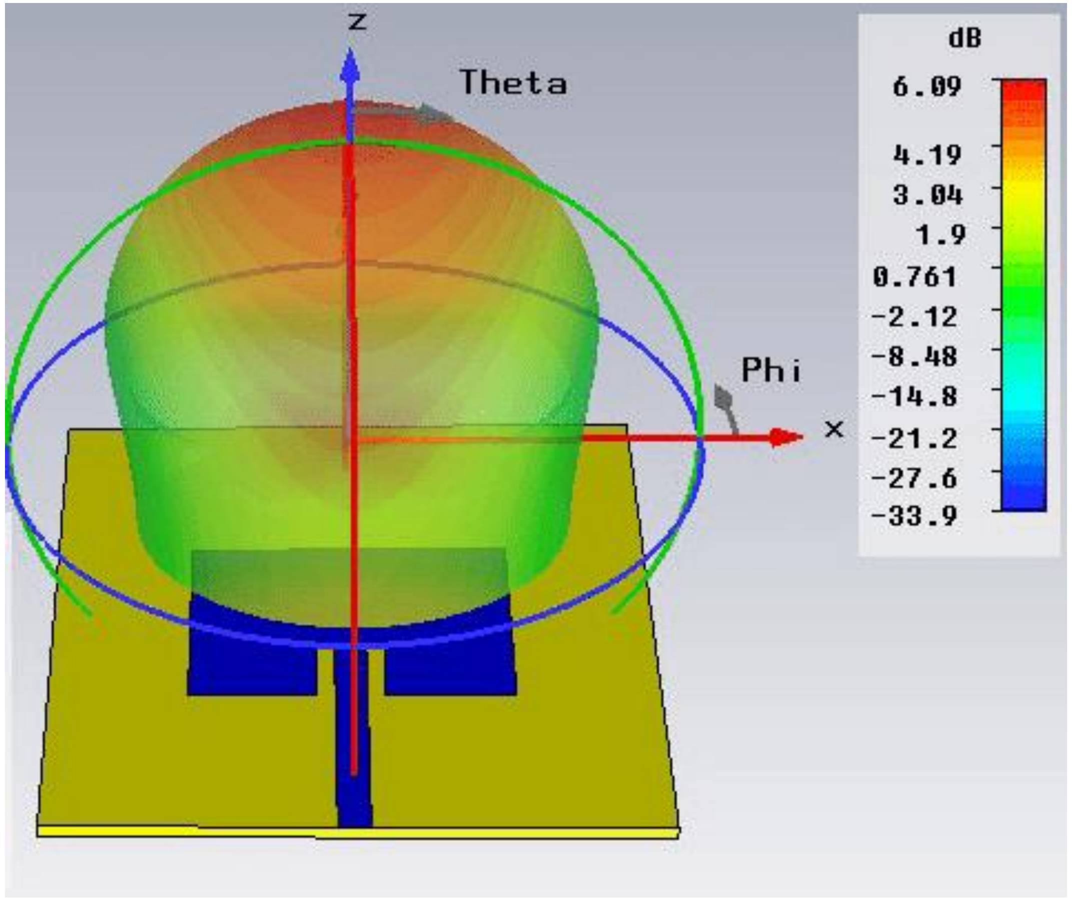


(b) Side view

Use and Advantages

- Microstrip antennas have become very popular in recent decades due to:
  - their thin planar profile which can be incorporated into the surfaces of consumer products, aircraft and missiles
  - their ease of fabrication using printed circuit techniques
  - the ease of integrating the antenna on the same board with the rest of the circuit,
  - and the possibility of adding active devices such as microwave integrated circuits to the antenna itself to make active antennas

Radiation Pattern (Patch Antenna)



Questions?? Thoughts??